A Simple Generalization of the CDMA Reverse Link Pole Capacity Formula

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***Abstract*—A formula that computes the maximum number of users supported per base station in a cellular radio network is generalized to consider the frequency reuse number and arbitrary processing gains. The generalization quantifies a cost associated with in-cell interference by accounting for the lack of interference from the desired user on the total interference and by considering the impact of the frequency reuse number on the out-of-cell interference. This interference cost results in an increase in the received Eb/Io relative to FDMA which should be weighted against a reduction in the Eb/Io requirement resulting from using CDMA.**

***Index Terms*—Discount, markup, template.**

I. Introduction

THE EFFICIENT use of the RF spectrum serves as a fundamental design goal for cellular radio network engineers. The more calls that can be supported by a base station at an acceptable quality, the less base stations that are needed to support a given subscriber demand. Since there are large fixed capital costs associated with base station deployment, it is desirable to maximize the number of subscribers that each base station can support.

A formula from [1] is sometimes used to estimate the maximum number of users supported by each base station. In [1], the number of users per CDMA carrier is given as

$$n = \frac{W_S}{R_b} F \left(\frac{1}{\gamma} \left(1 - \frac{1}{r} \right) \right) \tag{1}$$

where

W_S RF spread bandwidth;

R_b data rate;

$\gamma \cdots \dfrac{E_b}{(I_o + N_o)}$ signal-to-noise ratio (SNR) per bit;

$r \cdots \dfrac{I_o + N_o}{N_o}$ rise above thermal;

$F \cdots \dfrac{1}{1+f}$ frequency reuse efficiency.

This formula applies to CDMA networks such as IS-95 that are noncooperative in the sense that they do not exploit interference through multiuser detection. This formula has historically been associated with CDMA networks when interference rises to a level where users cannot compensate for less than the desired quality of service (QoS) by increasing their transmitted power. Such a condition establishes a maximum on the number of users supported for a given QoS objective and in theory a pole exists in the transmit power required to meet the QoS. The formula solves for the number of users when all users at all base stations are exactly at the required $E_b/(I_o + N_o)$ needed to meet a QoS objective such as a mean opinion score (MOS) or a frame error rate (FER). This is a pole condition since any additional user would create interference that could not be compensated for through increases in the transmitted power.

An assumption is commonly made in deriving various forms of this formula (e.g., [1]–[4]) that the number of interfering users in the serving cell creating in-cell interference (ICI) power is the same as the number of users in each of the other base stations that create (OCI) out-of-cell interference power. Such an assumption counts the desired signal as interference which becomes increasingly significant for lower processing gains. By removing this assumption, the number of users for arbitrary processing gains and frequency reuse numbers is found. The following generalization considers the impact of both allowing and prohibiting ICI in cellular system design.

II. Spreading With In-Cell Interference

Consider an idealized hexagonal lattice of base stations where the number of users supported by each base station is increased uniformly throughout the network until the interference plus noise power is just at a level required to meet a given QoS objective. At this point, the network ideally blocks additional calls due to quality considerations. Blocking due to resource limitations, a traditional blocking mechanism applying to any cellular technology, is assumed to be insignificant.

A bit stream after source coding of R_b bits per second has a bandwidth expansion due to channel coding with code rate, R_c, and a potential bandwidth change due to modulation with a spectral efficiency of modulation, η, as shown in Fig. 1. A spreading sequence of bandwidth W increases the bandwidth before spreading, B, by a spreading gain, $G = W/B$. The positive bandwidth of this signal at RF is doubled due to the shifting of the spectrum. Tradeoffs arising from using different combinations of spreading, modulation, and coding for a fixed bandwidth and spectrum efficiency are recent areas of research (e.g., [5]–[8]). Exploring these tradeoffs requires the consideration of not only the required $E_b/(I_o + N_o)$ needed to meet a given QoS, but also the effect that the bandwidth expansion/contraction has on the received $E_b/(I_o + N_o)$ when the number of users is held constant. The maximum number of users supported occurs when all of the users are exactly meeting the requirement since the addition of users beyond this maximum cannot be accomplished

