# Learning-based Attacks in Cyber-Physical Systems

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# Cloud robots and automation systems

# Security


We need to address physical security in addition to cyber security

# News reports

## Port of San Diego suffers cyber-attack, second port in a week after Barcelona

### Hacker jailed for revenge sewage attacks

Job rejection caused a bit of a stink

### HACKERS REMOTELY KILL A JEEP ON THE HIGHWAY—WITH ME IN IT



CYBERATTACK ON A GERMAN STEEL-MILL

# News reports

The Stuxnet outbreak

## A worm in the centrifuge

The Economist

*An unusually sophisticated cyber-weapon is mysterious but important*

**Computer virus Stuxnet a 'game changer,' DHS official tells Senate**

CNN

Symantec

"It has changed the way we view the security threat"

# The man in the middle

A fictitious plant for the controller


A malicious controller for the plant

# Mathematical formulation

- Linear dynamical system

$$X_{k+1} = aX_k + U_k + W_k$$

$$\{W_k\} \text{ are i.i.d. } \mathcal{N}(0, Var[W])$$

- The controller, at time $k$, observes $Y_k$ and generates a control signal $U_k$ as a function of all past observations $Y_1^k$.

$Y_k = X_k$ Under normal operation

$Y_k = V_k$ Under attack

- The attacker feeds a malicious input $\tilde{U}_k$ to the plant.

- How can the controller detect that the system is under attack?

# Anomaly detection

- The controller is armed with a detector that tests for anomalies in the observed history $Y_1^k$.
- Under legitimate system operation we expect

$$Y_{k+1} - aY_k - U_k(Y_1^k) \sim \text{ i.i.d. } \mathcal{N}(0, Var[W])$$

- The detector performs the variance test

$$\frac{1}{T}\sum_{k=1}^{T}\left[Y_{k+1} - aY_k - U_k(Y_1^k)\right]^2 \in (Var[W] - \delta, Var[W] + \delta).$$

- What kind of attacks can we detect?

# The man in the middle attack types

Replay attack

Stuxnet

[redacted]

Y. Mo, B. Sinopoli (2009)

Statistical-duplicate attack

$$X_{k+1} = aX_k + U_k + W_k$$

B. Satchidanandan,
P. R. Kumar (2017)
R. S. Smith (2011)

Learning-based attack

$$X_{k+1} = aX_k + U_k + W_k$$

**MJ Khojasteh** et al. (2020)

# Learning-based attack

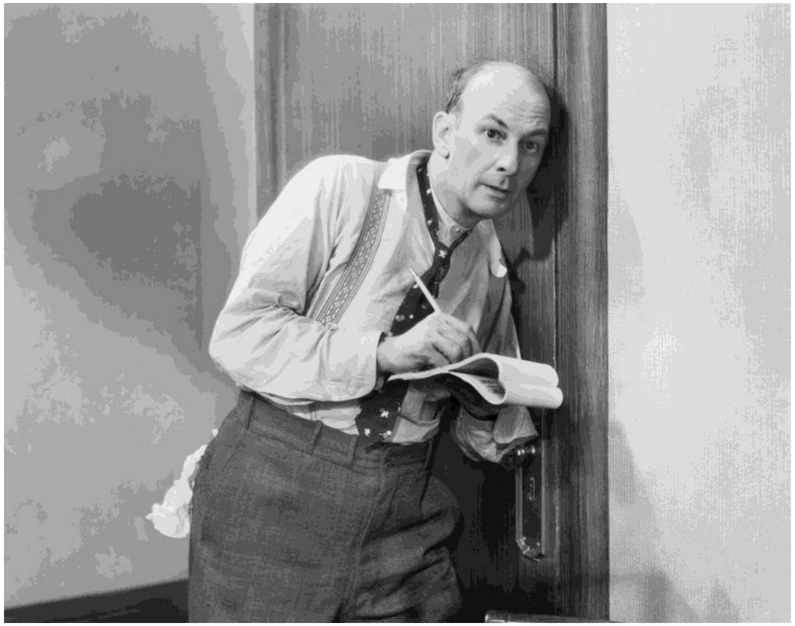
$$X_{k+1} = aX_k + U_k + W_k.$$

- The attacker has access to both $X_k$ and $U_k$ and knows the distribution of $W_k$ and of the initial condition $X_0$, but it should learn the open loop gain $a$ of the plant.
- For analysis purposes, we can assume the open loop gain of the plant is a random variable $A$ with a distribution known to the attacker and for any event $C$ we let

