

Scheduling of Multi-Antenna Broadcast Systems with Heterogeneous Users

Krishna Jagannathan, Sem Borst, Phil Whiting and Eytan Modiano

Abstract—We consider a two transmit antenna broadcast system with heterogeneous users, and tackle the problem of maximizing a *weighted* sum rate. We establish a novel upper bound for the weighted sum capacity, which we then use to show that the maximum expected weighted sum rate can be asymptotically achieved by transmitting to a suitably selected subset of at most $2C$ users, where C denotes the number of distinct user classes. Numerical experiments indicate that the asymptotic results are remarkably accurate and that the proposed schemes operate close to absolute performance bounds, even for a moderate number of users.

I. INTRODUCTION

In the present paper we consider the downlink transmission from a single base station equipped with M transmit antennas to K independent users each with a single receive antenna. In information-theoretic terms, this may be modeled as a multi-antenna Broadcast Channel (BC). Caire & Shamai [1] were the first to obtain the sum capacity expression for the Gaussian BC with two receivers, and to suggest the use of Dirty Paper Coding (DPC) [2] for transmitting over this channel. Viswanath & Tse [24] and Vishwanath *et al.* [23] extended the result for the sum capacity to an arbitrary number of users and receive antennas by exploiting a powerful duality relation with the multi-access channel which was further explored in Jindal *et al.* [12]. Recently, Weingarten *et al.* [28] showed that DPC in fact achieves the full capacity region of the multi-antenna Gaussian BC, thus providing a characterization of the entire capacity region.

Various researchers have investigated the sum capacity gains achievable in the above-described system by simultaneously transmitting to several users. In particular, Jindal & Goldsmith [10] show that the sum capacity gain over a TDMA strategy is approximately $\min\{M, K\}$, i.e., the minimum of the number of transmit antennas and the number of users. Jindal [8] demonstrates that the sum capacity grows with the SNR at rate $\min\{M, K\}$. In other words, multiple transmit antennas can potentially provide an M -fold gain in the sum capacity.

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The above capacity results rely on the assumption that perfect channel state information is available at the transmitter, which may involve a significant amount of feedback overhead. In addition, DPC is quite a sophisticated technique and challenging to implement in an actual system. Motivated by these issues, extensive efforts have been made to devise practical transmission and coding schemes and find ways to reduce the amount of channel feedback information required. Hochwald *et al.* [4], [5] describe an algorithm based on channel inversion and sphere encoding, and demonstrate that it closely approaches the sum capacity while being simpler to operate than DPC. Jindal [9] considers a multi-antenna BC with limited channel feedback information, and shows that the full sum capacity gain at high SNR values is achievable as long as the number of feedback bits grows linearly with the SNR (in dB).

As mentioned above, multiple transmit antennas can potentially yield an M -fold increase in the sum capacity. However, it is necessary that at least M users are served simultaneously in order to reap the full benefits. Transmitting to fewer than M users falls short of the maximum rate as it fails to fully exploit the available degrees of freedom. Transmitting to more than M users may be necessary to achieve the sum capacity in general, but the upper bound in [10] suggests that transmitting to a suitably selected subset of M near-orthogonal users is close to optimal. When the total number of users to choose from is sufficiently large, such a subset exists with high probability [19], [20].

Clearly, the above principle allows for a reduction of the amount of channel feedback and coding complexity. In particular, it suggests beam-forming schemes which construct M (random) orthogonal beams and serve the users with the largest channel gains on each of them with equal power. Transmission schemes along these lines are presented in Viswanath *et al.* [25], Sharif & Hassibi [15], and Vakili *et al.* [22]. Viswanathan & Kumaran [26] proposed fixed-beams and adaptive steerable-beams schemes grounded on that principle as well. Further related results may be found in Sharif & Hassibi [16], [17] who derive the asymptotic sum capacity for TDMA, DPC and beam-forming in the limit where the number of users grows large.

In [7], we considered a two transmit antenna broadcast system with homogeneous users, and derived an exact asymptotic characterization of the gap between the full sum capacity and the rate that can be achieved by transmitting to a suitably selected pair of users. In particular, we proposed a scheme that picks the user with the largest channel gain, and

then selects a second user from the next $L - 1$ strongest ones to form the best possible pair with it, taking channel orientations into account as well. We showed that the expected rate gap converges to $1/(L - 1)$ nats/symbol when the total number of users K tends to infinity. Allowing L to increase with K , we concluded that the gap asymptotically vanishes, and that the sum capacity is achievable by transmitting to a properly chosen pair of users.

In the present paper, we generalize the above results to a system with heterogeneous user characteristics. In this case, the sum capacity is no longer an appropriate performance measure, because it does not reflect the potential fairness issues that arise. Hence, we will focus on maximizing a *weighted* sum rate, where the users with weaker channels would typically be assigned higher weights. Leaving fairness considerations aside, maximizing a weighted sum rate is also of critical importance in so-called queue-based scheduling strategies where the user weights are taken to be functions of the respective queue lengths. Queue-based scheduling strategies are particularly attractive because under mild assumptions they are known to achieve stability whenever feasible without explicit knowledge of the system parameters, see for instance [14], [18], [21].

Although the sum rate expression for the multi-antenna Gaussian BC and associated bounds have been thoroughly investigated, the problem of maximizing a general function over the capacity region has not attracted nearly as much attention. To the best of our knowledge, Viswanathan *et al.* [27] are among the few authors who consider the problem of attaining more general points on the boundary of the capacity region. In particular, they present an algorithm for finding the power allocation to achieve any weighted sum rate maximizing point. However, the optimization procedure is computationally demanding, especially for large numbers of users, and requires perfect channel state information. Lee & Jindal [13] study the problem of obtaining the symmetric capacity, i.e., the maximum rate that can be provided to each of the users simultaneously.

In the present paper, we consider a two-antenna broadcast system with a user population that consists of C distinct classes, where each class is assigned a non-negative weight. We derive a generic upper bound for the weighted sum capacity, which includes as a special case the sum capacity bound in [10]. We then proceed to show that the upper bound is in fact attained for a particular ‘ideal’ configuration of $2C$ channel vectors. Finally, we prove that a nearly ideal configuration of such channel vectors exists with high probability, and that the maximum expected weighted sum rate can thus be asymptotically achieved, when the total number of users grows large.

