Chapter 1

COOPERATIVE ROUTING IN WIRELESS NETWORKS

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Abstract The joint problem of transmission-side diversity and routing in wireless networks is studied. It is assumed that each node in the network is equipped with a single omni-directional antenna and multiple nodes are allowed to coordinate their transmissions to achieve transmission-side diversity. The problem of finding the minimum energy route under this setting is formulated. Analytical asymptotic results are obtained for lower bounds on the resulting energy savings for both a regular line network topology and a grid network topology. For a regular line topology, it is possible to achieve energy savings of 39%. For a grid topology, it is possible to achieve energy savings of 56%. For arbitrary networks, we develop heuristics with polynomial complexity which result in average energy savings of $30\% - 50\%$ based on simulations.

Keywords: Wireless, Cooperation, Routing, Energy Efficiency, Diversity, Ad-Hoc Networks

1. Introduction

In this chapter, we study the problem of routing, cooperation and energy efficiency in wireless ad-hoc networks. In an ad-hoc network, nodes often spend most of their energy on communication [1]. In most applications, such as sensor networks, nodes are usually small and have limited energy supplies. In many cases, the energy supplies are non-replenishable and energy conservation is a determining factor in extending the life time of these networks. For this reason, the problem of energy efficiency and energy efficient communication in ad-hoc networks has received a lot of attention in the past several years. This problem, however, can be approached from two different angles: energy-efficient route selection algorithms at the network layer or efficient communication schemes at the physical layer. While each of these two areas has received a lot of attention separately, not much work has been done in jointly addressing these two problems. Our analysis in this chapter tackles this less studied area.

Motivated by results from propagation of electromagnetic signals in space, the amount of energy required to establish a link between two nodes is usually assumed to be proportional to the distance between the communicating nodes raised to a constant power. This fixed exponent, referred to as the path-loss exponent, is usually assumed to be between 2 to 4. Due to this relationship between the distance between nodes and the required power, it is usually beneficial, in terms of energy savings, to relay the information through multi-hop route in an ad-hoc network. Multi-hop routing extends the coverage by allowing a node to establish a multi-hop route to communicate with nodes that would have otherwise been outside of its transmission range. Finding the minimum energy route between two nodes is equivalent to finding the shortest path in a graph in which the cost associated with a link between two nodes is proportional to the distance between those nodes raised to the path-loss exponent. Figure 1.1 shows an example of a multi-hop route between two nodes.

The problem becomes more interesting once some special properties of the wireless medium are taken into account. In particular, there are

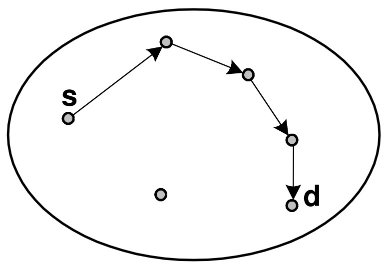

Figure 1.1. Multi-hop Relaying

three properties of the wireless physical layer that have motivated our work: the wireless broadcast property, the benefits of transmission side diversity, and multi-path fading.

A wireless medium is a broadcast medium in which signal transmitted by a node is received by all nodes within the transmission radius. For example, in figure 1.2, the signal transmitted by s is received by both nodes 1 and 2. This property, usually referred to as the *Wireless Broadcast Advantage* (WBA), was first studied in a network context in [3]. Clearly, this property of the wireless physical medium significantly changes many network layer route selection algorithm. The problem of finding the minimum energy multi-cast and broadcast tree in a wireless network is studied in [3] and [4]. This problem is shown to be NP-Complete in [5] and [6]. WBA also adds substantial complexity to route selection algorithms even in non-broadcast scenarios. For example, this model is used in [8] in the context of selecting the minimum energy link and node disjoint paths in a wireless network.

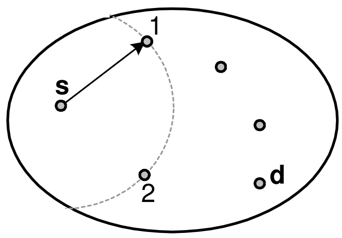

Figure 1.2. Wireless Broadcast Advantage

Another interesting property of the wireless medium is the benefit of space diversity at the physical layer. This type of diversity is achieved by employing multiple antennas on the transmitter or the receiver side. It is well known that transmission side diversity, i.e. using multiple antennas on the transmitter, results in significant energy savings (see [2]). In the network setting studied in this chapter, we assume that each node is only equipped with a single antenna. Hence, a straight forward extension of multiple-antenna results to a network setting is not possible. However, it might be possible that several nodes can cooperate with each other in transmitting the information to other nodes, and through this cooperation effectively achieve similar energy savings as a multiple antenna system. We call the energy savings due to cooperative transmission by several nodes the *Wireless Broadcast Advantage*. An overview of different transmission side diversity techniques is given in [2]. An architecture for achieving the required level of coordination among the cooperating nodes is discussed in [9].

In the problem studied in this chapter, we intend to take advantage of the wireless broadcast property and the transmission side diversity created through cooperation to reduce the end-to-end energy consumption in routing the information between two nodes. To make it clear, let's look at a simple example. For the network shown in figure 1.1, assume the minimum energy route from s to d is determined to be as shown. As discussed previously, the information transmitted by node s is received by nodes 1 and 2. After the first transmission, nodes s, 1 and 2 have the information and can cooperate in getting the information to d. For instance, these 3 nodes can cooperate with each other in transmitting the information to node 3 as shown in figure 1.3.

Several questions arise in this context: how much energy savings can be realized by allowing this type of cooperation to take place? What level of coordination among the cooperating nodes is needed? And how must the route selection be done to maximize the energy savings?