$$\mathbb{P}_a(C) = \mathbb{P}(C|A = a).$$

# Two phases of the learning-based attack

Learning (exploration) phase

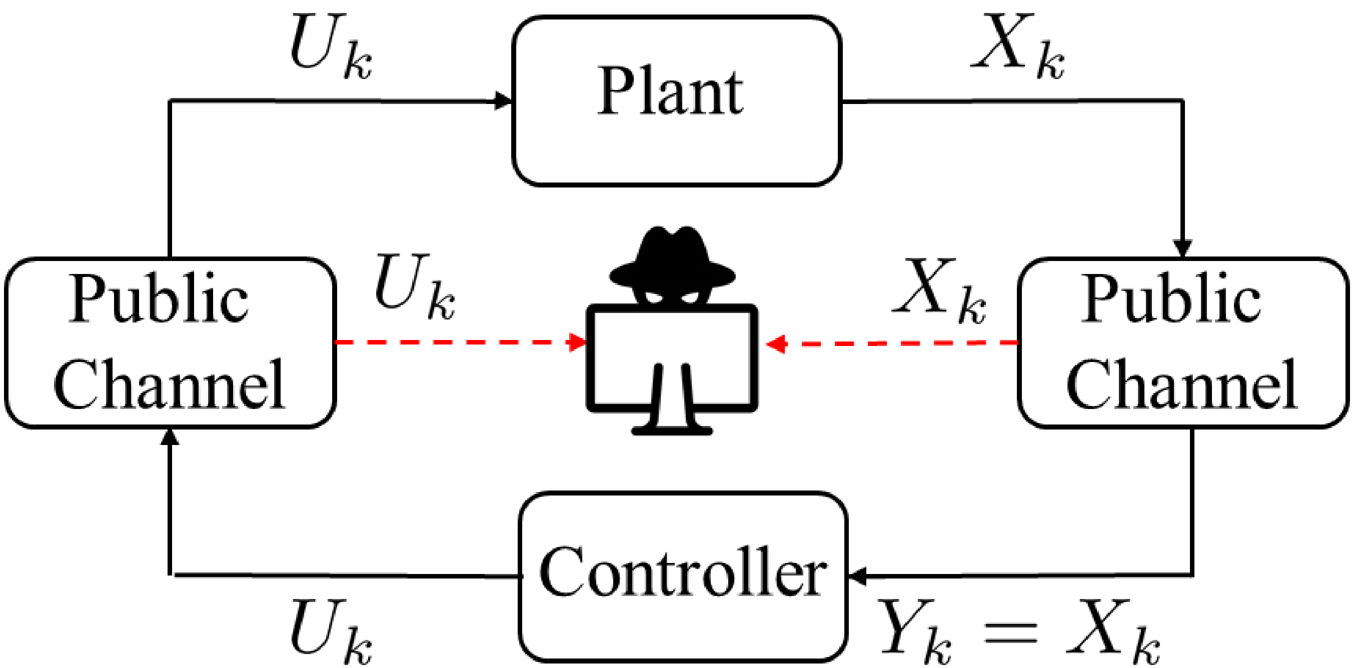


Eavesdropping and learning

Hijacking (exploitation) phase

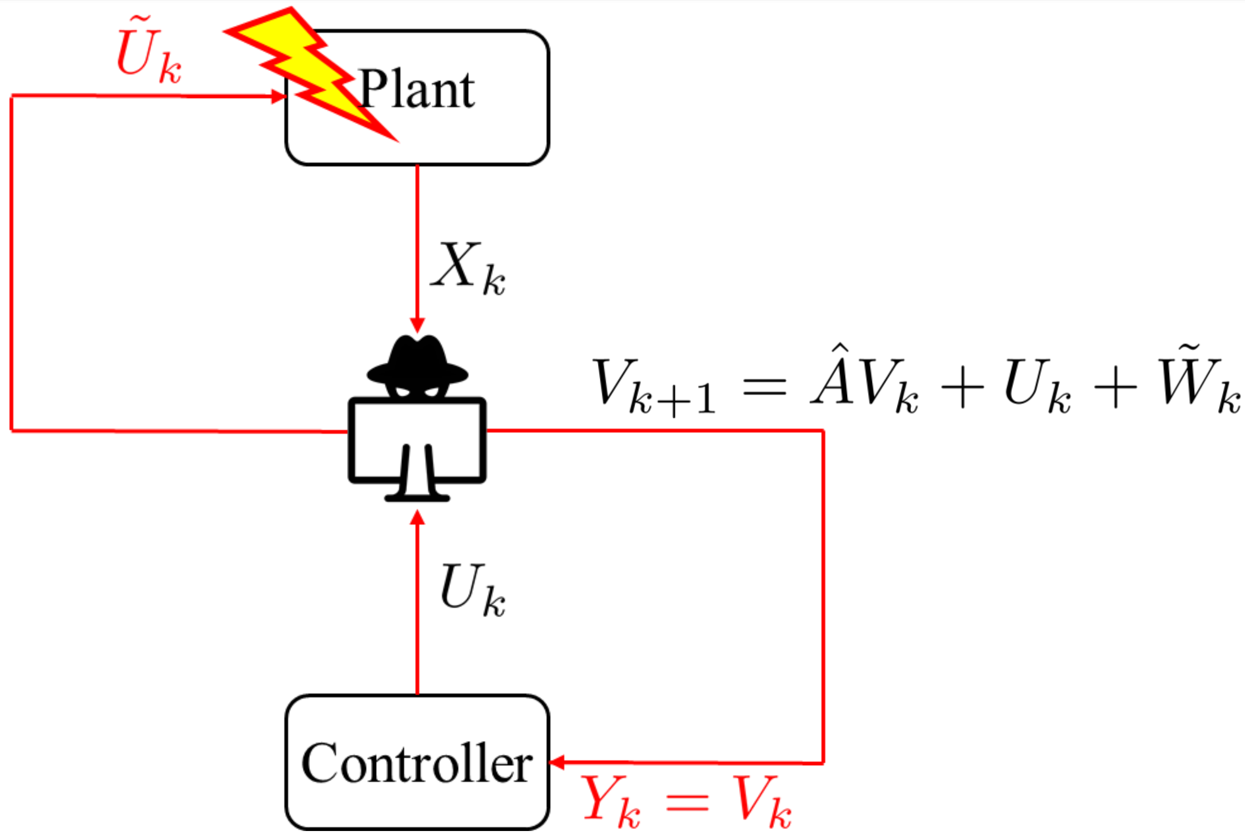
Hijacking the system

# Learning (exploration) phase




- For $k \in [0, L]$, the attacker observes the plant state and control input, and tries to learn the open-loop gain $a$.

# Hijacking (exploitation) phase




- For $k = L+1, \ldots, T$, the attacker feeds the fake signal $V_k$ to the controller, reads the next input $U_k$, and drives the system to an undesired state by feeding $\tilde{U}_k$ to the plant.