The above results, as well as their homogeneous counterparts in [7] have significant ramifications for the design of channel feedback mechanisms and scheduling strategies. Since the proposed schemes only transmit to a small fraction of the users, they provide significant scope for reducing the feedback overhead and operational complexity. We further

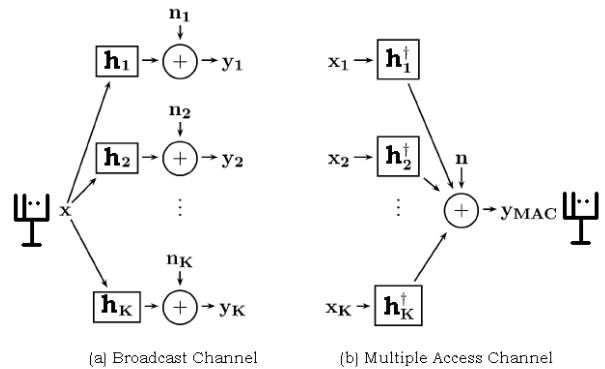


Fig. 1. The multi-antenna BC (left) and the MAC (right) have the same capacity region.

remark that even though this paper only treats the case of two transmit antennas in detail, the results extend naturally to an arbitrary number of antennas. Specifically, we can show that in a M -antenna system with C user classes, the weighted sum capacity can be asymptotically attained by transmitting to a suitably chosen set of MC users. See [6] for details.

The remainder of the paper is organized as follows. In Section II we present a detailed model description and review some relevant results for the capacity region of the Gaussian multi-antenna BC. In Section III we briefly review our results for a homogeneous system. Section IV addresses the weighted sum rate maximization problem in a scenario with heterogeneous users. In Section V we discuss the numerical experiments we conducted, which indicate that the asymptotics are surprisingly accurate, even for a moderate number of users. Throughout the paper, we omit most of the proofs due to length constraints.

II. MODEL DESCRIPTION AND KNOWN RESULTS

A. Model description

We consider a broadcast channel (BC) with $M > 1$ transmit antennas and K receivers each with a single antenna, as schematically represented in Figure 1(a).

Let $\mathbf{x} \in \mathbb{C}^{M \times 1}$ be the transmitted vector signal and let $\mathbf{h}_k \in \mathbb{C}^{1 \times M}$ be the channel gain vector of the k -th receiver. Denote by $\mathbf{H} = [\mathbf{h}_1^\dagger \mathbf{h}_2^\dagger \dots \mathbf{h}_K^\dagger]^\dagger$ the concatenated channel matrix of all K receivers. For now, the matrix \mathbf{H} is arbitrary but fixed. We assume that the transmitter has perfect channel state information, i.e., exact knowledge of the matrix \mathbf{H} . The circularly symmetric complex Gaussian noise at the k -th receiver is $n_k \in \mathbb{C}$ where n_k is distributed according to $\mathcal{CN}(0, 1)$. Thus the received signal at the k -th receiver is $y_k = \mathbf{h}_k \mathbf{x} + n_k$. The covariance matrix of the transmitted signal is $\Sigma_x = \mathbb{E}[\mathbf{x} \mathbf{x}^\dagger]$. The transmitter is subject to a power constraint P , which implies $\text{Tr}(\Sigma_x) \leq P$. (Here Tr denotes the trace operator, which is the sum of the diagonal elements of a square matrix.)

$$\mathcal{C}_{BC}^{\mathbf{w}}(\mathbf{H}, P) = \max_{\mathbf{R} \in \mathcal{C}_{BC}} \sum_{k=1}^K w_k R_k = \max_{\pi} \max_{\sum_{k=1}^K \text{Tr}(\Sigma_k) \leq P} \sum_{k=1}^K w_{\pi(k)} \log \left(\frac{1 + \mathbf{h}_{\pi(k)} (\sum_{l \leq k} \Sigma_{\pi(l)}) \mathbf{h}_{\pi(k)}^{\dagger}}{1 + \mathbf{h}_{\pi(k)} (\sum_{l < k} \Sigma_{\pi(l)}) \mathbf{h}_{\pi(k)}^{\dagger}} \right). \quad (1)$$

B. Known information-theoretic results

Let $\pi(k)$, $k = 1, \dots, K$, be a permutation of $k = 1, \dots, K$. As shown in [23], the following rate vector is achievable using Dirty Paper Coding (DPC), for $k = 1, \dots, K$:

$$R_{\pi(k)} = \log \left(\frac{1 + \mathbf{h}_{\pi(k)} (\sum_{l \leq k} \Sigma_{\pi(l)}) \mathbf{h}_{\pi(k)}^{\dagger}}{1 + \mathbf{h}_{\pi(k)} (\sum_{l < k} \Sigma_{\pi(l)}) \mathbf{h}_{\pi(k)}^{\dagger}} \right).$$

The DPC region is defined as the convex hull of the union of all such rate vectors, over all positive semi-definite covariance matrices that satisfy the power constraint $\sum_{k=1}^K \text{Tr}(\Sigma_k) \leq P$, and over all possible permutations $\pi(k)$. As shown in [1], [28], DPC in fact achieves the entire capacity region denoted as \mathcal{C}_{BC} . The weighted sum capacity $\mathcal{C}_{BC}^{\mathbf{w}}(\mathbf{H}, P)$ for any weight vector $\mathbf{w} \in \mathbb{R}_+^K$ can therefore be written as in equation (1).

Unfortunately, the maximization in (1) involves a non-concave function of the covariance matrices, which makes it hard to deal with analytically as well as numerically. However, in [23], [24], a duality is shown to exist between the BC and the Gaussian multiple-access channel (MAC) with a sum-power constraint P . That is, the dual MAC which is formed by reversing the roles of transmitters and receivers, as represented in Figure 1(b), has the same capacity region as the BC. Note that $\mathcal{C}_{BC}^{\mathbf{w}}(\mathbf{H}, P) = \sum_{k=1}^K \Delta w_k S_k$, with $S_k := \sum_{l=1}^k R_l$ the partial sum rate of the first k users and $\Delta w_k := w_k - w_{k+1}$, with the convention that $w_{K+1} = 0$. Without loss of generality we may assume that $w_1 \geq w_2 \geq \dots \geq w_K$. Using the duality result, the weighted sum capacity (1) of the BC can thus be expressed in terms of the dual MAC weighted sum rate as

$$\mathcal{C}_{BC}^{\mathbf{w}}(\mathbf{H}, P) = \max_{\sum_{k=1}^K P_k \leq P} \sum_{k=1}^K \Delta w_k \log \det \left(I_M + \sum_{l=1}^k P_l \mathbf{h}_l^{\dagger} \mathbf{h}_l \right), \quad (2)$$

where $P_k \geq 0$ denotes the power allocated to the k -th receiver. As a special case of (2) with $w_k = 1$, $k = 1, \dots, K$, the sum capacity is obtained as

$$\mathcal{C}_{BC}^{\text{sum}}(\mathbf{H}, P) = \max_{\sum_{k=1}^K P_k \leq P} \log \det \left(I_M + \sum_{k=1}^K P_k \mathbf{h}_k^{\dagger} \mathbf{h}_k \right). \quad (3)$$

Since $\log \det(\cdot)$ is a concave function on the set of positive-definite matrices, the problems in (2) and (3) only involve maximizing a concave objective function subject to convex constraints. Specialized algorithms have been developed to solve these problems [11], [27].

