Slovenian 'Dopuščati' and the Semantics of Epistemic Modals[*]

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There has been much recent interest in the analysis and distribution of embedded epistemic modals (Yalcin 2007, Anand and Hacquard 2013, a.o.). We present novel data using the embedding verb *dopuščati* ('to allow for the possibility that') from Slovenian, analysed as an existential doxastic attitude, and argue for a new analysis of epistemic modals that captures their restricted distribution under doxastic attitudes.

1 Introduction

Suppose you wake up late one morning. It's already bright outside but you are too lazy to open your eyes. You could entertain the following two thoughts about the light:

(1) a. Mislim, da utegne biti zunaj sončno.
        I.think that might be outside sunny
        'I think it might be sunny outside.'

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b. Mislim,   da   mora   biti zunaj   sončno.
    I.think   that   must   be   outside sunny
    'I think it must be sunny outside.'

As noted previously (Stephenson 2007, a.o.), doxastic attitudes like *think* can embed two kinds of modal verbs: possibility epistemic modals (e.g. *might*) and necessity epistemic modals (e.g. *must*).[1] The same facts hold in Slovenian, as illustrated above. Put differently then, universal doxastic quantifiers can embed existential as well as universal epistemic modals.

In addition to verbs like *misliti* ('think') or *verjeti* ('believe'), Slovenian has a weaker doxastic verb *dopuščati* ('to allow for the possibility'). The difference that I explore in this paper is the contrast between (1) above and (2) below, where an embedded necessity modal is odd.[2]

(2) a. Dopuščam, da   utegne biti sončno.
         I.allow   that might   be   sunny
         'I allow for the possibility that it might be sunny.'
    b. #Dopuščam, da   mora biti sončno.
         I.allow   that must be   sunny
         'I allow for the possibility that it must be sunny.'

This data yields the generalization that (in a situation where the evidence under consideration is the attitude holder's) it is odd to combine a strong embedded modal with a weak doxastic attitude, while the other three combinations are acceptable. This idea, that existential doxastic attitudes can only embed existential (and not universal) epistemic modals, is not new (see Anand & Hacquard 2013), but it has so far only been discussed in the context of attitude verbs involving preferences (*hope* and *fear*) or negative orientation (*doubt*). Since *dopuščati* involves neither, it delimits the space of possible analyses in an important way.

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[1] "Epistemic modal" is used for modals when under the epistemic construal.
[2] I use '#' descriptively, to signal that a sentence is odd in the given context (without making claims about grammaticality or interpretability in other contexts). The symbols '??' and '?' are used for weaker oddnesses.

The paper has two goals. The first one is empirical: to discuss *dopuščati* and the formally relevant data in a non-technical way (§2). This section can hopefully be of use to a broader audience. The second goal (§3-4) is to propose that (2b) is odd because it contextually expresses the same proposition as (1b), using a weak constituent (*dopuščati*) in comparison. The challenge lies in re-thinking epistemic modals and doxastic attitudes (§3) so that such an equivalence does not arise between (2a) and (1a).

2   Embedding under Doxastic Attitudes

I first discuss some properties of *dopuščati* and argue that it is a weak doxastic attitude (§2.1). Building on the data in (1) and (2), I show that *dopuščati* cannot embed epistemic necessity modals or their equivalents (negated possibility modals) (§2.2). Section §2.3 discusses matrix clause negation with doxastic attitudes – embedded necessity under *don't think* and negated *dopuščati* is odd. Finally, *dopuščati* reveals an important difference in acceptability between embedding epistemic modals and embedding doxastic attitudes (§2.4).

2.1   What it Means to 'Dopuščati'
Consider (3). The sentence conveys that Othello considers it possible that Desdemona loves Cassio, but he leaves it open as to whether or not she actually does. That is, the proposition that Desdemona loves Cassio is consistent with Othello's beliefs, but he is understood to not have made up his mind as to whether he should believe it.

(3) Othello dopušča, da   Desdemona ljubi Cassija.
     Othello allows   that Desdemona loves Cassio
     'Othello allows for the possibility that Desdemona loves Cassio.'

Below are some natural occurring examples to the same point. Example (4c) illustrates that *dopuščati* can appear with the (always optional) noun *možnost* ('possibility').[3]

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[3] The Russian cognate *dopuskat'*, which is also used as a weak doxastic attitude, does not seem to do this as naturally. The Russian National Corpus (http://www.ruscorpora. ru/en/index.html, last accessed in May 2017) contains 406 tokens of *dopuskaju* ('I allow') immediately followed by a *čto* ('that') clause, compared to 20 tokens of

(4) a. Dopuščam da je vaša laž posledica   neznanja in ne
         I.allow   that is your lie consequence   ignorance and not
         zlonamernosti.
         malevolence.
         'I allow for the possibility that your lie follows from ignorance
         and not malevolence.' (web)
    b. To je   seveda   le moje mnenje, nikakor ga ne
         this is   of.course only my   opinion in.no.way it not
         vsiljujem, tudi dopuščam, da se motim.
         I.impose   also I.allow   that refl I.err
         'This is of course only my opinion, I definitely do not impose it,
         I even allow for the possibility that I'm wrong.' (web)
    c. Tusk dopušča možnost,   da brexita ne bo.
         Tusk allows   possibility that Brexit not will.be
         'Tusk allows for the possibility that there will be no Brexit.'
         (web)

In English, some speakers use the verb *allow* (without *for the possibility*)
in a related way, as in (5).[4] The difference, however, is that *allow* is more
"discursive" (used, for example, to admit something to be true for the
sake of the argument). *Dopuščati*, on the other hand, carries no such
implication – it is used merely to report on your mental state.[5]

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dopuskaju immediately followed by a noun, of which 1 is 'possibility' and 19 are
'thought' (Rafael Abramovitz, p.c.).