# Detecting the attack

- Let $\Theta_T$ be the indicator of the attack at any time before $T$
- The controller uses $Y_1^T$ to construct an estimate $\hat{\Theta}_T$ of $\Theta_T$ according to the variance test
- Define the deception probabilities

$$P_{dec}^{a,T} \triangleq \mathbb{P}_a \left( \hat{\Theta}_T = 0 \middle| \Theta_T = 1 \right)$$

$$P_{dec}^{T} \triangleq \mathbb{P} \left( \hat{\Theta}_T = 0 \middle| \Theta_T = 1 \right) = \int_{-\infty}^{\infty} P_{dec}^{a,T} f_A(a) da$$

- Assume the power of the fictitious sensor reading converges a.s.

$$\lim_{T\to\infty} \frac{1}{T} \sum_{k=L+1}^{T} V_k^2 = \frac{1}{\beta} < \infty$$

# Results

- We provide lower and upper bounds on the deception probability
- The lower bound is based on a given learning algorithm and holds for any measurable control policy
- The upper bound holds for any learning algorithm, and any measurable control policy

# Lower bound

- Assuming the attacker uses a least-square learning algorithm to learn the plant, such that

$$\hat{A} = \arg\min_{A} \|X_{k+1} - AX_k - U_k\| = \frac{\sum_{k=1}^{L-1}(X_{k+1} - U_k)X_k}{\sum_{k=1}^{L-1} X_k^2}$$

- This algorithm is consistent, namely

$$\hat{A} \xrightarrow{P} a \quad \text{as} \qquad L \to \infty$$

K. J. Åström, P. Eykhoff (1971), L Ljung (1982)

# Lower bound

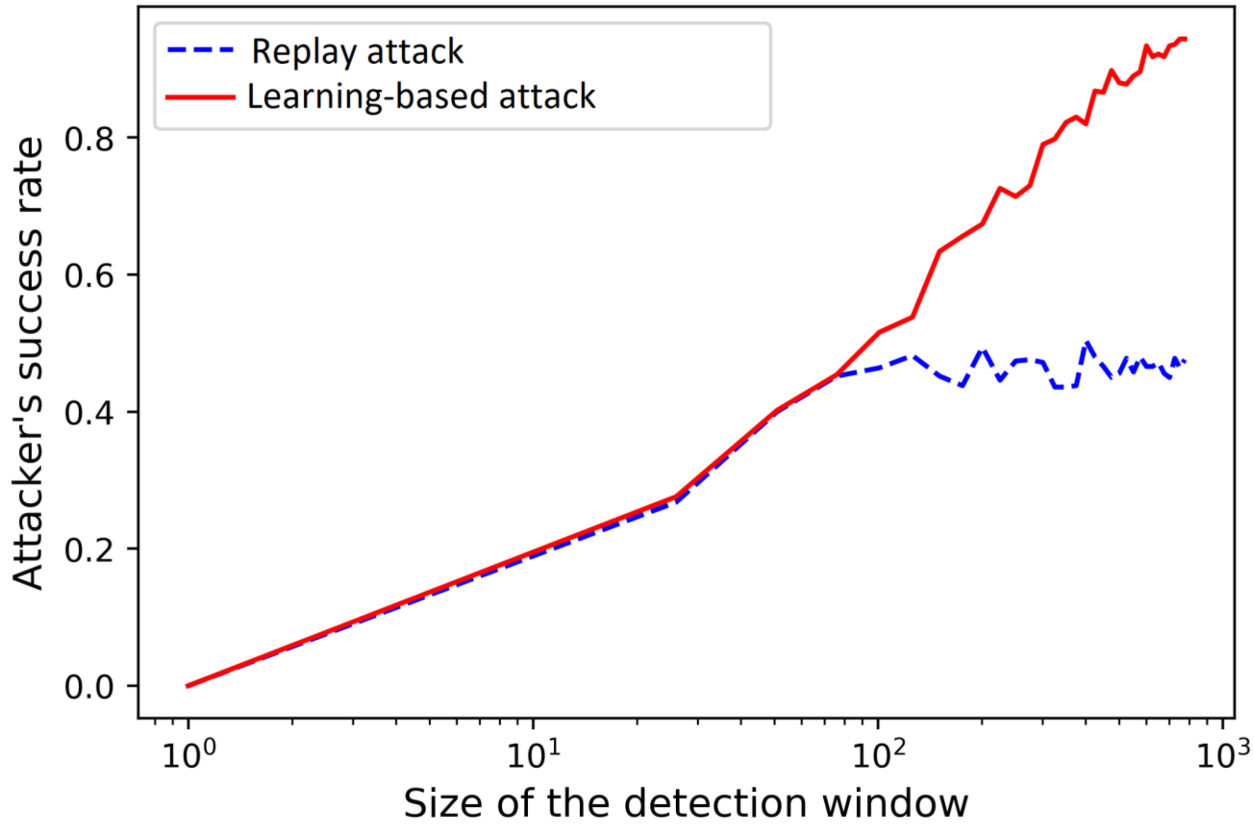
- On the other hand, for any fixed L the deception probability depends on the ability to learn the plant, and we can show