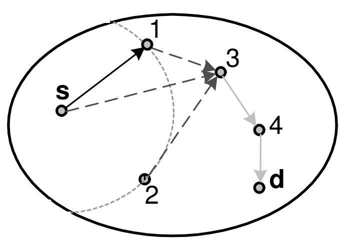

Figure 1.3. Cooperative Transmission

These are the problems that we look at here. We develop a formulation that captures the benefit of cooperative transmission and develop an algorithm for selecting the optimal route under this setting. We formulate the problem of finding the minimum energy cooperative route as two separate minimization problems. First, we look at the problem of optimal transmission of information between two sets of nodes. A separate problem is how to decide which nodes must be added to the reliable set in each transmission such that the information is routed to the final destination with minimum overall energy. We use dynamic programming to solve this second minimization problem. We present analytical results for the lower-bound of savings in networks with regular line or grid topology. We also propose two heuristics for finding the optimal path in arbitrary networks and present simulation results for the average energy savings of those heuristics.

2. Cooperative Transmission

Consider a wireless ad-hoc network consisting of arbitrarily distributed nodes where each node has a single omni-directional antenna. We assume that each node can dynamically adjust its transmitted power to control its transmission radius. It is also assumed that multiple nodes cooperating in sending the information to a single receiver node can precisely delay their transmitted signal to achieve perfect phase synchronization at the receiver. Under this setting, the information is routed from the source node to the destination node in a sequence of transmission slots, where each transmission slot corresponds to one use of the wireless medium. In each transmission slot/stage, either a node is selected to broadcast the information to a group of nodes or a subset of nodes that have already received the information cooperate to transmit that information to another group of nodes. As explained shortly, under our assumption it is only reasonable to restrict the size of the receiving set to one node when multiple nodes are cooperating in the transmission. So, each transmission is either a broadcast, where a single node is transmitting the information and the information is received by multiple nodes, or a cooperative, where multiple node simultaneously send the information to a single receiver. We refer to the first case as the *Broadcast Mode* and the second case at the *Cooperative Mode*. In the *Broadcast Mode*, we take advantage of the known *Wireless Broadcast Advantage*. In the *Cooperative Mode*, we benefit from the newly introduced concept of *Wireless Cooperative Advantage*.

The routing problem can be viewed as a multi-stage decision problem, where at each stage the decision is to pick the transmitting and the receiving set of nodes as well as the transmission power levels among all nodes transmitting in that stage. The objective is to get the information to the destination with minimum energy. The set of nodes that have the information at the k^{th} stage is referred to as the k^{th}-stage *Reliable Set*, S_k, and the routing solution may be expressed as a sequence of expanding reliable sets that starts with only the source node and terminates as soon as the reliable set contains the destination node. We denote the transmitting set by S and the receiving set by T. The link cost between S and T, $LC(S, T)$, is the minimum power needed for transmitting from S to T.

In this chapter, we make several idealized assumptions about the physical layer model. The wireless channel between any transmitting node, labeled s_i, and any receiving node, labeled t_j, is modeled by two parameters, its magnitude attenuation factor α_{ij} and its phase delay θ_{ij}. We assume that the channel parameters are estimated by the receiver and fed back to the transmitter. This assumption is reasonable for slowly varying channels, where the channel coherence time is much longer than the block transmission time. We also assume a free space propagation model where the power attenuation α_{ij}^2 is proportional to the inverse of the square of the distance between the communicating nodes s_i and t_j. For the receiver model, we assume that the desired minimum transmission rate at the physical layer is fixed and nodes can only decode based on the signal energy collected in a single channel use. We also assume that the received information can be decoded with no errors if the received Signal-to-Noise ration, SNR, level is above a minimum threshold SNR_{min}, and that no information is received otherwise. Without loss of generality, we assume that the information is encoded in a signal $\phi(t)$ that has unit power $P_\phi = 1$ and that we are able to control the phase and magnitude of the signal arbitrarily by multiplying it by a complex scaling factor w_i before transmission. The transmitted power by node i is $|w_i|^2$. The noise at the receiver is assumed to be additive, and the noise signal and power are denoted by $\eta(t)$ and P_η, respectively. This simple model allows us to find analytical results for achievable energy savings in some simple network topologies.

Link Cost Formulation

In this section, our objective is to understand the basic problem of optimal power allocation required for successful transmission of the same information from a set of source nodes $S = \{s_1, s_2, \cdots, s_n\}$ to a set of target nodes $T = \{t_1, t_2, \cdots, t_m\}$. In order to derive expressions for the link costs, we consider 4 distinct cases:

1 *Point-to-Point Link:* $n = 1, m = 1$: In this case, only one node is transmitting within a time slot to a single target node.

2 *Point-to-Multi-Point, Broadcast Link:* $n = 1, m > 1$: This type of link corresponds to the broadcast mode introduced in the last section. In this case, a single node is transmitting to multiple target nodes.

3 *Multi-Point-to-Point, Cooperative Link:* $n > 1, m = 1$: This type of link corresponds to the cooperative mode introduced in the last section. In this case, multiple nodes cooperate to transmit the same information to a single receiver node. We will assume that coherent reception, i.e. the transmitters are able to adjust their phases so that all signals arrive in phase at the receiver. In this case, the signals simply add up at the receiver and complete decoding is possible as long as the received SNR is above the minimum threshold SNR_{min}. Here, we do not address the feasibility of precise phase synchronization. The reader is referred to [9] for a discussion of mechanisms for achieving this level of synchronization.

4 *Multi-Point-to-Multi-Point Link:* $n > 1, m > 1$: This is not a valid option under our assumptions, as synchronizing transmissions for coherent reception at multiple receivers is not feasible. Therefore, we will not be considering this case.