[4] Thanks to Matt Mandelkern (p.c.) for first pointing this out to me.

[5] A reviewer notes that the Czech *připouštět* seems to mean *concede* (not allowing for
a possibility before but allowing for it now) rather than *allow for the possibility*. In
Slovenian, this is not the case for *dopuščati*, but it is worth pointing out that *dopustiti*
(its perfective version) is used like this. What is curious is that the Czech version,
which is said to embody this change of state, looks imperfective in the examples
provided by the reviewer.

There is in fact other micro-variation between the cognates of *dopuščati* in Slavic that I
will not have the space to explore. For example, a speaker of Bulgarian and a speaker
of Ukranian have pointed out to me that they are unable to do (6a), which suggests that
their version of *dopuščati* might be somewhat stronger (at least when the embedded
proposition is not modal). On the other hand, a speaker of Serbian and an anonymous
reviewer of Polish report that they cannot find equivalents of *dopuščati* in their
languages/dialects.

(5) a.   Othello allows that Desdemona might love Cassio.
    b.   I'll allow that I'm wrong.

Unlike vanilla doxastic attitude (*think*/*believe*), *dopuščati* is **weak** in the sense that one can *dopuščati* something as well as its opposite, as in (6a).

(6) a.   Dopuščam, da   je notri,   in   dopuščam, da je zunaj.
         I.allow   that is inside   and I.allow   that is outside
         'I allow that he's inside and I allow that he's outside.'
    b. #Mislim,   da   je notri,   in   mislim, da je zunaj.
         I.think   that   is inside   and I.think that is outside
         'I think that he's inside and I think that he's outside.'

The relationship between *dopuščati* and attitudes like *think/believe* resembles that of *some* compared to *all*.[6] In (7), we see that a *dopuščati* claim can be strengthened into a belief claim (cf. *some students passed the exam, in fact all of them did*).

(7) In a debate with Flat-Earthers, a scientist is asked: *Ali dopuščate, da je Zemlja okrogla?* (Do you allow for the possibility that the Earth is round?) The scientist replies:
    Seveda   dopuščam, da je – trdno   verjamem, da je!
    of.course I.allow   that is   firmly believe   that is
    'Of course I allow for the possibility that it is – I firmly believe it!'

The reason why one might think that *dopuščati* talks about something that is consistent with our **belief**s, rather than knowledge, is that it is commonly assumed that there is no such thing as false knowledge. There are, however, false *dopuščati* states:

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[6] The analogy with *some* and *all* runs into trouble with the example below, which is not odd (contradictory). There is plausibly, however, a contextual shift involved – I think x but I allow for the possibility that I'm wrong in which case not x. Alternatively, *think*/*believe* are in fact weaker than usually assumed (Hawthorne et al. 2016), in which case the analogy might be closer to, for example, *some* and *most*.
(i) Marija misli/verjame, da   je notri, ampak dopušča, da   je zunaj.
     Mary thinks/believes that is inside but   allows   that is outside
     'Mary thinks/believes he's inside but allows for the possibility that he's outside.'

(8) Dežuje,   ampak Janez ne dopušča, da dežuje.
     rains   but   John not allows   that rains
     'It's raining but John doesn't allow for the possibility that it's raining.'

In §3 I will assume that *dopuščati* is an existential quantifier over the doxastic (belief) state.

2.2   Embedding Epistemic Modals
Epistemic **possibility modals** can be embedded under strong doxastics like *misliti* or *think*, (1a), but also under weak ones like *dopuščati*, (2a). The examples in the introduction involved the verb *utegniti* but the same point can be made with an adverb like *mogoče* ('maybe'), as in (9a).

(9) Othello is asked whether he thinks that Desdemona is cheating on him. He replies:
    a. Dopuščam, da me (mogoče) vara.
       I.allow   that me maybe cheats.on
       'I allow for the possibility that she might be cheating on me.'
    b. Mislim,   da me mogoče vara.
       I.think   that me maybe cheats.on
       'I think she might be cheating on me.'

Is there a difference between (9a) and (9b)? The two are very close, but speakers report Othello to have perhaps some reason for suspecting Desdemona of cheating in (9b), while (9a) merely expresses that she might in principle be unfaithful. Speakers report the modal in (9a) to be somewhat redundant, i.e. not needed for conveying that she might in principle be unfaithful, but they note that its presence adds tentativeness.[7]

In contrast to possibility modals, epistemic **necessity modals** do not behave uniformly with respect to the strength of the embedding verb, as illustrated in (1b) and (2b). What leads to oddness in (2b), however, seems to be the embedding of a wide-scope necessity meaning. Consider:

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[7] The tentativeness effect seems to occur also with *must* under *think*. I leave it aside here.