$$\lim_{T\to\infty} P^a_{\mathrm{dec}} = \mathbb{P}_a\left(|\hat{A} - a| < \sqrt{\delta\beta}\right)$$

$$\geq 1 - \frac{2}{(1+\delta\beta)^{L/2}}$$

Using concentration bound of A. Rantzer 2018

# Comparison with a replay attack




**MJ Khojasteh** et al. (2020)

# Upper bound on the deception probability

- If $A$ is distributed uniformly in $[-R, R]$, then letting $Z_1^k = (X_1^k, U_1^k)$, we have

$$\lim_{T\to\infty} P_{dec} \leq \frac{I(A; Z_1^L) + 1}{\log(R/\sqrt{\delta\beta})}.$$

- The numerator represents the information revealed about $A$ from the observation of the random variable $Z$.
- The denominator represents the intrinsic uncertainty of $A$ when it is observed at resolution $\epsilon = \sqrt{\delta\beta}$ corresponding to the entropy of the quantized random variable $H(A_\epsilon)$.

# Upper bound on the deception probability

- In addition, if $A \rightarrow (X_k, Z_1^{k-1}) \rightarrow U_k$ is a Markov chain for all $k \in \{1, \ldots, L\}$ , then

$$\lim_{T\to\infty} P_{dec} \leq \frac{I(A; Z_1^L) + 1}{\log(R/\sqrt{\delta\beta})}$$

$$\leq \frac{\sum_{k=1}^{L} D\left(\mathbb{P}_{X_k|Z_1^{k-1},A} \middle\| \mathbb{Q}_{X_k|Z_1^{k-1}} \middle| \mathbb{P}_{Z_1^{k-1},A}\right) + 1}{\log(R/\sqrt{\delta\beta})}$$

any sequence of probability measures $\left\{\mathbb{Q}_{X_k|Z_1^{k-1}}\right\}$, provided

$\mathbb{P}_{X_k|Z_1^{k-1}} \ll \mathbb{Q}_{X_k|Z_1^{k-1}}$ for all $k \in \{1, \ldots, L\}$.

# The Gaussian case

- The freedom in choosing the auxiliary probability measure $\left\{\mathbb{Q}_{X_k|Z_1^{k-1}}\right\}$ make the second bound  a useful bound.
- Gaussian plant disturbance $W_k \sim \mathcal{N}(0, Var[W])$
- By choosing $\mathbb{Q}_{X_k|Z_1^{k-1}} \sim \mathcal{N}(0, Var[W])$ we have

$$\lim_{T\to\infty} P_{dec} \leq G(Z_1^L),$$

where $$G(Z_1^L) \triangleq \frac{\frac{\log e}{2\sigma^2}\sum_{k=1}^{L}\mathbb{E}(AX_{k-1}+U_{k-1})^2+1}{\log\left(R/\sqrt{\delta\beta}\right)}.$$