***Point-to-Point Link:* $\mathbf{n = 1, m = 1}$.** In this case, $S = \{s_1\}$ and $T = \{t_1\}$. The channel parameters may be simply denoted by α and θ, and the transmitted signal is controlled through the scaling factor w. Although in general the scaling factor is a complex value, absorbing both power and phase adjustment by the transmitter, in this case we can ignore the phase as there is only a single receiver. The model assumptions made in Section 2 imply that the received signal is simply:

$$r(t) = \alpha e^{j\theta} w\phi(t) + \eta(t)\cdot$$

where $\phi(t)$ is the unit-power transmitted signal and $\eta(t)$ is the receiver noise with power P_η. The total transmitted power is $P_T = |w|^2$ and the SNR ratio at the receiver is $\frac{\alpha^2|w|^2}{P_\eta}$. For complete decoding at the receiver, the SNR must be above the threshold value SNR_{min}. Therefore the minimum power required, $\hat{P_T}$, and hence the point-to-point link cost $LC(s_1, t_1)$, is given by:

$$LC(s_1, t_1) \equiv \hat{P_T} = \frac{SNR_{min} P_\eta}{\alpha^2}\cdot \quad (1.1)$$

In equation 1.1, the point-to-point link cost is proportional to $\frac{1}{\alpha^2}$, which is the power attenuation in the wireless channel between $\mathsf{s_1}$ and $\mathsf{t_1}$, and therefore is proportional to the square of the distance between $\mathsf{s_1}$ and $\mathsf{t_1}$ under our propagation model.

Point-to-Multi-Point, Broadcast Link: $\mathbf{n = 1, m > 1.}$ In this case, $\mathsf{S} = \{\mathsf{s_1}\}$ and $\mathsf{T} = \{\mathsf{t_1}, \mathsf{t_2}, \cdots, \mathsf{t_m}\}$, hence m simultaneous SNR constraints must be satisfied at the receivers. Assuming that omni-directional antennas are being used, the signal transmitted by node $\mathsf{s_1}$ is received by all nodes within a transmission radius proportional to the transmission power. Hence, a broadcast link can be treated as a set of point-to-point links and the cost of reaching a set of node is the maximum over the costs for reaching each of the nodes in the target set. Thus the minimum power required for the broadcast transmission, denoted by $\mathsf{LC}(\mathsf{s_1}, \mathsf{T})$, is given by:

$$\mathsf{LC}(\mathsf{s}, \mathsf{T}) = \max\{\mathsf{LC}(\mathsf{s_1}, \mathsf{t_1}), \mathsf{LC}(\mathsf{s_1}, \mathsf{t_2}), \cdots, \mathsf{LC}(\mathsf{s_1}, \mathsf{t_m})\}\cdot \tag{1.2}$$

Multi-Point-to-Point, Cooperative Link: $\mathbf{n > 1, m = 1.}$ In this case $\mathsf{S} = \{\mathsf{s_1}, \mathsf{s_2}, \cdots, \mathsf{s_n}\}$ and $\mathsf{T} = \{\mathsf{t_1}\}$. We assume that the n transmitters are able to adjust their phases in such a way that the signal at the receiver is:

$$\mathsf{r}(\mathsf{t}) = \sum_{\mathsf{i}}^{\mathsf{n}} \alpha_{\mathsf{i1}} |\mathsf{w_i}| \phi(\mathsf{t}) + \eta(\mathsf{t})\cdot$$

The total transmitted power is $\sum_{\mathsf{i=1}}^{\mathsf{n}} |\mathsf{w_i}|^2$ and the received signal power is $|\sum_{\mathsf{i=1}}^{\mathsf{n}} \mathsf{w_i} \alpha_{\mathsf{i1}}|^2$. The power allocation problem for this case is simply

$$\begin{aligned} \min \quad & \sum_{\mathsf{i=1}}^{\mathsf{n}} |\mathsf{w_i}|^2 \\ \text{s.t.} \quad & \frac{|\sum_{\mathsf{i=1}}^{\mathsf{n}} \mathsf{w_i} \alpha_{\mathsf{i1}}|^2}{\mathsf{P}_\eta} \geq \mathsf{SNR}_{\mathsf{min}}\cdot \end{aligned} \tag{1.3}$$

Lagrangian multiplier techniques may be used to solve the constrained optimization problem above. The resulting optimal allocation for each node i is given by

$$|\hat{\mathsf{w}}_{\mathsf{i}}| = \frac{\alpha_{\mathsf{i1}}}{\sum_{\mathsf{i}}^{\mathsf{n}} \alpha_{\mathsf{i1}}^2} \sqrt{\mathsf{SNR}_{\mathsf{min}} \mathsf{P}_\eta}\cdot \tag{1.4}$$

The resulting cooperative link cost $\mathsf{LC}(\mathsf{S}, \mathsf{t}_1)$, defined as the optimal total power, is therefore given by

$$\begin{aligned} \mathsf{LC}(\mathsf{S}, \mathsf{t}_1) &= \hat{\mathsf{P}}_\mathsf{T} \\ &= \sum_{\mathsf{i}=1}^{\mathsf{n}} |\hat{\mathsf{w}}_\mathsf{i}|^2 \\ &= \frac{1}{\sum_{\mathsf{i}=1}^{\mathsf{n}} \frac{\alpha_{\mathsf{i}1}^2}{\mathsf{SNR}_{\mathsf{min}} \mathsf{P}_\eta}}. \end{aligned} \tag{1.5}$$

It is easy to see that it can be written in terms of the point-to-point link costs between all the source nodes and the target nodes (see Equation 1.1) as follows:

$$\mathsf{LC}(\mathsf{S}, \mathsf{t}_1) = \frac{1}{\frac{1}{\mathsf{LC}(\mathsf{s}_1,\mathsf{t}_1)} + \frac{1}{\mathsf{LC}(\mathsf{s}_2,\mathsf{t}_2)} + \cdots + \frac{1}{\mathsf{LC}(\mathsf{s}_\mathsf{n},\mathsf{t}_1)}}. \tag{1.6}$$

A few observations are worth mentioning here. First, based on equation 1.4, the transmitted signal level is proportional to the channel attenuation. Therefore, in the cooperative mode *all* nodes in the reliable set cooperate to send the information to a single receiver. In addition, based on equation 1.6, the cooperative cost is smaller than each point-to-point cost. This conclusion is intuitively plausible and is a proof on the energy saving due to the *Wireless Cooperative Advantage*.