(10)   Situation as in (1) and (2).
       #Dopuščam, da ne more deževati.
        I.allow   that not can   rain
       'I allow for the possibility that it can't be raining.'

On a fairly standard assumption, the force of a negated possibility modal, as in (10), equals that of a necessity modal with a negated complement. Given that (10) is odd, the culprit in (2b) is plausibly not *morati* ('must') per se, but the overall force in the embedded clause of *dopuščati*.[8]

2.3   Negated Doxastic Attitudes
Consider the following example:[9]

(11)   Situation: You, me, and John see Bob go home from work early. We sit down on some couches in front of Bob's office. John has his back turned to Bob's door. He puts on some headphones and starts cheating on the latest homework. After a while, Bob, who has a secret entry to his office, which he used to come back, creeps out of his office and comes up behind John's back. John, still immersed in cheating, does not notice this. I nudge you and whisper, with both of us staring at Bob:
       a.   John does not think that Bob might be behind his back.
       b.   ??John does not think that Bob must be behind his back.

This data is in line with Anand and Hacquard's (2013, fn. 27) finding for Romance that main clause negation makes the embedding of a necessity

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[8] It is difficult to find negated necessity modals to check whether those are acceptable, by analogy to (2a). Here is a potential candidate:
(i) Dopuščam možnost, da ni   nujno,   da sem prinesel "tako" hude
     I.allow   possibility that is.not necessary that aux brought such   bad
     poškodbe in bil zato   lahek plen MOMa [...]
     injuries   and aux therefore easy pray BPD
     'I allow for the possibility that it is not necessarily the case that having such bad wounds made me easy prey to BPD [...]' (web)
[9] Judgments in this section vary somewhat; I mark the most cheritable interpretation (e.g. "??" should be read as "?? or worse").

modal under a doxastic attitude degraded.[10,11] The same pattern occurs in Slovenian with ***misliti***, which is not neg-raising.

(12) Situation: as in (11).
　　a. Janez　ne　misli,　da je Bob mogoče za　　njegovim
　　　　John　 not thinks that is Bob maybe 　behind his
　　　　hrbtom.
　　　　back
　　　　'John does not think that Bob might be behind his back.'
　　b.??Janez　ne　misli,　da mora biti Bob za　　njegovim
　　　　John　 not thinks that must be　Bob behind his
　　　　hrbtom.
　　　　back
　　　　'John does not think that Bob must be behind his back.'

The situation in (11) is constructed so that the only evidence that the embedded epistemic can be felicitously sensitive to is the attitude holder's, i.e. John's (since you, me, and Bob know/see that Bob is behind John's back). The statements in (11b)/(12b) also express something that is intuitively true, cf. (13). While (13) may be a somewhat awkward way of putting it, it does not feel odd in the same way as (11b)/(12b) does.

(13) It's not the case that John thinks that Bob must be behind his back.

A possible fault with the scenario in (11) is that it gives John no reason for entertaining the thought that Bob must be behind his back.[12] Given the contrast between (11b)/(12b) and (13), it is unclear why this should play a role. Nevertheless, consider a different scenario:

(14) Situation: You and I have had the opportunity to work as assistants to Sherlock Holmes, who is investigating a recent murder.

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[10] See also Crnič (2014) and Ippolito (2017).
[11] Homer (2015) makes this observation for American English with (i) below. He notes that the British English *must* is acceptable under *don't think* and argues that this is because it can participate in double neg-raising (i.e. *think>must>not* in (ii) below).
(i) #*I don't think that John must*epis *be very intelligent*. (AmE)
(ii) *I don't think that John must*epis *be very intelligent*. (BrE)
[12] Thanks to Kai von Fintel (p.c.) for raising this issue.

Sherlock has taken an interest in the gardener and the butler. You and I are discussing what Sherlock thinks about who the murderer is. I say:
?Sherlock does not think that the gardener must be innocent (since he followed him around this morning).

Speakers still find something a bit odd about this example.[13] Here, however, Sherlock presumably thinks that the gardener might, or possibly must, be guilty. It should therefore be felicitous, given his behaviour, to deny that he thinks that the gardener must be innocent.

Interestingly, this contrast between embedded possibility and embedded necessity modals is maintained with *dopuščati*, as illustrated in (15). Put differently, negation over *dopuščati* does not seem to play a role in the embedding of epistemic modals under *dopuščati*.

(15)   Situation: as in (11).
       a. Janez   ne dopušča, da je Bob mogoče za   njegovim
           John   not allows   that is Bob maybe   behind his
           hrbtom.
           back
           'John does not allow for the possibility that Bob might be
       behind his back.'
       b.??Janez   ne dopušča, da mora biti Bob za   njegovim
           John   not allows   that must be Bob behind his
           hrbtom.
           back
           'John does not allow for the possibility that Bob must be
       behind his back.'