# Privacy-enhancing signal

Impede the learning process of the attacker

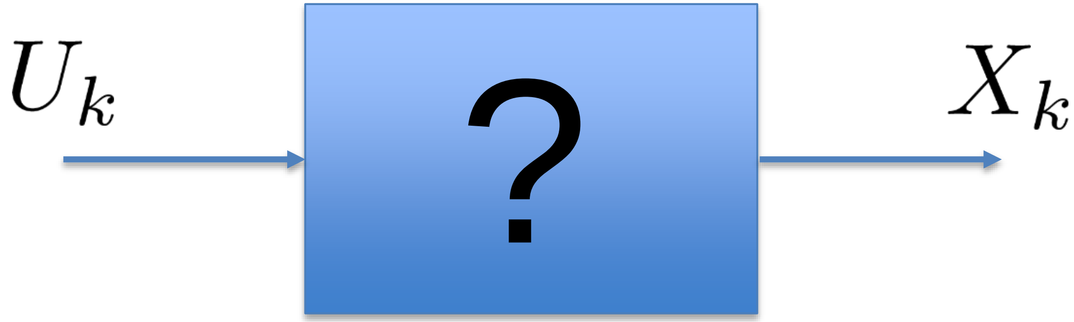
Privacy-enhancing signal

$$U_k = \bar{U}_k + \Gamma_k$$

Nominal control policy

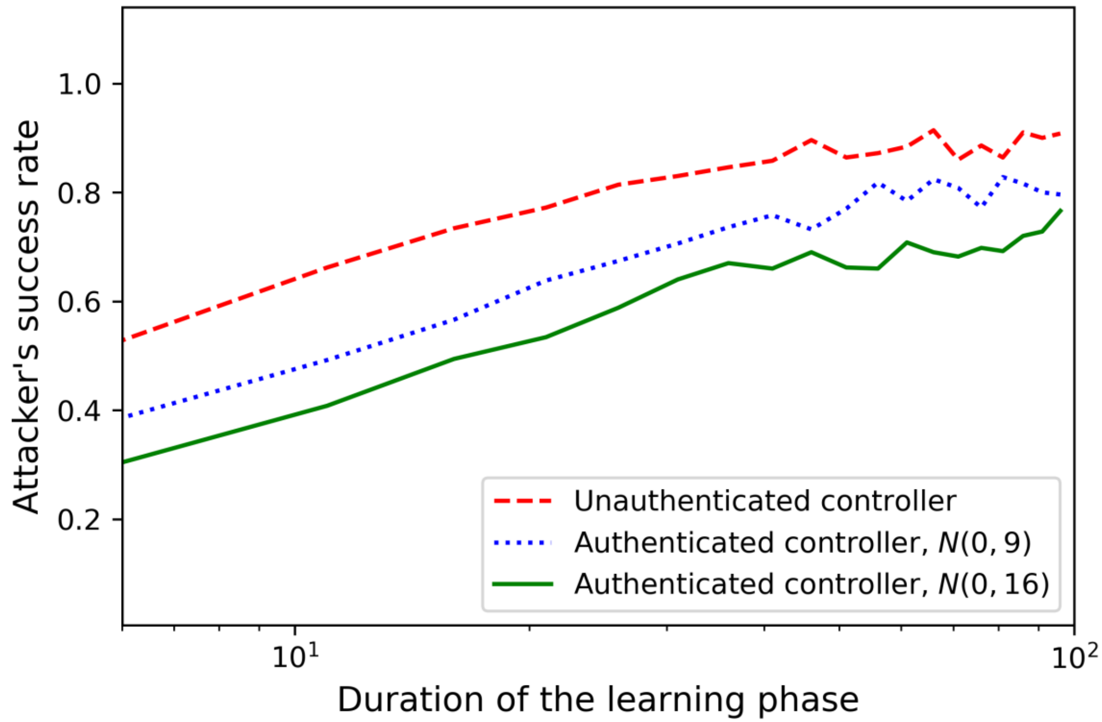
# Privacy-enhancing signal

- Injecting a strong noise may in fact speed up the learning process




- Carefully crafted watermarking signals provide better guarantees on the deception probability

# Defense against learning-based attack




**MJ Khojasteh** et al. (2020)

# Vector systems

$$A \longrightarrow \mathbf{A} = \left[\ ?\ \right]$$

$$\mathrm{Var} \longrightarrow \mathrm{Cov}$$

$$\hat{A} = \frac{\sum_{k=1}^{L-1} (X_{k+1} - U_k) X_k}{\sum_{k=1}^{L-1} X_k^2}$$