Optimal Cooperative Route Selection

The problem of finding the optimal cooperative route from the source node s to the destination node d, formulated in Section 2, can be mapped to a Dynamic Programming (DP) problem. The state of the system at stage k is the reliable set S_k, i.e. the set of nodes that have completely received the information by the k^{th} transmission slot. The initial state S_0 is simply $\{\mathsf{s}\}$, and the termination states are all sets that contain d. The decision variable at the k^{th} stage is U_k, the set of nodes that will be added to the reliable set in the next transmission slot. The dynamical system evolves as follows:

$$\mathsf{S}_{\mathsf{k}+1} = \mathsf{S}_\mathsf{k} \cup \mathsf{U}_\mathsf{k} \quad \mathsf{k} = 1, 2, \cdots \tag{1.7}$$

The objective is to find a sequence $\{\mathsf{U}_\mathsf{k}\}$ or alternatively $\{\mathsf{S}_\mathsf{k}\}$ so as to minimize the total transmitted power P_T, where

$$\mathsf{P}_\mathsf{T} = \sum_\mathsf{k} \mathsf{LC}(\mathsf{S}_\mathsf{k}, \mathsf{U}_\mathsf{k}) = \sum_\mathsf{k} \mathsf{LC}(\mathsf{S}_\mathsf{k}, \mathsf{S}_{\mathsf{k}+1} - \mathsf{S}_\mathsf{k}). \tag{1.8}$$

We will refer to the solution to this problem as the optimal transmission policy. The optimal transmission policy can be mapped to finding the shortest path in the state space of this dynamical system. The state space can be represented by as graph with all possible states, i.e. all possible subsets of nodes in the network, as its nodes. We refer to this graph as the *Cooperation Graph*. Figure 1.6 show the cooperation graph corresponding to the 4-node network shown in Figure 1.1.

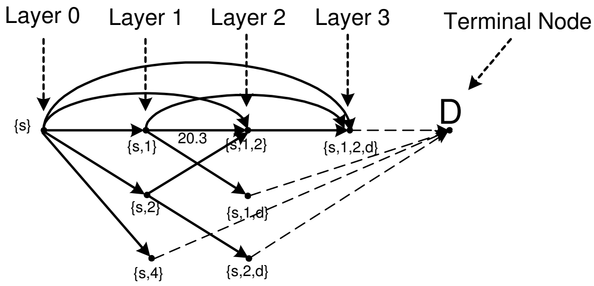

Figure 1.4. Cooperation Graph for a 4-Node Network

Nodes in the cooperation graph are connected with arcs representing the possible transitions between states. As the network nodes are allowed only to either fully cooperate or broadcast, the graph has a special layered structure as illustrated by Figure 1.6. All nodes in the k^{th} layer are of size $k+1$, and a network with $n+1$ nodes the cooperation graph has n layers, and the k^{th} layer has $\binom{n}{k}$ nodes. Arcs between nodes in adjacent layers correspond to cooperative links, whereas broadcast links are shown by cross-layer arcs. The costs on the arcs are the link costs defined in Section 2.0. All terminal states are connected to a single artificial terminal state, denoted by D, by a zero-cost arc. The optimal transmission policy is simply the shortest path between nodes s and D. There are 2^n nodes in the cooperation graph for a network with $n+1$ nodes. Therefore standard shortest path algorithms will in general have a complexity of $O(2^{2n})$. However, by taking advantage of some special properties of the cooperation graph, we are able to come up with an algorithm with complexity reduced to $O(n2^n)$. This algorithm is based on scanning the cooperation graph from left to right and constructing the shortest path to each nodes at the k^{th} layer based on the shortest path to nodes in the pervious layers. The *Sequential Scanning Algorithm* is outlined below.

Sequential Scanning Algorithm This is the algorithm for finding the optimal cooperative route in an arbitrary network based on finding the shortest path in the corresponding cooperation graph.

Initialize Initialize the cooperation graph data structure. Initialize the layer counter k to $\mathsf{k} = 1$.

Repeat Construct to the shortest path to all nodes at the k^{th} layer based on the shortest path to all nodes in the previous layers. Increment the counter.

Stop Stop when D is reached. i.e. when $\mathsf{k} = \mathsf{n} + 1$.

For a network with $\mathsf{n} + 1$ nodes, the main loop in this algorithm is repeated n times and at the k^{th} stage the shortest path to $\binom{n}{k}$ nodes must be calculated. This operation has a complexity of order $\mathsf{O}(2^{\mathsf{n}})$, hence finding the optimal route is of complexity $\mathsf{O}(\mathsf{n}2^{\mathsf{n}})$.

Although the *Sequential Scanning Algorithm* substantially reduces the complexity for finding the optimal cooperative route in an arbitrary network, its complexity is still exponential in the number of nodes in the wireless network. For this reason, finding the optimal cooperative route in an arbitrary network becomes computationally intractable for larger networks. We will focus on developing computationally simpler and relatively efficient heuristics and on assessing their performance through simulation.

Example

Having developed the necessary mathematical tools, we now present a simple example that illustrates the benefit of cooperative routing. Figure 1.5 shows a simple network with 4 nodes. The arcs represent links and the arc labels are point-to-point link costs. The diagrams below show the six possible routes, P_0 through P_5. P_0 corresponds to a simple 2-hop, non-cooperative minimum energy path between s and d. P_1, P_2, and P_3 are 2-hop cooperative routes, whereas P_4 and P_5 are 3-hop cooperative routes. Figure 1.6 shows the corresponding cooperation graph for this network. Each transmission policy corresponds to a distinct path between $\{\mathsf{s}\}$ and D in this graph and the minimum energy policy of P_3 corresponds to the shortest path. Table 1.1 lists the costs of the six policies.