In this section we showed that embedding a necessity modal under a negated doxastic, be it a strong one like *think/misliti* or a weak one like

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[13] Some speakers feel that (14) is as bad as (11b). The difference for the others could be related to the ability of *because/since* to suspend implicatures, e.g. *Some students passed the exam because all of them did*, bringing *doesn't think* closer to *it's not the case that he thinks*.

dopuščati, leads to a certain degree of oddness. This does not occur with embedded possibility modals.[14]

2.4   Embedding Doxastic Attitudes

I want to briefly touch upon a difference between attitudes and modals, most influentially discussed in Yalcin (2007). I will not review his data here, but I will make a similar point with different data. Consider again example (2), repeated below, which showed that embedding a weak epistemic modal under a weak attitude is acceptable, in (2a), while embedding a strong epistemic modal under a weak attitude leads to oddness, in (2b), repeated below. By contrast, embedding either kind of attitude strength (under a weak attitude) is odd, as illustrated in (17).

(2) a.   Dopuščam, da   utegne biti sončno.
          I.allow   that might   be sunny
          'I allow for the possibility that it might be sunny.'
    b. #Dopuščam, da   mora biti sončno.
          I.allow   that must be sunny
          'I allow for the possibility that it must be sunny.'

(17) a.   #Dopuščam, da dopuščam, da sem se zmotila.
              I.allow   that I.allow   that am   self err
              'I allow that I allow that I made a mistake.'
     b.   #Dopuščam, da verjamem, da sem se zmotila.
              I.allow   that I.believe   that am   self err
              'I allow that I believe that I made a mistake.'

Intuitively, what goes wrong in (17) is that the speaker (more generally, the attitude holder) fails to be an **authority** on his own beliefs. We take belief to be something that we all 'have privileged and immediate access to' (Klein et al. 2015); see also Dorr and Hawthorne (2013: 897–898).

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[14] A reviewer points out that the typology of doxastic attitudes could be extended along the lines of negative attitude verbs, such as *rule out* or the Polish *wykluczać* ('rule out'). Polish provides an interesting test case since it does not have a verb like *dopuščati* but uses *nie wykluczać* ('not rule out') in its place. The reviewer points out that (6a) is good with *nie wykluczać*, suggesting weakness, and that *nie wykluczać* behaves like *dopuščati* in examples like (2), while *wykluczać* behaves like *ne dopuščati* in (15) above. I hope to explore this suggestion in future work.

The point about this is minor but important: (2a) and (17a) both involve embedding under a weak expression, but only the case of embedded attitudes leads to oddness. (On a related note, the oddness in (17b) feels distinctively different from that in (2b).) Informally, we can conclude from this that while people are assumed to be authorities on their beliefs, they are not assumed to be authorities of the same kind on their evidence. Nevertheless, a tight connection needs to be maintained between belief and epistemic modals embedded under belief, for reasons discussed in Yalcin (2007).[15]

3   A New Semantics for Doxastic Attitudes and Epistemic Modals

The generalization that we want to model is that embedded universal epistemic force is odd under *dopuščati* and negated doxastic attitudes (be it *misliti* or *dopuščati*). There are three key notions to the interaction: (i) doxastic states are structured in terms of prominence and the choice of what is made salient depends on the attitude verb, (ii) epistemic modals are "local" (Mandelkern, forthcoming), and (iii) epistemic modals are "total". The rest of the interaction is carried by the assumptions about presupposition projection that I make.

Formulas will be evaluated with respect to an index that consists of three parameters: (i) the information state (a set of worlds), as in Yalcin (2007), (ii) the salient state (a subset of the information state), and (iii) the world of evaluation. I will use intensional semantics à la von Fintel and Heim (2016) over these evaluation triples.

(18) a.   extension: $[\![\cdot]\!]^{c,g,\langle s,s',w\rangle}$
      b.   intension: $\lambda\langle s, s', w\rangle[\![\cdot]\!]^{c,g,\langle s,s',w\rangle}$ (abbreviated as $[\![\cdot]\!]^{c,g}_{\cent}$)

In (18), c is the context set, g the assignment function, s the information state, s' the salient state, and w the world of evaluation. We can follow

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[15] The standard analysis (Hintikka (1962), Kratzer (1977, 1981)) predicts (2a) and (2b) to both be good, while a simple extension of Yalcin's (2007) revised semantics predicts both to be odd. Our goal is to arrive at something intermediate for embedded modals, while keeping the common assumptions about belief introspection that explain the oddnesses in (17).

Lewis (1980) in assuming that assertions would simply be a set of worlds, letting the context close off the values of *s* and *s'*.[16] Here is then the proposed meaning for the relevant two doxastics in Slovenian:[17]

(19) **Semantics of doxastic attitudes**:

  a. $[\![misliti]\!]^{g,\langle s,s',w\rangle} = \lambda p \lambda x \forall w' \in \mathcal{B}_x^w[\, p(\langle \mathcal{B}_x^w, \mathcal{B}_x^w, w'\rangle) = 1]$

  b. $[\![dopuščati]\!]^{g,\langle s,s',w\rangle} = \lambda p \lambda x \exists w' \in \mathcal{B}_x^w[\, p(\langle \mathcal{B}_x^w, \{w'\}, w'\rangle) = 1]$

Following Hintikka (1962), the verbs in (19) are analysed as quantifiers over the set of worlds compatible with what the attitude holder *x* believes in *w*. Following Yalcin (2007), they both shift the information state parameter to the doxastic set. The difference lies in what is made salient: *misliti* is neutral in prominence in that it makes the whole doxastic state salient, while *dopuščati* brings to attention the witness(es) to its existential statement. This distinction is passed on to the embedded proposition (evaluated with respect to an updated point of evaluation). Notice then that the two verbs are duals for any expression that is not sensitive to the second coordinate (the salience parameter).