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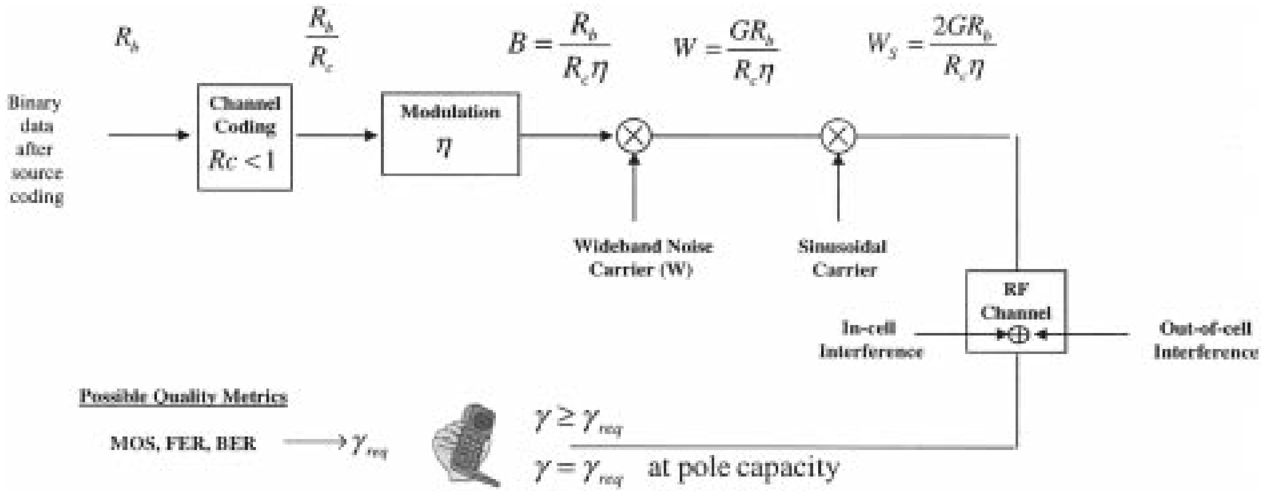

Fig. 1. System diagram and signal bandwidths for a generic communications link in a cellular network that employs direct sequence spread spectrum.

without degrading the received $E_b/(I_o + N_o)$ and correspondingly the QoS.

The total number of users, n_T, in a given available bandwidth at each base station is

$$n_T = \frac{W_A}{KW_S}n \tag{2}$$

where

- n number of users supported for each carrier at a base station;
- W_A available bandwidth to the cellular operator;
- K frequency reuse number (or cluster size).

The number of users per carrier can be found by directly writing the carrier-to-(interference plus noise) power ratio of each user assuming that the interference is ideally spread and despread as

$$\Gamma \cdots \frac{\text{Carrier Power}}{\text{ICI} + \text{ OCI} + \text{Noise Power}} = \frac{C}{\dfrac{C(n-1)d}{G} + \dfrac{nfCd}{G} + N} \tag{3}$$

where

- C received carrier power of each user;
- N noise power in the despread signal bandwidth (N_oB);
- f total interference from one out-of-cell user in all of the other cells normalized to the carrier power;
- G spreading gain;
- d reduction in interference due to the voice duty cycle.

The processing gain, G_p, defined by W_S/R_b can differ from the amount of bandwidth expansion resulting from direct sequence spreading, and thus arises the need for a spreading gain term. Also, reverse link (mobile to base station) values for f with power control can be found for $K = 1$ in [9]–[17] and for $K > 1$ in [16] and [17].

Equation (3) can be written in terms of $E_b/(I_o + N_o)$ by noting the carrier power in the numerator is E_bR_b and the total interference plus noise power in the despread bandwidth in the denominator is $(I_o + N_o)B$. Utilizing the bandwidth relationship in Fig. 1 for $(R_b/B) = \eta R_c$ gives

$$\gamma \cdots \frac{E_b}{I_o + N_o} = \frac{1}{\dfrac{n-1+nf\,\eta R_c d}{G} + \dfrac{N_o}{E_b}}. \tag{4}$$

Solving for n in (4) and substituting it into (2) gives

$$n_T = \frac{W_A GF}{\eta R_c K W_S\, d}\left(\frac{1}{\gamma}\left(1 - \frac{1}{r}\right) + \frac{d\eta R_c}{G}\right). \tag{5}$$

When $(n-1)$ in the denominator of (3) is replaced by n, the second term in (5) goes away and (5) reduces to forms in [1], [3], and [4] with $W_A = W_S$, $G = W_s/R_b$, $\eta = 1$, $R_c = 1$ and $K = 1$.

The rise above thermal is sometimes used to measure reverse link load [4] with respect to the total number of users at a pole in the carrier power when considering reverse link QoS with power control. The pole capacity is the interference limited form of (5) since at the pole, the E_b/N_o goes to infinity with the power. The pole can be seen by equating (4) with the required $E_b/(I_o+N_o)$ needed to meet the QoS objective, γ_{req}, and solving for E_b/N_o as

$$\frac{E_b}{N_o} = \frac{1}{\dfrac{1}{\gamma_{\text{req}}} - \dfrac{(n-1+nf)\eta R_c\, d}{G}}. \tag{6}$$

As the number of users per base station per carrier produces interference that approaches the requirement, the received energy per bit goes to infinity. Setting the denominator of (6) equal to zero, solving for n, and using (2) gives the interference limited form of (5). Note that (5) includes no assumptions about power control or access technique. It simply computes the number of users supported as a function of the received $E_b/(I_o + N_o)$. The interference limited form of (5) happens to also be the pole capacity since at the pole capacity noise becomes insignificant.