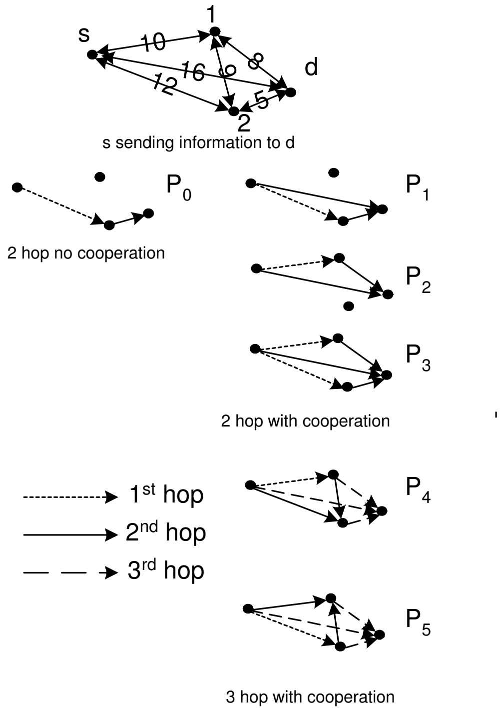

Figure 1.5. 4-Node Network Example

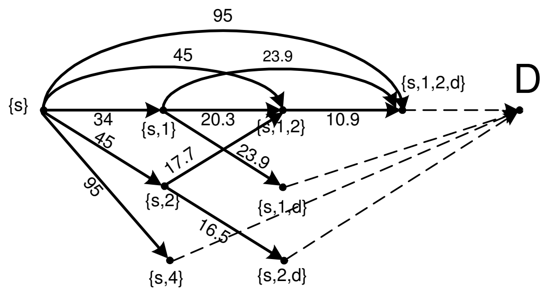

Figure 1.6. 4-Node Cooperation Graph

No.	Policy	Cost
P_0	$NonCooperative$	65
P_1	$(\{s\},\{s,2\},\{s,2,d\})$	≈ 61.5
P_2	$(\{s\},\{s,1\},\{s,1,d\})$	≈ 57.9
P_3	$(\{s\},\{s,1,2\},\{s,1,2,d\})$	≈ 55.9
P_4	$(\{s\},\{s,2\},\{s,1,2\},\{s,1,2,d\})$	≈ 73.6
P_5	$(\{s\},\{s,1\},\{s,1,2\},\{s,1,2,d\})$	≈ 65.2

Table 1.1. Transmission Policies for Figure 1.5

3. Analytical Results for Line and Grid Topologies

In this section, we develop analytical results for achievable energy savings in line and grid networks. In particular, we consider a *Regular Line* Topology (see Figure 1.7) and a *Regular Grid* Topology (see Figure 1.8) where nodes are equi-distant from each other. Before proceeding further, let us define precisely what we mean by energy savings for a cooperative routing strategy relative to the optimal non-cooperative strategy:

$$\mathsf{Savings} = \frac{\mathsf{P_T(Non-cooperative) - P_T(Cooperative)}}{\mathsf{P_T(Non-cooperative)}}. \tag{1.9}$$

where $\mathsf{P_T(strategy)}$ denotes the total transmission power for the strategy.

Line Network-Analysis

Figure 1.7) shows a regular line where nodes are located at unit distance from each other on a straight line. In our proposed scheme, we restrict the cooperation to nodes along the optimal non-cooperative route. That is, at each transmission slot, all nodes that have received the information cooperate to send the information to the next node along the minimum energy non-cooperative route. This cooperation strategy is referred to as the CAN (*Cooperation Along the Minimum Energy Non-Cooperative Path*) strategy.

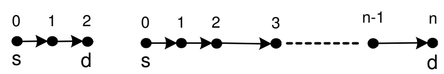

Figure 1.7. Regular Line Topology

For the 3-node line network in Figure 1.7, it is easy to show that the optimal non-cooperative routing strategy is to relay the information through the middle node. Since a longer line network can be broken down into short 2-hop components, it is clear that the optimal non-cooperative routing strategy is to always send the information to the next nearest node in the direction of the destination until the destination node is reached. From Equation 1.1, the link cost for every stage is $\frac{\mathsf{SNR_{min}P_\eta}}{\alpha^2}$, where α is the magnitude attenuation between two adjacent nodes 1-distance unit apart. Under our assumptions, α^2 is proportional to the inverse of the distance squared. Therefore,

$$\mathsf{P_T(Non-cooperative)} = \mathsf{n}\frac{\mathsf{SNR_{min}P_\eta}}{\alpha^2}. \tag{1.10}$$

With the CAN strategy, after the $\mathsf{m^{th}}$ transmission slot, the reliable set is $\mathsf{S_m} = \{\mathsf{s}, 1, \cdots, \mathsf{m}\}$, and the link cost associated with the nodes in $\mathsf{S_m}$ cooperating to send the information to the next node $(\mathsf{m}+1)$ follows from Equation 1.6 and is given by

$$\mathsf{LC(S_m, m+1)} = \frac{\mathsf{SNR_{min}P_\eta}}{\sum_{\mathsf{i}=1}^{\mathsf{m}+1}\frac{\alpha^2}{\mathsf{i}^2}}. \tag{1.11}$$

Therefore, the total transmission power for the CAN strategy is

$$\begin{aligned} \mathsf{P_T(CAN)} &= \sum_{\mathsf{m}=0}^{\mathsf{n}-1}\mathsf{LC(S_m, m+1)} \\ &= \frac{\mathsf{SNR_{min}P_\eta}}{\alpha^2}\sum_{\mathsf{m}=0}^{\mathsf{n}-1}\frac{1}{\mathsf{C(m+1)}}, \qquad (1.12) \\ &\text{where } \mathsf{C(m)} = \sum_{\mathsf{i}=1}^{\mathsf{m}}\frac{1}{\mathsf{i}^2}. \qquad (1.13) \end{aligned}$$

Before moving to find the savings achieved by *CAN* in a line, we need to proves the following simple lemma regarding the existence of the average of terms for a decreasing sequence.