(20) **Semantics of epistemic modals**:

  a. $[\![morati]\!]^{g,\langle s,s',w\rangle} = [\![must]\!]^{g,\langle s,s',w\rangle} =$
      $= \lambda R: \forall v \in s[R(v) \subseteq s] \& \exists v \in s'[s \subseteq R(v)].\lambda p.\forall w'' \in R(w)[p(\langle s, s', w''\rangle) = 1]$

  b. $[\![utegniti]\!]^{g,\langle s,s',w\rangle} = [\![might]\!]^{g,\langle s,s',w\rangle} =$
      $= \lambda R: \forall v \in s[R(v) \subseteq s] \& \exists v \in s'[s \subseteq R(v)].\lambda p.\exists w'' \in R(w)[p(\langle s, s', w''\rangle) = 1]$

The entries in (20) follow the standard truth-conditional content of modals as quantifiers over a contextually-determined set of accessible worlds (Kratzer, 1977, 1981). The modal combines with a free (modal base) variable of type *sst* (von Fintel, 1994), which maps the world of evaluation to a set of accessible worlds (sloppily switching between function-talk and set-talk). The crucial component in (20) is a two-part presupposition on the modal base. I discuss the two parts ("Locality" and "Totality") in turn.

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[16] Assertion: $\lambda w.[\![\cdot]\!]^{c,g,\langle s_c,s_c,w\rangle}$ where s_c is the contextually determined *s*, etc.

[17] Thanks to Irene Heim (p.c.) for suggesting a simplification.

Following Mandelkern (2017, forthcoming), who builds on Yalcin (2007), epistemic modals carry a definedness condition called **Locality** (the first presupposition).[18] Under belief, this constraint requires the modal base function to map belief worlds onto subsets of the doxastic state. Locality, which constrains admissible modal bases, ensures that epistemic modals only access the information that is locally provided to them.[19] A way to intuitively understand Mandelkern's and Yalcin's idea is to think of our beliefs as pieces of evidence that we use to navigate the world. Epistemic modals under belief predicates are constrained by this kind of evidence.

There is a second constraint on epistemic modals, called **Totality**. Under belief, it requires that the modal base function finds at least one world (v) in the salient part of the belief state (s') and map the modal base from that world ($R(v)$) to a superset of the belief state (s). (Together with Locality, this means that the modal base from that world and the doxastic state coincide.[20]) Intuitively, epistemic modals under belief predicates are not allowed to be completely constrained by the outside evidence – there are salient worlds from which only belief-evidence is accessed.

I will write the two presuppositions as intermediately accommodated into the restrictor of the attitude predicate (as is customary, restrictors will be assumed to be non-empty). This is only crucial when Totality ends up containing a variable bound by the attitude (as with *dopuščati*), in which case we need intermediate accommodation to derive the right truth-conditions. The formulas are, however, more transparent if Locality and Totality are kept together (it is easier to see how the negation is "pushed in", for example).

Consider first examples like (1b), repeated in (21) with *Janez* ('John') as the subject (*J* in the LFs). Here is how we derive the truth-conditions using the entries in (19a) and (20a).

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[18] This constraint is weaker than Yalcin's but can account for Yalcin's data (and more).
[19] Like Yalcin (2007) and Mandelkern (2017, forthcoming), I would need to assume a covert shifting operator for the cases in which the embedded epistemic modal is not sensitive to the attitude holder's evidence.
[20] Thanks to Daniel Rothschild (p.c.) for suggesting I adopt this weaker version of Totality.

(21) Janez misli, da   mora   biti   sončno.
      John thinks that   must   be   sunny
      'John thinks it must be sunny.'

$[\![(1b)]\!]^{g,\langle s,s',w\rangle} = 1$ iff

$[\![misli]\!]^{g,\langle s,s',w\rangle}([\![mora_i \; biti \; sončno]\!]^g_{\mathfrak{C}})(\text{John}) = 1$ iff

$\forall w' \in \mathcal{B}^w_{\text{John}}[[\![mora_i \; biti \; sončno]\!]^g_{\mathfrak{C}}(\langle \mathcal{B}^w_{\text{John}}, \mathcal{B}^w_{\text{John}}, w'\rangle) = 1]$ iff

$\forall w' \in \mathcal{B}^w_{\text{John}}[[\![mora_i \; biti \; sončno]\!]^{g,\langle \mathcal{B}^w_{\text{John}}, \mathcal{B}^w_{\text{John}}, w'\rangle} = 1]$ iff

$\forall w' \in \mathcal{B}^w_{\text{John}}[[\![mora]\!]^{g,\langle \mathcal{B}^w_{\text{John}}, \mathcal{B}^w_{\text{John}}, w'\rangle}([\![i]\!]^{g,\langle \mathcal{B}^w_{\text{John}}, \mathcal{B}^w_{\text{John}}, w'\rangle})([\![biti \; sončno]\!]^g_{\mathfrak{C}}) = 1]$ iff