III. HOMOGENEOUS USERS

In this section, we take a synoptic look at the problem of maximizing the sum rate in a system with two transmit antennas and statistically homogeneous user population. A more detailed treatment of this problem can be found in [7].

The sum capacity is a key metric of interest for the BC as it measures the maximum achievable total rate. Since it only considers the aggregate throughput, it does not reflect potential fairness issues that arise when users with widely disparate channel characteristics obtain vastly different throughput portions. In the present section, however, we focus on the case of statistically identical users, which by symmetry will obtain equal long-term throughput shares, so that fairness is not a major issue. In the next section, we will address the problem of maximizing a *weighted* sum rate in a system where the users may have different characteristics.

We will show that the sum capacity can be closely approached by transmitting to a suitably selected pair of users as the total number of users grows large. In preparation for that, we first present some useful lower and upper bounds for the sum capacity.

A. Bounds for the sum capacity

Denote by $\mathbf{h}_{(k)}$ the channel vector of the receiver with the k -th largest norm, i.e., $\|\mathbf{h}_{(1)}\|^2 \geq \|\mathbf{h}_{(2)}\|^2 \geq \dots \geq \|\mathbf{h}_{(K)}\|^2$. The next upper bound for the sum capacity is established in [10]:

$$\mathcal{C}_{BC}^{\text{sum}}(\mathbf{H}, P) \leq M \log \left(1 + \frac{P}{M} \|\mathbf{h}_{(1)}\|^2 \right). \quad (4)$$

Observe that the above bound can be achieved when there are M receivers with orthogonal channel vectors tied for the maximum norm $\|\mathbf{h}_{(1)}\|^2$. For a two-antenna system, the above bound becomes

$$\mathcal{C}_{BC}^{\text{sum}}(\mathbf{H}, P) \leq 2 \log \left(1 + \frac{P}{2} \|\mathbf{h}_{(1)}\|^2 \right). \quad (5)$$

Taking $P_i = P_j = P/2$ and $P_k = 0$ for all $k \neq i, j$ in Equation (3), we obtain a simple lower bound for the sum capacity

$$\mathcal{C}_{BC}^{\text{sum}}(\mathbf{H}, P) \geq C(\mathbf{h}_i, \mathbf{h}_j, P) := \log \det \left(I_2 + \frac{P}{2} (\mathbf{h}_i^{\dagger} \mathbf{h}_i + \mathbf{h}_j^{\dagger} \mathbf{h}_j) \right), \quad (6)$$

which corresponds to transmitting to users i and j at equal power.

For any two vectors $\mathbf{g}, \mathbf{h} \in \mathbb{C}^2$, let $U(\mathbf{g}, \mathbf{h}) := \frac{|\langle \mathbf{g}, \mathbf{h} \rangle|^2}{\|\mathbf{g}\|^2 \|\mathbf{h}\|^2}$ be the squared normalized inner product. By expanding the determinant in (6), we obtain

$$C(\mathbf{h}_i, \mathbf{h}_j, P) = \log \left(1 + \frac{P}{2} (\|\mathbf{h}_i\|^2 + \|\mathbf{h}_j\|^2) + \frac{P^2}{4} \|\mathbf{h}_i\|^2 \|\mathbf{h}_j\|^2 V_{ij} \right), \quad (7)$$

with $V_{ij} = 1 - U(\mathbf{h}_i, \mathbf{h}_j)$.

The lower bound expression (7) reflects the fact that the sum rate for two users critically depends on the norms of the respective channel vectors and their degree of orthogonality. In particular, the sum rate is large when the channel vectors are nearly orthogonal and have large norms. Indeed, the lower bound coincides with the upper bound (5) when users i and j are orthogonal and tied for the maximum norm, i.e., $\|\mathbf{h}_i\|^2 = \|\mathbf{h}_j\|^2 = \|\mathbf{h}_{(1)}\|^2$ and $\langle \mathbf{h}_i, \mathbf{h}_j \rangle = 0$.

B. Large- K asymptotics

The lower and upper bounds for the sum capacity in the previous subsection hold for any arbitrary but fixed set of channel vectors. In order to derive meaningful asymptotic results, we will in the remainder of the section assume the channel vectors to be random and focus on the *expected* sum rate. We will adhere to the common assumption that the components of the channel vectors of the various users are independent and distributed according to $\mathcal{CN}(0, 1)$, which corresponds to independent Rayleigh fading. As it turns out, this specific assumption is actually not essential for most of the results to hold. We will not pursue this thread in any detail here, but revisit the issue when we later consider heterogeneous user scenarios.

As mentioned earlier, the upper bound in (5) for the sum capacity can be achieved when there is a pair of orthogonal users tied for the maximum channel norm $\|\mathbf{h}_{(1)}\|^2$ by granting equal power to each of them. Intuitively, when the total number of users is large, there exists with high probability a pair of users which are nearly orthogonal and have norms close to the maximum. This suggests that the sum capacity can be closely approached by transmitting to such a pair of users and allocating equal power to each of them.

We are now ready to formalize the above assertion. We will consider three heuristic selection schemes for scheduling a pair of users with equal power. Scheme I picks two arbitrary users among the L strongest ones. Scheme II selects an arbitrary user among the L strongest ones, and a second one from the same group to form the best pair, i.e., the pair that maximizes the sum rate. Scheme III picks the best pair among the L strongest users, i.e., the pair that maximizes the sum rate. Note that scheme II dominates scheme I and that scheme III in turn dominates scheme II, and that all three schemes coincide when $L = 2$.