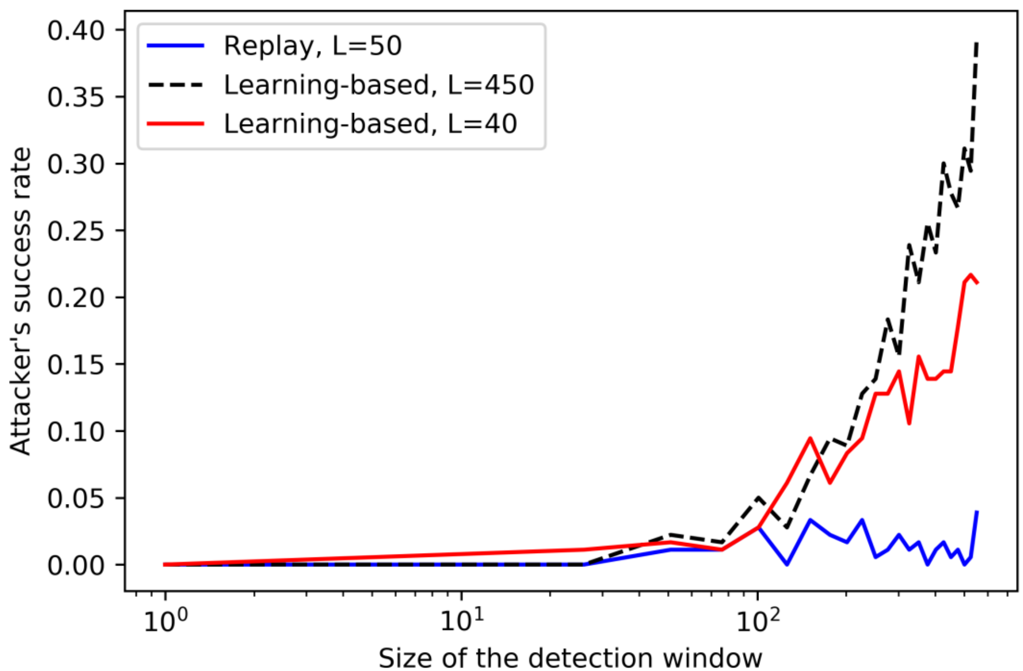
$$\Downarrow$$

$$\mathbf{A} = \begin{cases} \mathbf{0}_{n \times n}, & \det(\mathbf{G}_{L-1}) = 0; \\ \sum_{k=1}^{L-1} \left( (\mathbf{X}_{k+1} - \mathbf{U}_k) \mathbf{X}_k^{\dagger} \right) \mathbf{G}_{L-1}^{-1}, & \text{otherwise.} \end{cases}$$