$\forall w' \in \mathcal{B}^w_{\text{John}}[[\![mora]\!]^{g,\langle \mathcal{B}^w_{\text{John}}, \mathcal{B}^w_{\text{John}}, w'\rangle}(g(i))([\![biti \; sončno]\!]^g_{\mathfrak{C}}) = 1]$ iff

$\forall w' \in \mathcal{B}^w_{\text{John}}[[\lambda R : \forall v \in s[R(v) \subseteq s] \& \exists v \in s'[s \subseteq R(v)].\lambda p.\forall w'' \in R(w')[p(\langle \mathcal{B}^w_{\text{John}}, \mathcal{B}^w_{\text{John}}, w''\rangle)=1]](g(i))([\![biti \; sončno]\!]^g_{\mathfrak{C}})=1]$ iff

$\forall w' \in \mathcal{B}^w_{\text{John}}[(\forall v \in \mathcal{B}^w_{\text{John}}[g(i)(v) \subseteq \mathcal{B}^w_{\text{John}}] \& \exists v \in \mathcal{B}^w_{\text{John}}[\mathcal{B}^w_{\text{John}} \subseteq g(i)(v)]) \rightarrow \forall w'' \in g(i)(w')[\text{it's sunny at } w'']]$

<div align="center">Figure 1: Truth-conditions of (21)/(1b)</div>

Given non-empty restrictors, the doxastic state *B* should be non-empty. Notice that this extends to satisfying Locality and Totality in the antecedent.[21] Above, the two require the modal base *g(i)* function to be such that it maps belief worlds onto subsets of the doxastic state while there being a belief world that it maps onto the doxastic state itself. The rest of the truth-conditions are as expected: every belief world is such that it is sunny at every world in the modal base from it. Notice that since the modal base function from some world maps onto the whole doxastic state, it follows that it is sunny at every belief world.[22]

We now show that (22), based on (2b), is equivalent to (21).

(22) #Janez dopušča, da   mora biti sončno.
       John allows   that   must be sunny
      'John allows for the possibility that it must be sunny.'

$[\![(2b)]\!]^{g,\langle s,s',w\rangle} = 1$ iff

$\exists w' \in \mathcal{B}^w_{\text{John}}[[\![mora]\!]^{g,\langle \mathcal{B}^w_{\text{John}}, \{w'\}, w'\rangle}(g(i))([\![biti \; sončno]\!]^g_{\mathfrak{C}}) = 1]$ iff

$\exists w' \in \mathcal{B}^w_{\text{John}}[\forall v \in \mathcal{B}^w_{\text{John}}[g(i)(v) \subseteq \mathcal{B}^w_{\text{John}}] \& \exists v \in \{w'\}[\mathcal{B}^w_{\text{John}} \subseteq g(i)(v)] \& \forall w'' \in g(i)(w')[\text{it's sunny at } w'']]$ iff

$\exists w' \in \mathcal{B}^w_{\text{John}}[\forall v \in \mathcal{B}^w_{\text{John}}[g(i)(v) \subseteq \mathcal{B}^w_{\text{John}}] \& [\mathcal{B}^w_{\text{John}} \subseteq g(i)(w')] \& \forall w'' \in g(i)(w') : \text{it's sunny at } w'']$

<div align="center">Figure 2: Truth-conditions of (22)/(2b)</div>

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[21] Locating them in the antecedent of the conditional statement is logically equivalent to writing them as conditions on the set.

[22] On this semantics *must* is strong: Bp and $B\Box p$ are equivalent.

In Figure 2, Locality is as before whereas Totality contains the variable *w'*, bound by the existential quantification contributed by *dopuščati*. This means that it is the world that *dopuščati* talks about ('there is a world in the belief state such that...') that is such that the modal base maps it onto the doxastic state. Since the truth-conditions require it to be sunny in every world from that modal base (*g(i)(w')*), it again follows that the attitude holder believes that it is sunny. It is then easy to see how the two entail each other.[23]

Let us now turn to embedded existential modals from (1a) and (2a), used in (23) and (24) with the subject *Janez* ('John') (*J* in the LFs).

(23) Janez misli, da   utegne biti sončno.
       John thinks that   might   be sunny
     'John thinks it might be sunny.'

(24) Janez dopušča, da   utegne biti sončno.
       John allows   that   might   be sunny
     'John allows for the possibility that it might be sunny.'

$[\![(1a)]\!]^{g,s,s',w} = 1$ iff
$\forall w' \in \mathcal{B}_J^w[(\forall v \in \mathcal{B}_J^w[g(i)(v) \subseteq \mathcal{B}_J^w]\&\exists v \in \mathcal{B}_J^w[\mathcal{B}_J^w \subseteq g(i)(v)]) \to \exists w'' \in g(i)(w')[p(w'') = 1]]$

$[\![(2a)]\!]^{g,s,s',w} = 1$ iff
$\exists w' \in \mathcal{B}_J^w[(\forall v \in \mathcal{B}_J^w[g(i)(v) \subseteq \mathcal{B}_J^w] \ \& \ \mathcal{B}_J^w \subseteq g(i)(w')]) \ \& \ \exists w'' \in g(i)(w')[p(w'') = 1]]$

<div align="center">Figure 3: Truth-conditions of (23)/(1a) and (24)/(2a)</div>

The situation in Figure 4 shows that (23) and (24) are not equivalent since (24) does not entail (23). That is, (24) is true in Figure 4 while (23) is false (given the truth-conditions in Figure 3). The modal base from both worlds in Figure 4 does *not* map outside the belief state, so Locality is satisfied for both (23) and (24) and we can leave it aside. To see that (24) is true, consider the belief world on the left hand side. The modal base from it is indeed a superset of the belief state and there is a world in

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[23] Similarly for the truth-conditions of (10), which mirror (22).