Our main theorems in this section consider the asymptotic gap between the upper bound in (5) and the sum rate achievable by scheme II.

Theorem 3.1: For any fixed value of $L \geq 2$, $l \leq L$, the difference

$$\mathbb{E} \left[2 \log \left(1 + \frac{P}{2} \|\mathbf{h}_{(1)}\|^2 \right) \right] - \mathbb{E} \left[\max_{k=1, \dots, L, k \neq l} C(\mathbf{h}_{(l)}, \mathbf{h}_{(k)}, P) \right]$$

converges to $1/(L-1)$ as $K \rightarrow \infty$.

Theorem 3.2: For any fixed value l and sequence $L(K)$ with $\lim_{K \rightarrow \infty} L(K) \rightarrow \infty$,

$$2\mathbb{E} \left[\log \left(1 + \frac{P}{2} \|\mathbf{h}_{(1)}\|^2 \right) \right] - \mathbb{E} \left[\max_{k=1, \dots, L(K), k \neq l} C(\mathbf{h}_{(l)}, \mathbf{h}_{(k)}, P) \right] \rightarrow 0$$

as $K \rightarrow \infty$.

The next corollaries follow as immediate consequences from Theorems 3.1 and 3.2.

Corollary 3.1: For any fixed value of L , $l \leq L$,

$$\mathbb{E} [C_{BC}^{\text{sum}}(\mathbf{H}, P)] - \mathbb{E} \left[\max_{k=1, \dots, L, k \neq l} C(\mathbf{h}_{(l)}, \mathbf{h}_{(k)}, P) \right] \rightarrow \frac{1}{L-1}$$

as $K \rightarrow \infty$.

The above corollary shows that the asymptotic performance gap of scheme II decays as $1/(L-1)$, which suggests that a relatively moderate value of L may be adequate for most practical purposes.

Corollary 3.2: For any fixed value l and sequence $L(K)$ with $\lim_{K \rightarrow \infty} L(K) = \infty$,

$$\mathbb{E} [C_{BC}^{\text{sum}}(\mathbf{H}, P)] - \mathbb{E} \left[\max_{k=1, \dots, L(K), k \neq l} C(\mathbf{h}_{(l)}, \mathbf{h}_{(k)}, P) \right] \rightarrow 0$$

as $K \rightarrow \infty$.

The above corollary shows that scheme II is asymptotically optimal when sufficiently many users are considered, and thus implies that the dominating scheme III is asymptotically optimal as well. As a by-product, we conclude that the upper bound (5) is asymptotically tight.

Corollary 3.3:

$$\mathbb{E} [C_{BC}^{\text{sum}}(\mathbf{H}, P)] - \mathbb{E} [C(\mathbf{h}_{(1)}, \mathbf{h}_{(2)}, P)] \rightarrow 1$$

as $K \rightarrow \infty$.

This corollary corresponds to a special case of scheme I with $L = 2$, and shows that simply selecting the two strongest users leaves a performance gap of 1 nat/symbol.

In conclusion, the above results show that scheme II is asymptotically optimal in the sense that the absolute gap to the sum capacity vanishes to zero provided $L(K) \rightarrow \infty$ as $K \rightarrow \infty$. Thus, transmitting to a suitably selected pair of users is asymptotically optimal, where one of them may in fact be arbitrarily chosen from a fixed short list. The gain from considering all pairs of users, as in scheme III, is asymptotically negligible. However, picking an arbitrary pair of users, as in scheme I, is not optimal even when the users are the two strongest ones.

IV. HETEROGENEOUS USERS

In this section, we extend our results in [7] to a system in which the user channels are not identically distributed. In particular, we turn our attention to a system where users may have different statistical characteristics, and focus on the problem of maximizing a *weighted* sum rate expression. We will demonstrate that transmitting to a properly selected group of users asymptotically achieves the maximum expected weighted sum rate, although scheduling just two users will no longer be sufficient in general.

A. Bounds for the weighted sum rate

We first establish a generic upper bound for the weighted sum rate for an arbitrary number of M transmit antennas. Let w_k be the weight associated with the k -th user. For notational convenience, define $\Delta w_k := w_k - w_{k+1}$ with the convention that $w_{K+1} = 0$. Without loss of generality, we assume that the users are indexed such that $w_1 \geq w_2 \geq \dots \geq w_K$.

Theorem 4.1: For any given set of channel vectors,

$$C_{BC}^w(\mathbf{H}, P) \leq \max_{\sum_{k=1}^K P_k \leq P} \Delta w_1 \log(1 + P_1 \|\mathbf{h}_1\|^2) + M \sum_{k=2}^K \Delta w_k \log \left(1 + \sum_{l=1}^k \frac{P_l}{M} \|\mathbf{h}_l\|^2 \right). \quad (8)$$

Proof

Equation (2) yields that $C_{BC}^w(\mathbf{H}, P) = \sum_{k=1}^K \Delta w_k S_k$, with

$$S_k = \log \det \left(I_M + \sum_{l=1}^k P_l \mathbf{h}_l^\dagger \mathbf{h}_l \right).$$

Clearly,

$$S_1 = \log(1 + P_1 \|\mathbf{h}_1\|^2). \quad (9)$$

Using Hadamard's inequality for Hermitian positive semi-definite matrices [3], p. 502, and the concavity of the log function, we obtain

$$S_k \leq \sum_{m=1}^M \log \left(1 + \sum_{l=1}^k P_l h_{lm}^\dagger h_{lm} \right) \leq \log \left(1 + \sum_{l=1}^k \frac{P_l}{M} \|\mathbf{h}_l\|^2 \right) \quad (10)$$

for all $k = 2, \dots, K$.

Substituting inequalities (9) and (10), the statement of the theorem follows. \square

The next upper bound follows as a straightforward corollary of Theorem 4.1.