LEMMA 1.1 *Let* $\mathsf{a_n}$ *be a decreasing sequence with a finite limit* c*, then* $\lim_{\mathsf{m}\to\infty}\frac{\sum_{\mathsf{n}=1}^{\mathsf{m}}\mathsf{a_n}}{\mathsf{m}} = \mathsf{c}$.

Proof: For any value of m, let $\mathsf{m_0}$ be an arbitrary integer less than m:

$$\begin{aligned}
\lim_{m\to\infty} \frac{\sum_{n=1}^{m} a_n}{m} &= \lim_{m\to\infty} \frac{1}{m}\left(\sum_{n=1}^{m_0} a_n + \sum_{n=m_0+1}^{m} a_n\right) \\
&= \lim_{m\to\infty} \frac{1}{m}\sum_{n=1}^{m_0} a_n + \lim_{m\to\infty} \frac{1}{m}\sum_{n=m_0+1}^{m} a_n \\
&= 0 + \lim_{m\to\infty} \frac{1}{m}\frac{m-(m_0+1)}{m-(m_0+1)}\sum_{n=m_0+1}^{m} a_n \\
&= \lim_{m\to\infty} \frac{m-(m_0+1)}{m}\frac{1}{m-(m_0+1)}\sum_{n=m_0+1}^{m} a_n \\
&= \lim_{m\to\infty} \frac{m-(m_0+1)}{m}\lim_{m\to\infty}\frac{1}{m-(m_0+1)}\sum_{n=m_0+1}^{m} a_n \\
&= \lim_{m\to\infty} \frac{1}{m-(m_0+1)}\sum_{n=m_0+1}^{m} a_n \,.
\end{aligned} \tag{1.14}$$

Since $\mathsf{a_n}$ is a decreasing sequence, all terms in the final sum are less than $\mathsf{a_{m_0}}$. Furthermore, $\lim_{n\to\infty} a_n = c$. So, all terms in the final sum are greater than c. Hence:

$$c \le \lim_{m\to\infty} \frac{\sum_{n=1}^{m} a_n}{m} = \lim_{m\to\infty} \frac{1}{m-(m_0+1)}\sum_{n=m_0+1}^{m} a_n \le a_{m_0}.$$

For increasing values of m, $\mathsf{m_0}$ may be chosen such that $\mathsf{a_{m_0}}$ is arbitrarily close to c and the proof is established.

THEOREM 1.2 *For a regular line network as shown in Figure 1.7, the CAN strategy results in energy savings of* $(1 - \frac{1}{n}\sum_{m=1}^{n}\frac{1}{C(m)})$. *As the number of nodes in the network grows, the energy savings value approaches* $(1 - \frac{6}{\pi^2}) \approx 39\%$.

Proof: The minimum energy non-cooperative routing a regular line network with n hops has cost equal to n. The cost of the optimal cooperation scheme, i.e. the CAN strategy, is:

$$P_T(\text{Cooperative}) = \sum_{m=1}^{n} LC(\{s, \cdots, m-1\}, m) = \sum_{m=1}^{n} \frac{1}{C(m)} \tag{1.15}$$

where $\mathsf{C(m)}$ is defined by equation 1.13. The energy savings achieved, as defined by equation 1.11, is:

$$\mathsf{Savings(n)} = \frac{\mathsf{P_T(Non-Cooperative) - P_T(Cooperative)}}{\mathsf{P_T(Non-Cooperative)}} \quad (1.16)$$

$$= \frac{\mathsf{n} - \sum_{\mathsf{m=1}}^{\mathsf{n}} \frac{1}{\mathsf{C(m)}}}{\mathsf{n}} \quad (1.17)$$

$$= 1 - \frac{1}{\mathsf{n}} \sum_{\mathsf{m=1}}^{\mathsf{n}} \frac{1}{\mathsf{C(m)}} \quad (1.18)$$

$\frac{1}{\mathsf{C(m)}}$ is a decreasing sequence with limit of $\frac{6}{\pi^2}$. So, based on lemma 1.1 we have:

$$\lim_{\mathsf{n}\to\infty} \mathsf{Savings(n)} = 1 - \lim_{\mathsf{n}\to\infty} \frac{1}{\mathsf{n}} \sum_{\mathsf{m=1}}^{\mathsf{n}} \frac{1}{\mathsf{C(m)}} = 1 - \frac{6}{\pi^2} \quad (1.19)$$

This establishes the claim and completes the proof.

Grid Network

Figure 1.8 shows a regular $\mathsf{n} \times \mathsf{n}$ grid topology with s and d located at opposite corners. A $\mathsf{n} \times \mathsf{n}$ grid can be decomposed into many 2×2 grid. Assuming that the nodes are located at a unit distance from each other, in a 2×2 grid, a diagonal transmission has a cost of 2 units, equal to the cost of one horizontal and one vertical transmission. For this reason, in an $\mathsf{n} \times \mathsf{n}$ grid there are many non-cooperative routes with equal cost. Figure 1.8 shows two such routes for an $\mathsf{n} \times \mathsf{n}$ grid.