# Learning-based attack: vector systems

$$A \longrightarrow \mathbf{A} = \begin{bmatrix} ? \end{bmatrix}$$

$$\mathrm{Var} \longrightarrow \mathrm{Cov}$$

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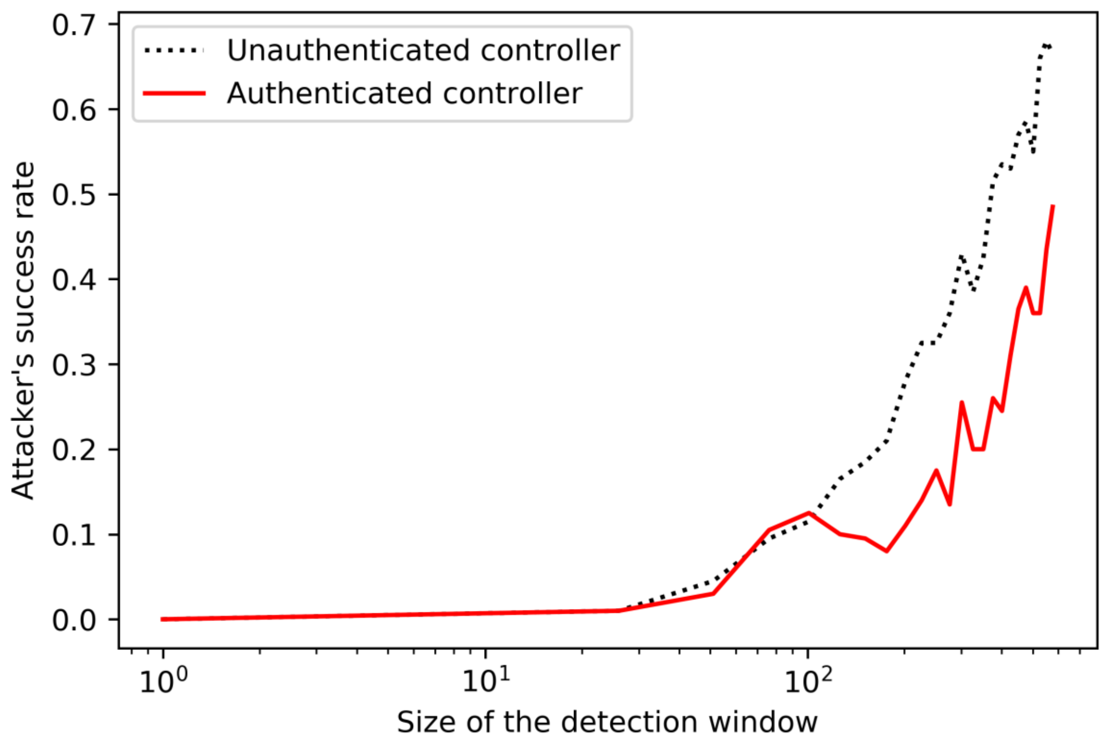
# Defense against vector learning-based attack

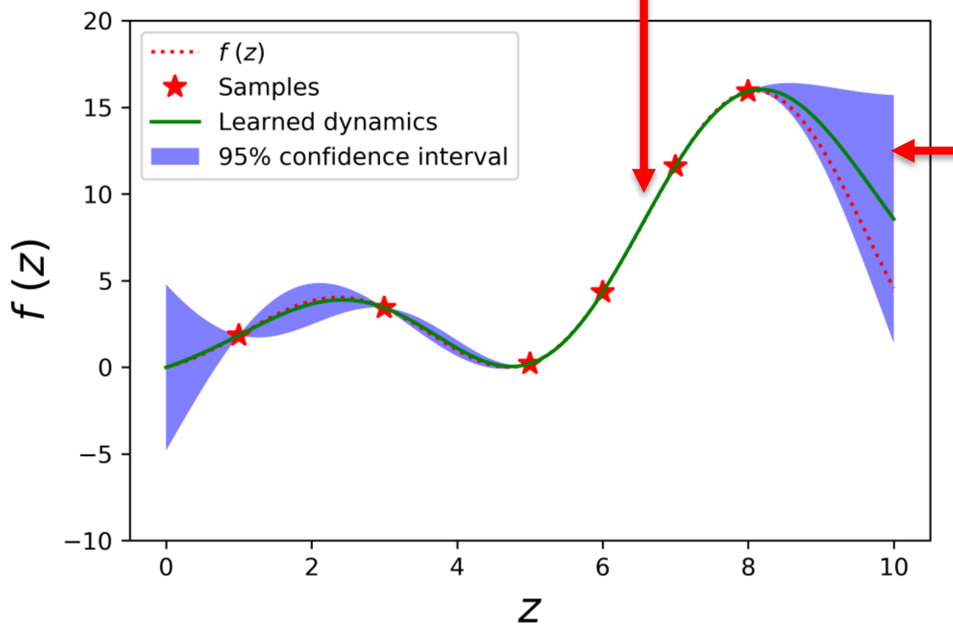


# Nonlinear learning-based attack

$A \longrightarrow f(X, U) \in$ Reproducing Kernel Hilbert Space (RKHS)

Linear regression $\longrightarrow$ Bayesian learning: Gaussian processes (GP)

Vulnerable region




Lower attacker's success rate

# References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