Corollary 4.1: For any given set of channel vectors,

$$C_{BC}^w(\mathbf{H}, P) \leq M \max_{\sum_{k=1}^K P_k \leq P/M} \sum_{k=1}^K \Delta w_k \log \left(1 + \sum_{l=1}^k P_l \|\mathbf{h}_l\|^2 \right). \quad (11)$$

In order to develop a suitable asymptotic framework, we assume that there are C classes of users, with K_c the number of class- c users and $\sum_{c=1}^C K_c = K$. Let $\mathbf{h}_k^{(c)}$ be the channel vector of the k -th class- c user. With minor abuse of notation, we let w_c be the weight associated with class c , and define $\Delta w_c := w_c - w_{c+1}$, with the convention that $w_{C+1} = 0$ as before. Let T_c be the total rate received by class c . Thus the weighted sum rate is $T := \sum_{c=1}^C w_c T_c$. Without loss of generality, we assume that the classes are indexed such that $w_1 \geq w_2 \geq \dots \geq w_C$. Let $\mathbf{h}_{(k)}^{(c)}$ be the channel vector of the class- c user with the k -th largest norm, i.e., $\|\mathbf{h}_{(1)}^{(c)}\|^2 \geq \|\mathbf{h}_{(2)}^{(c)}\|^2 \geq \dots \geq \|\mathbf{h}_{(K_c)}^{(c)}\|^2$.

The next corollary specializes the upper bound in (11) to a class-based system.

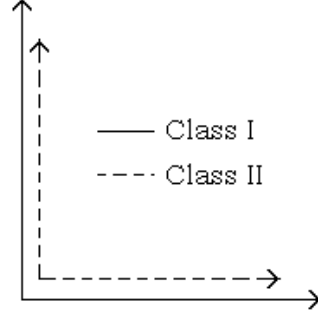


Fig. 2. The optimal channel configuration for two user classes.

Corollary 4.2: For any given set of channel vectors,

$$\sum_{c=1}^C w_c T_c \leq M \max_{\sum_{c=1}^C P_c \leq P/M} \sum_{c=1}^C \Delta w_c \log \left(1 + \sum_{d=1}^c P_d \|\mathbf{h}_{(1)}^{(d)}\|^2 \right). \quad (12)$$

Note that when all weights are taken equal to one, the upper bound in (11) reduces to that in Equation (4) for the sum rate. Recall that the upper bound in (4) is tight in the sense that it can actually be achieved when there are M users with orthogonal channel vectors tied for the maximum norm. Likewise, the upper bound in (12) can be attained for a particular configuration of channel vectors. Specifically, assume that there are M unit orthogonal vectors $\mathbf{u}_m \in \mathbb{C}^M$, $m = 1, \dots, M$, i.e., $\|\mathbf{u}_m\| = 1$ for all m , $\langle \mathbf{u}_m, \mathbf{u}_n \rangle = 0$, $m \neq n$, and MC users, M from each class, with channel vectors $\mathbf{h}_{u_c^m}^{(c)}$, $c = 1, \dots, C$, $m = 1, \dots, M$, that satisfy the following two properties:

- (i) within each class, all M users are tied for the maximum norm, i.e., $\|\mathbf{h}_{u_c^m}^{(c)}\|^2 = \|\mathbf{h}_{(1)}^{(c)}\|^2$ for all $c = 1, \dots, C$, $m = 1, \dots, M$;
- (ii) the channel vector of one of the users of each class is parallel to \mathbf{u}_m and thus orthogonal to \mathbf{u}_n , $m \neq n$, i.e., $\langle \mathbf{u}_m, \mathbf{h}_{u_c^m}^{(c)} \rangle = \|\mathbf{h}_{u_c^m}^{(c)}\|$ and $\langle \mathbf{u}_n, \mathbf{h}_{u_c^m}^{(c)} \rangle = 0$ for all $c = 1, \dots, C$.

The second property implies that all the \mathbf{u}_m -users are orthogonal to all the \mathbf{u}_n -users, i.e., $\langle \mathbf{h}_{u_c^m}^{(c)}, \mathbf{h}_{u_d^n}^{(d)} \rangle = 0$ for all $c, d = 1, \dots, C$, $m \neq n$. For brevity, the above-described constellation of channel vectors will be referred to as the optimal configuration. Figure 2 provides a pictorial representation of the optimal configuration for the case of $C = 2$ user classes and $M = 2$ transmit antennas.

Let $P_1^*(K), \dots, P_C^*(K)$ be the optimizing power levels of the upper bound in (12) for given values of $\|\mathbf{h}_{(1)}^{(c)}\|^2$, $c = 1, \dots, C$, i.e.,

$$P^*(K) = (P_1^*(K), \dots, P_C^*(K)) \\ =: \arg \max_{\sum_{c=1}^C P_c \leq P/M} \sum_{c=1}^C \Delta w_c \log \left(1 + \sum_{d=1}^c P_d \|\mathbf{h}_{(1)}^{(d)}\|^2 \right).$$

Now, by assigning power $P_c^*(K)$ to all M class- c users in the optimal configuration, and arranging the users in order

of increasing class index in the DPC sequence, we can show that the upper bound in (12) is indeed achievable.

From now on, we focus on the case of $M = 2$ transmit antennas. The upper bound in (12) then becomes:

$$\sum_{c=1}^C w_c T_c \leq U(w_c; \|\mathbf{h}_{(1)}^{(c)}\|^2; P) := 2 \max_{\sum_{c=1}^C P_c \leq P/2} \sum_{c=1}^C \Delta w_c \log \left(1 + \sum_{d=1}^c P_d \|\mathbf{h}_{(1)}^{(d)}\|^2 \right). \quad (13)$$

The next lemma provides a lower bound. Consider a scheme that assigns power P_c^* to class- c users u_c and v_c with channel vectors $\mathbf{h}_{u_c}^{(c)}$ and $\mathbf{h}_{v_c}^{(c)}$, respectively, $c = 1, \dots, C$, and arranges users in order of increasing class index in the DPC sequence. Let \widehat{T}_c and $\widehat{S}_c := \sum_{d=1}^c \widehat{T}_d$ be the resulting total rate received by class c and the partial sum rate of the first c classes, respectively.

Lemma 4.1: Let $V_c := \min_{d,e=1,\dots,c} V^{(d),(e)}$, with $V^{(d),(e)} := 1 - U(\mathbf{h}_{u_d}^{(d)}, \mathbf{h}_{v_e}^{(e)})$. Then $\sum_{c=1}^C w_c \widehat{T}_c = \sum_{c=1}^C \Delta w_c \widehat{S}_c$, with

$$\begin{aligned} \widehat{S}_c &\geq \log \left(1 + \sum_{d=1}^c P_d^* \|\mathbf{h}_{u_d}^{(d)}\|^2 \right) \\ &+ \log \left(1 + \sum_{d=1}^c P_d^* \|\mathbf{h}_{v_d}^{(d)}\|^2 \right) + \log(V_c). \end{aligned}$$

Note that the above lower bound coincides with the upper bound in (13) if $\mathbf{h}_{u_c}^{(c)}, \mathbf{h}_{v_c}^{(c)}$, $c = 1, \dots, C$, form the optimal configuration of channel vectors, i.e., $\|\mathbf{h}_{u_c}^{(c)}\|^2 = \|\mathbf{h}_{v_c}^{(c)}\|^2 = \|\mathbf{h}_{(1)}^{(c)}\|^2$ for all $c = 1, \dots, C$, and $\langle \mathbf{h}_{u_c}^{(c)}, \mathbf{h}_{v_d}^{(d)} \rangle = 0$, so that $V^{(d),(e)} = 1$ for all $d, e = 1, \dots, C$.