$[\![(10)]\!]^{g,s,s',w} = 1$ iff

$\exists w' \in \mathcal{B}_{John}^w[[\![more]\!]^{g,\langle\mathcal{B}_{John}^w,\{w'\},w'\rangle}(g(i))([\![deževati]\!]_{\mathfrak{c}}^g) = 0]$ iff

$\exists w' \in \mathcal{B}_{John}^w[\forall v \in \mathcal{B}_{John}^w[g(i)(v) \subseteq \mathcal{B}_{John}^w] \ \& \ [\mathcal{B}_{John}^w \subseteq g(i)(w')] \ \& \ \neg\exists w'' \in g(i)(w')[\text{it's raining at } w'']]$

its modal base, namely itself, that satisfies *p*. By contrast, the truth-conditions of (23) require every belief world to have a modal base in which there is a *p*-world. This is not the case for the world on the right hand side.
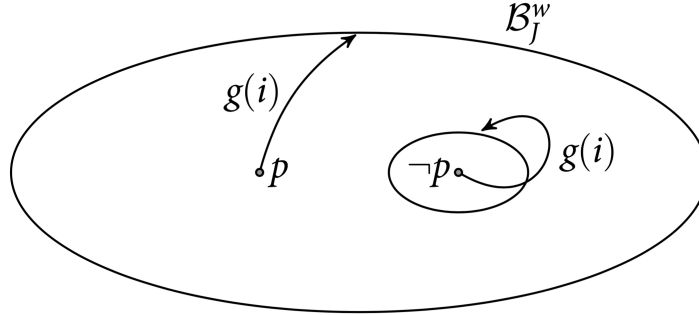

Figure 4: $D\Diamond_i p \not\models B\Diamond_i p$

It is easy to see that the converse holds: (23) entails (24), assuming non-empty restrictors. Thus, when *dopuščati* embeds an existential modal, the sentence is strictly weaker than the one obtained with *misliti*. This effect was indirectly observed in (9) where *misliti* suggested that Othello had some evidence for suspecting Desdemona of cheating while *dopuščati* triggered no such inference. This can be linked up to the properties of the modal base function in Figure 4 but I will not explore this here.

In sum, I proposed a semantics on which embedded universal modals collapse the distinction in the attitude force, while embedded existential modals preserve it. I will not go through the proofs, but this property is maintained under negation, as illustrated schematically in Figure 6.[24] Figure 5 provides the remaining truth-conditions.

$$[\![\neg D_J\Box_i p]\!]^{g,s,s',w}=1 \text{ iff } \forall w' \in \mathcal{B}_J^w[(\forall v \in \mathcal{B}_J^w[g(i)(v) \subseteq \mathcal{B}_J^w]\&[\mathcal{B}_J^w \subseteq g(i)(w')]) \to \exists w'' \in g(i)(w')[p(w'')=0]])$$
$$[\![\neg B_J\Box_i p]\!]^{g,s,s',w}=1 \text{ iff } \exists w' \in \mathcal{B}_J^w[\forall v \in \mathcal{B}_J^w[g(i)(v) \subseteq \mathcal{B}_J^w]\&\exists v \in \mathcal{B}_J^w[\mathcal{B}_J^w \subseteq g(i)(v)]\&\exists w'' \in g(i)(w')[p(w'')=0]]$$
$$[\![\neg B_J\Diamond_i p]\!]^{g,s,s',w}=1 \text{ iff } \exists w' \in \mathcal{B}_J^w[\forall v \in \mathcal{B}_J^w[g(i)(v) \subseteq \mathcal{B}_J^w]\&\exists v \in \mathcal{B}_J^w[\mathcal{B}_J^w \subseteq g(i)(v)]\&\forall w'' \in g(i)(w')[p(w'')=0]]$$
$$[\![\neg D_J\Diamond_i p]\!]^{g,s,s',w}=1 \text{ iff } \forall w' \in \mathcal{B}_J^w[(\forall v \in \mathcal{B}_J^w[g(i)(v) \subseteq \mathcal{B}_J^w] \& \mathcal{B}_J^w \subseteq g(i)(w')) \to \forall w'' \in g(i)(w')[p(w'')=0]]$$

Figure 5: Truth-conditions for modals under negated attitudes
(D=*dopuščati*, B=*misliti*, J=attitude holder, *i*=modal base)

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[24] Figure 4 can be re-used ($\neg B_J\Diamond_i p$ is true while $\neg D_J\Diamond_i p$ is false).