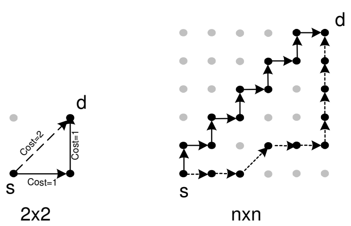

Figure 1.8. Regular Grid Topology

The minimum-energy non-cooperative route is obtained by a stair-like policy (illustrated in Figure 1.8), and its total power is $\mathsf{2n}$. We will base our analysis for deriving the bound for saving based on this stair-like non-cooperative path. The following theorem stated the energy savings achieved by the CAN strategy applied to this non-cooperative route.

THEOREM 1.3 *For a regular grid network as shown in Figure 1.8, the energy savings achieved by using the CAN strategy approaches* 56% *for large networks.*

Proof: Figure 1.9 shows an intermediate step in routing the information is a regular grid. At this stage, all the nodes with a darker shade, nodes 1 through 8, have received the information. In the next step, the information must be relayed to node 9. The cooperative cost of this stage is

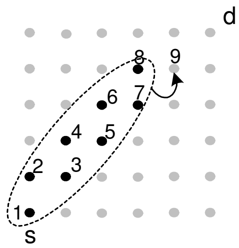

Figure 1.9. Cooperative Routing in a Grid Topology

$$LC(\{1,\cdots,8\},9) = \frac{1}{\sum_{i=1}^{8}\frac{1}{LC(i,9)}}$$

$$= \frac{1}{\frac{1}{1}+\frac{1}{2}+\frac{1}{5}+\frac{1}{8}+\frac{1}{13}+\frac{1}{18}+\frac{1}{25}+\frac{1}{32}} \quad (1.20)$$

$$= \frac{1}{\underbrace{\frac{1}{1}+\frac{1}{5}+\frac{1}{13}+\frac{1}{25}}+\underbrace{\frac{1}{2}+\frac{1}{8}+\frac{1}{18}+\frac{1}{32}}} \quad (1.21)$$

In general, the cooperative cost of the m^{th} stage of the proposed strategy is

$$\mathsf{C_{grid}}(\mathsf{m}) = \mathsf{LC}(\{1,\cdots,\mathsf{m}\},\mathsf{m}+1) = \frac{1}{\sum_{\mathsf{i}=1}^{\mathsf{m}}\frac{1}{\mathsf{LC}(\mathsf{i},\mathsf{m})}} \quad (1.22)$$

It is not too hard to see that the point-to-point costs have the following form

$$\mathsf{LC}(\mathsf{i},\mathsf{m}) = \left(\left\lceil\frac{\mathsf{m}-\mathsf{i}}{2}\right\rceil\right)^2 + \left(\left\lfloor\frac{\mathsf{m}-\mathsf{i}}{2}\right\rfloor\right)^2 \quad (1.23)$$

Using Equation 1.23, Equation 1.22 can be written as

$$\begin{aligned} C_{grid}(m) &= \frac{1}{\sum_{i=1}^{m} \frac{1}{LC(i,m)}} \\ &= \frac{1}{\sum_{i=1}^{m} \frac{1}{(\lceil \frac{m-i}{2} \rceil)^2 + (\lfloor \frac{m-i}{2} \rfloor)^2}} \\ &= \frac{1}{\sum_{k=1}^{\lceil \frac{m}{2} \rceil} \frac{1}{2k^2-2k+1} + \sum_{k=1}^{\lfloor \frac{m}{2} \rfloor} \frac{1}{2k^2}} \qquad (1.24) \end{aligned}$$

Comparing Equation 1.21 and Equation 1.24, it is easy to see that the first group of terms is generated by the first sum term and the second group is generated by the second sum term. $C_{grid}(m)$ is a decreasing sequence of numbers and can be shown, using Maple, to have a limit equal to 0.44.

The total cost for the cooperative route in an $n \times n$ grid is

$$P_T(\text{Cooperative}) = \sum_{m=1}^{2n} C_{grid}(m) \qquad (1.25)$$

The energy saving, as defined by equation 1.9, is

$$\begin{aligned} \text{Savings}(n) &= \frac{P_T(\text{Non}-\text{Cooperative}) - P_T(\text{Cooperative})}{P_T(\text{Non}-\text{Cooperative})} \\ &= \frac{2n - \sum_{m=1}^{2n} C_{grid}(m)}{2n} \\ &= 1 - \frac{1}{2n} \sum_{m=1}^{2n} C_{grid}(m) \qquad (1.26) \end{aligned}$$

Since $C_{grid}(m)$ is a decreasing sequence and $\lim_{m\to\infty} C_{grid}(m) = 0.44$, by lemma 1.1, the savings in the case of a regular grid, as calculated in equation 1.26, approaches $1 - 0.44 = 56\%$. This establishes the claim and completes the proof for the lower bound of achievable savings in a regular grid.

4. Heuristics & Simulation Results

We present two possible general heuristic schemes and related simulation results. The simulations are over a network generated by randomly placing nodes on an 100×100 grid and randomly choosing a pair of nodes to be the source and destination. For each realization, the minimum energy non-cooperative path was found. Also, the proposed heuristic were used to find co-operative paths. The performance results reported are the energy savings of the resulting strategy with respect to the optimal non-cooperative path averaged over $100,000$ simulation runs.

The two heuristics analyzed are outlined below.

CAN-L Heuristic **C***ooperation* **A***long the* **N***on-Cooperative Optimal Route*:

This heuristic is based on the CAN strategy described Section 3. CAN-L is a variant of CAN as it limits the number of nodes allowed to participate in the cooperative transmission to L. In particular, these nodes are chosen to be the last L nodes along the minimum energy non-cooperative path. As mentioned before, in each step the last L nodes cooperate to transmit the information to the next node along the optimal non-cooperative path. The only processing needed in this class of algorithm is to find the optimal non-cooperative route. For this reason, the complexity of this class of algorithms is the same as finding the optimal non-cooperative path in a network or $\mathsf{O}(\mathsf{N}^2)$.