B. Random channel vectors

The lower and upper bounds for the weighted sum rate in the previous subsection hold for any arbitrary but fixed set of channel vectors. In order to derive asymptotic results, we will as before assume the channel vectors to be random and focus on the *expected* weighted sum rate. Within each class we assume the channel vectors to be independent and identically distributed, i.e., $\mathbf{h}_1^{(c)}, \mathbf{h}_2^{(c)}, \dots$ are i.i.d. copies of some random vector $\mathbf{h}^{(c)} \in \mathbb{C}^2$. Among the various classes, the channel vectors may however have different statistical characteristics. To be specific, we assume that each class- c channel is Rayleigh faded with parameter β_c . In other words, $\mathbf{h}^{(c)} = \beta_c \mathbf{h}$, $c = 1, \dots, C$, where the components of \mathbf{h} are independent and distributed according to $\mathcal{CN}(0, 1)$ as in the homogeneous case. The numbers of users of the various classes are assumed to grow large in fixed proportions, i.e., $K_c = \alpha_c K$ for fixed coefficients $\alpha_1, \dots, \alpha_C$ with $\sum_{c=1}^C \alpha_c = 1$.

C. Large- K asymptotics

We now proceed to show that the upper bound in (13) is asymptotically achievable by transmitting to a judiciously chosen subset of $2C$ users. Analogous to the homogeneous case, there exists with high probability a group of $2C$ users

with channel vectors close to the optimal configuration in the heterogeneous case when the total number of users is large. Thus, we will show that selecting such a group of $2C$ users and allocating power P_c^* to both class- c users, where

$$P^* = (P_1^*, \dots, P_C^*) := \arg \max_{\sum_{c=1}^C P_c \leq P/2} \sum_{c=1}^C \Delta w_c \log \left(\sum_{d=1}^c P_d \beta_d^2 \right)$$

asymptotically achieves the upper bound in (13). We remark that the power levels (P_1^*, \dots, P_C^*) are the limiting values of the sequence of random variables $(P_1^*(K), \dots, P_C^*(K))$ when the norms $\|\mathbf{h}_{(1)}^{(c)}\|^2$ grow large. It may in fact be shown that $(P_1^*(K), \dots, P_C^*(K))$ converge to (P_1^*, \dots, P_C^*) in probability, as $K \rightarrow \infty$.

We will now prove that transmitting to a carefully selected subset of $2C$ users asymptotically achieves the upper bound (13) and thus maximizes the expected weighted sum rate. Motivated by the knowledge of the optimal channel configuration, we will consider the following two user selection schemes which will be referred to as the ‘list’ scheme and the ‘cone’ scheme, respectively.

List scheme

The ‘list’ scheme first identifies for each class the users with norms close to the maximum, and then selects a nearly orthogonal pair of users among these. Specifically, the list scheme first selects the class-1 user with the largest norm $\|\mathbf{h}_{(1)}^{(1)}\|^2$. Let the channel vector of this user be $\mathbf{h}_{v_1}^{(1)} = \mathbf{h}_{(1)}^{(1)}$. It then considers the class-1 users with the L_1 next largest norms, and selects the user whose channel vector is most orthogonal to $\mathbf{h}_{(1)}^{(1)}$, i.e., the user that minimizes $U(\mathbf{h}_{(k)}^{(1)}, \mathbf{h}_{(1)}^{(1)})$. Let the channel vector of this user be $\mathbf{h}_{u_1}^{(1)}$, and $U_1 := U(\mathbf{h}_{u_1}^{(1)}, \mathbf{h}_{(1)}^{(1)})$. Next, it identifies the class- c users with the $2L_c$ largest norms and divides these in two groups of size L_c each, say odd ones and even ones. Within the first group, it selects the user whose channel vector is most parallel to $\mathbf{h}_{(1)}^{(1)}$, i.e., the user that maximizes $U(\mathbf{h}_{(2k-1)}^{(1)}, \mathbf{h}_{(1)}^{(1)})$. Let the channel vector of this user be $\mathbf{h}_{v_c}^{(c)}$, and $V_c := 1 - U(\mathbf{h}_{(1)}^{(1)}, \mathbf{h}_{v_c}^{(c)})$. Finally, it selects within the second group of class- c users the user whose channel vector is most orthogonal to $\mathbf{h}_{(1)}^{(1)}$, i.e., the user that minimizes $U(\mathbf{h}_{(2k)}^{(1)}, \mathbf{h}_{(1)}^{(1)})$. Let the channel vector of this user be $\mathbf{h}_{u_c}^{(c)}$, and $U_c := U(\mathbf{h}_{(1)}^{(1)}, \mathbf{h}_{u_c}^{(c)})$.

Cone scheme

The ‘cone’ scheme first identifies users that are close to orthogonal, and then selects the ones with the largest norms among these. Specifically, it first picks two orthogonal vectors $\mathbf{u}, \mathbf{v} \in \mathbb{C}^2$, i.e., $\langle \mathbf{u}, \mathbf{v} \rangle = 0$ and some small tolerance margin $\delta > 0$. Then it finds the class- c user with the largest norm among those with $U(\mathbf{u}, \mathbf{h}_k^{(c)}) \geq 1 - \delta$. Let the channel vector of this user be $\mathbf{h}_{u_c}^{(c)}$. Similarly, it selects the class- c user with the largest norm among those with $U(\mathbf{v}, \mathbf{h}_k^{(c)}) \geq 1 - \delta$. Let the channel vector of this user be $\mathbf{h}_{v_c}^{(c)}$.