$$B\square p$$

$$D\diamond p \longleftarrow B\diamond p$$

$$\#D\square p$$

$$\#\neg D\square p$$

$$\neg B\diamond p \longleftarrow \neg D\diamond p$$
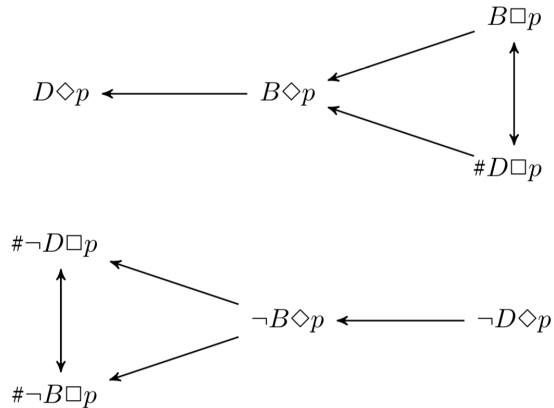
$$\#\neg B\square p$$

<center>Figure 6: Summary of the entailments</center>

4   How to Derive Oddness?

As illustrated in Figure 6, the sentences with embedded strong modals are equivalent, regardless of what the embedding doxastic attitude is. In this section I want to give an idea as to why this might trigger oddness, but I leave much of the work for future research.

(24) a.   #Some Italians come from a warm country.
      b.   #Some students got an A. (Situation: the professor is known to assign the same grade to all of his students)

<div align="right">(Magri 2009, 2011)</div>

Roughly, these sentences are odd because they are equivalent to their scalar alternatives (*All Italians come...*, *All students got....*). In a situation where everyone gets the same grade, if some students get an A, then they all do, and vice versa.[25] The semantics we set up in the previous section

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[25] Magri derives the oddness with an exhaust operator that generates the uncancellable inference *some but not all*, which yields a contextual contradiction. A reviewer points out that (24a) is acceptable with the continuation *in fact all of them do* and suggests a parallel to (7). The *in fact* data is a more general challenge for Magri-like theories, so I will not attempt to address it here, but the parallel is there. To show that *dopuščati* does not semantically encode *dopuščati but not think*, we used a context where 'think' was granted (the person in (7) is a scientist) and showed that we can strengthen *dopuščati* into *think* without oddness, which is what we can in principle do in (24) as well.

gives us some equivalences, as in Figure 6. Can we exploit those to explain why it is odd to use *dopuščati* with 'it must be sunny outside'?

Let's have a rule that says that the sentence in (24a) is **deviant** in a context where Italians come from the same country because there is a Horn competitor *All Italians come from a warm country* (obtained by substituting *some* with *all*) such that (i) the two propositions are contextually equivalent and, importantly, (ii) (24a) is "structurally weaker" than its *all*-version. More precisely, there is a shared constituent *come from a warm country* that could be replaced by another structure, such as *smiled,* where *All Italians smiled* entails that *Some Italians smiled* and not vice versa.[26] In other words, we have access to the make-up of (24a), which uses a weaker expression than needed (to convey the same information).

This works well for the base case in (2b) ('I allow that it must be sunny') or (22) above. We say that (2b) is deviant in that context because there is a Horn competitor (1b) ('I think it must be sunny'), which is equivalent to it and, furthermore, (2b) is structurally weaker than (1b). Consider replacing the embedded clause (a shared constituent) with a non-modal proposition, e.g. 'it's sunny'. Intuitively as well as theoretically (such simple propositions are not sensitive to prominence) *Dopuščam, da je sončno* ('I allow that it's sunny') is entailed by but does not itself entail *Mislim, da je sončno* ('I think it's sunny'). Thus, (2b) uses a weaker expression than needed to convey the same message as (1b), so it is odd.

The explanation works less well for the difference between embedded necessity epistemics under *don't think* and *it's not the case that think*, in (12b) and (13), respectively. To solve this, we need to appeal to meta-linguistic negation for (13). The more serious issue is (15b), for which we need the deviance principle to apply locally and percolate upward.[27] We can stipulate that a constituent or a sentence is deviant if it contains a deviant subconstituent, and leave the details of this for future work.

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[26] We would probably need a more fine-grained notion of what kind of structure makes for a good replacement test.

[27] Magri (2009, 2011) achieves this effect by making the exhaust operator obligatory at every scope site, so also in embedded positions such as under negation.

5   Conclusion

This paper examined the behaviour of epistemic modals (*might*, *must*) when embedded under two types of doxastic attitude verbs: strong verbs like *misliti* ('think') and weak verbs like the previously-undiscussed *dopuščati* ('to allow for the possibility') from Slovenian. I analysed the latter as an existential belief verb and showed that a semantics can be designed for doxastic attitudes and epistemic modals that captures the intuitive oddness of sentences like *Dopuščam, da mora deževati* ('I allow for the possibility that it must be raining'). I proposed a semantics where embedded universal modals collapse the distinction in the attitudinal force, while embedded existential modals preserve it. I concluded with some thoughts on why this leads to oddness in some cases, leaving the pragmatic details for future work.

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(4a)   http://www.delo.si/novice/slovenija/repe-zombiji-bodo-ukradeno-drzavo-priborili-nazaj.html   (last accessed Dec17)
(4b)   http://www.delo.si/kultura/knjizevni-listi/recenzija-knjige-vcasih-se-zdi-da-je-cas-za-posmeh.html   (last accessed Dec17)
(4c)   http://www.delo.si/svet/evropa/tusk-dopusca-moznost-da-brexita-ne-bo.html   (last accessed Dec17)
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