PC-L Heuristic **P***rogressive* **C***ooperation*:

Initialize Initialize *B*est Path to the optimal non-cooperative route. Initialize the *S*uper Node to contain only the source node.

Repeat Send the information to the first node along the current *B*est Path. Update the *S*uper Node to include all past L nodes along the current *B*est Path. Update the link costs accordingly, i.e. by considering the *S*uper Node as a single node and by using equation 1.6. Compute the optimal non-cooperative route for the new network/graph and update the *B*est Path accordingly.

Stop Stop as soon as the destination node receives the information.

For example, with $\mathsf{L} = 3$, this algorithm always combines the last 3 nodes along the current *B*est Route into a single node, finds the shortest path from that combined node to the destination and send the information to the next node along that route. This algorithm turns out to have a complexity of $\mathsf{O}(\mathsf{N}^3)$ since the main loop is repeated $\mathsf{O}(\mathsf{N})$ times and each repetition has a complexity of $\mathsf{O}(\mathsf{N}^2)$.

A variant of this algorithm keeps a window W of the most recent nodes, and in each step all subsets of size L among the last W nodes are examined and the path with the least cost is chosen. This variant has a complexity of $\mathsf{O}\left(\begin{pmatrix} W \\ L \end{pmatrix} \times \mathsf{N}^3\right)$, where W is the window size. We refer to this variant as *Progressive Cooperation with Window.*

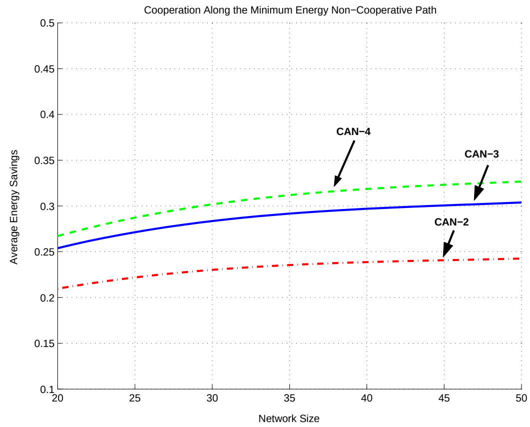

Figure 1.10. Performance of CAN

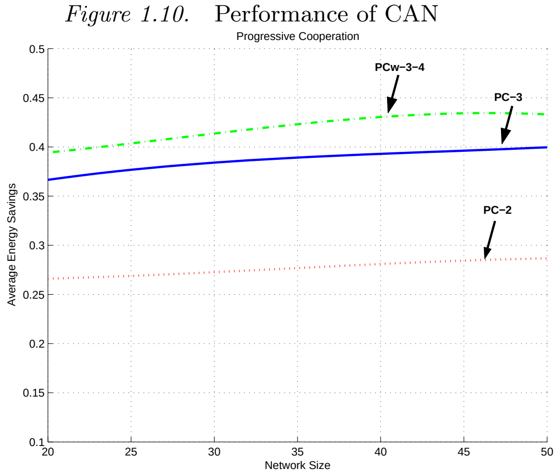

Figure 1.11. Performance of PC

Figures 1.10 and 1.11 show average energy savings ranging from 20% to 50% for CAN and PC algorithms. It can be seen that PC-2 performs almost as well as CAN-3 and PC-3 performs much between than CAN-4. This show that the method for approximating the optimal route is very important factor in increasing the savings. Figures 1.12 compares CAN, PC, and PC-W on the same chart. It is seen that PC-3-4 performs better than PC-3, which performs substantially better than CAN-4. In general, it can be seen that the energy savings increase with L, and that improvements in savings are smaller for larger values of L. As there is a trade-off between the algorithm complexity and the algorithm performance, these simulation results indicate that it would be reasonable to chose L to be around 3 or 4 for both the CAN and PC heuristics.

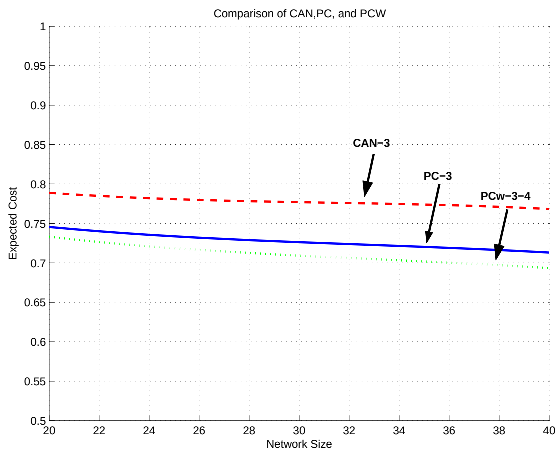

Figure 1.12. Comparison

5. Conclusions

In this chapter we formulated the problem of finding the minimum energy cooperative route for a wireless network under idealized channel and receiver models. Our main assumption were that the channel states are known at the transmitter and precise power and phase control, to achieve coherent reception is possible. We focused on the optimal transmission of a single message from a source to destination through sets of nodes, that may act as cooperating relays. Fundamental to the understanding of the routing problem was the understanding of the optimal power allocation for a single message transmission from a set of source nodes to a set of destination nodes. We presented solutions to this problem, and used these as the basis for solving the minimum energy cooperative routing problem. We used Dynamic Programming (DP) to formulate the cooperative routing problem as a multi-stage decision problem. However, general shortest algorithms are not computationally tractable and are not appropriate for large networks. For a Regular Grid Topology and a Regular Grid Topology, we analytically obtained the energy savings due to cooperative transmission, demonstrating the benefits of the proposed cooperative routing scheme. For general topologies, we proposed two heuristics and showed significant energy savings (close to 50%) based on simulation results.

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