After selecting the users in the above-described fashion,

both the list and the cone schemes allocate power P_c^* to both class- c users. Define \widehat{T}_c as the rate received by class c under the list scheme, i.e., the sum rate of the two class- c users selected, and denote by $\widehat{T} := \sum_{c=1}^C w_c \widehat{T}_c$ the total weighted sum rate. The next two theorems show that the list scheme achieves a finite rate gap that vanishes to zero as the list size grows large, and thus asymptotically maximizes the expected weighted sum rate. In a similar fashion, it can be shown that the cone scheme asymptotically achieves the maximum weighted sum rate.

Theorem 4.2: Assume that $L_c(K)$ is such that $L_c \leq L_c(K) \leq o(K^\delta)$ for any $\delta > 0$. Then

$$\limsup_{K \rightarrow \infty} \mathbb{E} \left[U(w_c; \|\mathbf{h}_{(1)}^{(c)}\|^2; P) \right] - \mathbb{E} \left[\widehat{T} \right] \leq D(L),$$

with $L := \min\{L_1, \dots, L_C\}$,

$$D(L) := 4w_1 C^2 \left[2L^{-\alpha} + C_1(A(1-A/4))^{2(L-1)} + C_2 e^{-AL^{1-\alpha}} \right],$$

$A := \min\{A_1, A_2\}$, $\alpha > 0$, and $C_1, C_2 > 0$ constants independent of L_1, \dots, L_C .

The next result follows by letting $L \rightarrow \infty$ in Theorem 4.2 and observing that $\lim_{L \rightarrow \infty} D(L) = 0$ for any $\alpha \in (0, 1)$.

Theorem 4.3: Assume that $L_c(K)$ is such that $\lim_{K \rightarrow \infty} L_c(K) = \infty$ and $L_c(K) \leq o(K^\delta)$ as $K \rightarrow \infty$ for any $\delta > 0$. Then

$$\lim_{K \rightarrow \infty} \mathbb{E} \left[U(w_c; \|\mathbf{h}_{(1)}^{(c)}\|^2; P) \right] - \mathbb{E} \left[\widehat{T} \right] = 0.$$

The above theorem shows that scheduling a suitably selected group of $2C$ users asymptotically achieves the upper bound (13) and thus maximizes the expected weighted sum rate. In fact, it shows that scheduling two users of each of the classes $c \in C^*$ is sufficient to asymptotically achieve the maximum expected weighted sum rate, where $C^* := \{c : P_c^* > 0\}$.

While we have focused only on the case of two transmit antennas, it can be shown along similar lines that in general the upper bound (12) is asymptotically achievable by transmitting to a suitably selected subset of MC users. See [6] for a treatment of the general M -transmit antenna case.

V. NUMERICAL RESULTS

In this section, we discuss the numerical experiments that we conducted for a two transmit antenna broadcast system with a heterogeneous user population. These simulations indicate that our asymptotic results tend to be remarkably accurate, even for moderate population sizes. Similar results for the homogeneous case were presented in [7].

A. Background for the numerical results

The simulation results which are provided below are for a two-class system. The weights are taken to be $w_1 = 2$, $w_2 = 1$ (although we equivalently normalized these to sum to 1 over the users), and the coefficients $\beta_1 = 0.5$, $\beta_2 = 1.0$ determine the mean SNRs. The two populations of users are of equal size, $K_1 = K_2 = 10$. Under these circumstances,

the asymptotically optimal power values are $P_1^* = 1/3$, $P_2^* = 1/6$, scaling out P , which is varied in most of the results below. We will state its value when necessary.

We now describe the schemes themselves. As far as the list and cone schemes are concerned, these are detailed in the text. Throughout, the asymptotically optimal power settings will be used, no power optimization is being employed. We will also consider TDMA, by which we mean the scheme that picks the user which has the maximum weighted rate when assigned full power, over all the users. Thus, it selects the k -th class- c user which maximizes

$$\max_{c=1, \dots, C} w_c \log(1 + P \|\mathbf{h}_{(1)}^{(c)}\|^2).$$

Finally, we consider two beamforming (BF) versions. The first version (referred to as BeamForm 2 in the figure) schedules one user in each beam, with the powers equally split and the user with the maximum weighted rate as determined by the SINR being the one selected for each beam. The second version (referred to as BeamForm 4) schedules one user from each class in each of the beams. In this case each user is assigned half its classes asymptotic power. The latter scheme is not expected to perform well as the interference between users on the same beam cannot be resolved except by using DPC or some equivalent approach.

B. Graphs for basic schemes

Figure 3(a) shows results for all the main schemes as well as the upper bound and the average maximum weighted capacity limit. $L = 5$ was set for the list scheme and $\delta = 0.2$ for the cone scheme. (Further numerical experiments indicated that the performance of the list scheme is quite robust with respect to the list size L , so that the exact value is not that critical.) As expected, the upper bound (13) is loose and the list and cone schemes perform well at high SNR values. For low SNR values, TDMA outperforms these schemes. The BF schemes fall off at very high SNR as the figure shows. All rates are given in nats.

As far as the list and cone schemes are concerned, good performance at high SNR is expected. However, at low SNR TDMA is close to optimal. (This latter conclusion follows from the linearity of the log.) Thus for low to moderate SNR's one could make up for the loss of rate in the list scheme by optimizing the powers. Similarly, the cone scheme does well at high SNR but not at low SNR. This loss in performance can also be addressed by assigning the powers optimally. This is a concave optimization in three independent variables, and is therefore potentially a time-consuming calculation, since we have no explicit formula for determining the optimal powers.

Figure 3(b) shows the same results, but gives the ratio to TDMA. Note that unlike the homogeneous case [15], BF is not asymptotically optimal as the number of users is increased at fixed SNR. However, at low SNR's (below 0 dB) BeamForm 2 does better than cone or list. Figure 3(b) shows that BeamForm 2 performs consistently worse than TDMA, which was also observed in the homogeneous example in

[7] where we had a similar number of users. The results for BeamForm 4 are worse than those for BeamForm 2 as expected.

C. Additional compound schemes

We now look at simpler enhancements to avoid power optimization. One such enhancement to the list scheme is to identify the best possible pair among the already selected four users. Consider the two-user weighted sum rates obtained by scheduling all possible pairs of these users. The power is split equally while scheduling two users of the same class, but when scheduling one user from each class, we allocate them powers $2P_1^*$ and $2P_2^*$ respectively. The two-user scheme picks the pair that corresponds to the highest weighted sum rate among the six possible pairs.

We thus arrive at the following heuristic schemes. Compound scheme I selects the better among TDMA and the list schemes. Compound scheme II goes further and selects the best among TDMA, the two-user scheme above, and the original list scheme. A three-user heuristic scheme was also considered, but since it did not provide any appreciable improvement, it has been omitted from the results.

In Figure 4 we compare the list scheme with the two heuristic schemes, Compound I and Compound II. These results are more clearly seen as a ratio to TDMA rather than the absolute rates which are difficult to distinguish. Since Compound I takes the best of TDMA and the list scheme, it cannot do worse than TDMA at any point and list at any point. Hence, it does well at low SNRs and at high SNRs. There is nevertheless a significant rate gap for this scheme for moderate SNR's, roughly in the range 0–5dB. Here TDMA falls off, but the list scheme is not yet in its most advantageous range. However, Compound II closes most of this gap as can be seen. The results in Figures 3 and 4 were averaged over 50 channel realizations.

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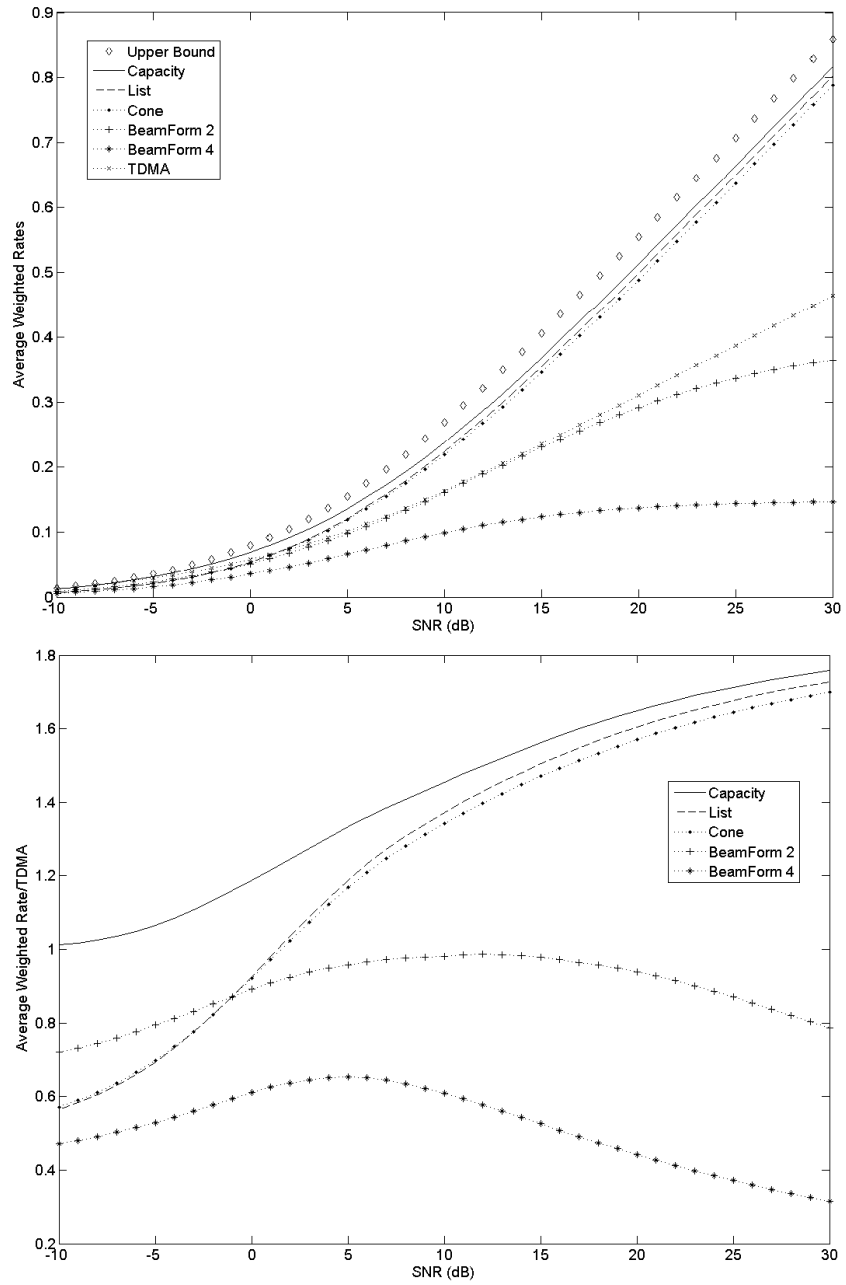


Fig. 3. (a) Absolute weighted rates for various schemes and upper bound (b) Ratio to TDMA for various schemes.

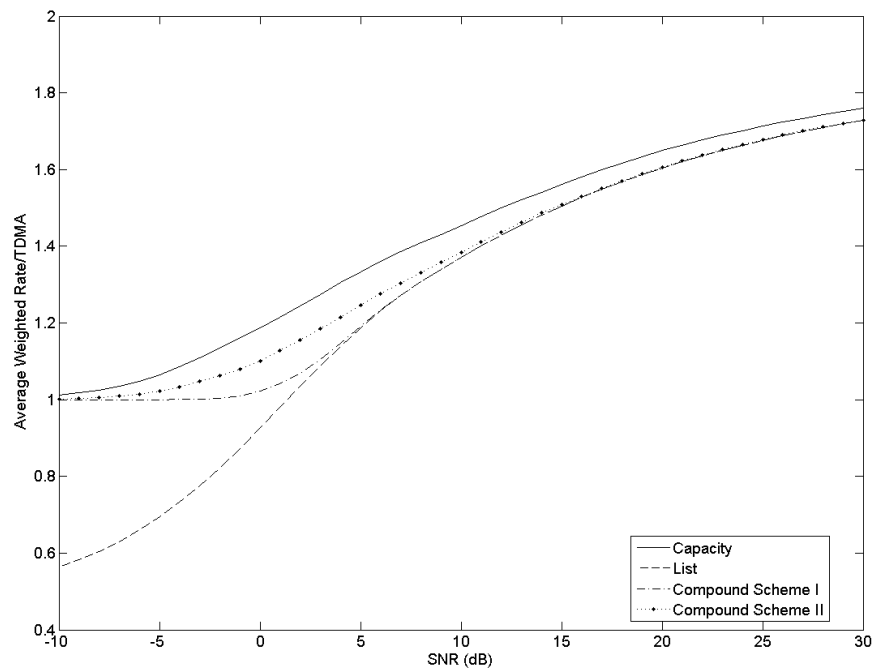


Fig. 4. Relative weighted sum rates for compound schemes compared to TDMA.