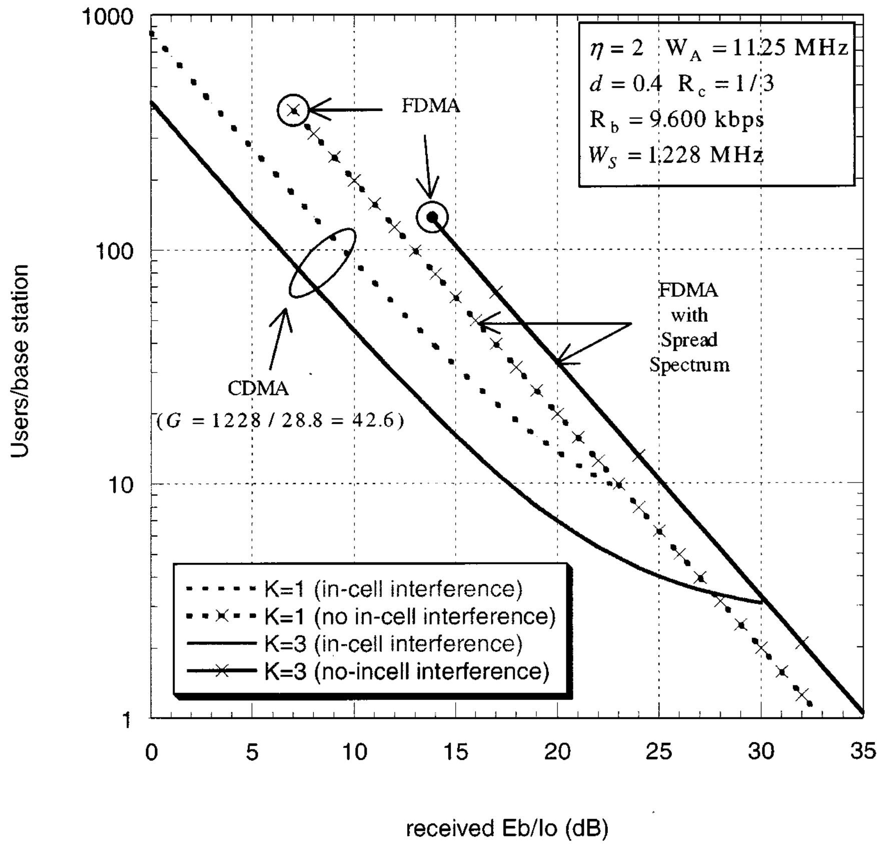

Fig. 2. Maximum number of users supported versus received E_b/I_o for the generalized pole capacity formula.

III. Spreading Without In-Cell Interference

The following equation is a lower limit on (2) by considering only a single user per carrier in each base station as:

$$n_T = \frac{W_A}{KW_S} = \frac{W_A \eta R_c}{K2GR_b}. \quad (7)$$

This is an FDMA limiting case when there is no reuse of the same channel within a base station. When $W_s = 2B$, the spreading gain is unity and this lower limit is conventional cellular FDMA with a frequency reuse number of K. For nonunity spreading gains, $E_b/(I_o + N_o)$ can be increased at the cost of a reduction in the number of users supported by increasing the spreading gain. The spreading gain from (4) when $n = 1$ is

$$G_{\text{FDMA}} = \frac{r\gamma R_c \eta d}{(r-1)} f > 1. \quad (8)$$

Substituting (8) into (7) gives the total number of users supported when spread spectrum is used with FDMA and a frequency reuse strategy prohibiting ICI as

$$n_T = \frac{1}{\gamma}\left(1 - \frac{1}{r}\right)\frac{W_A}{2Kf\,dR_b}. \quad (9)$$

At this limit, a value for f may require a different calculation than with CDMA. The OCI will be due to a smaller number of users making the interference that is averaged throughout a cell less indicative of the actual interference. Additionally, a soft handover solution may become difficult to achieve or infeasible. Various studies have computed f under a variety of conditions in [9]–[17] with primary considerations being factors such as shadowing margin, path loss slope, and the number of base stations in soft handover. Representative values for f are chosen for the purpose of illustration.

The interference limited forms of (5) and (9) are plotted in Fig. 2 for two frequency reuse numbers with $f = 0.74$ for $K = 1$ using the mean value from [9] and a value of $f = 0.148$ for $K = 3$ from [17]. A value of 0.4 for the voice duty cycle is from [3]. The spreading gain, code rate, and the spectrum efficiency of modulation match that of IS-95 reverse link traffic channels excluding orthogonal modulation. The available spectrum considered is that of an 800-MHz cellular operator deploying nine CDMA carriers. The plot shows that ICI degrades the E_b/I_o for CDMA. A hypothetical FDMA system with spread spectrum and no ICI is plotted until the FDMA spreading gain is unity giving conventional FDMA at two points that correspond to the two frequency reuse numbers. For an E_b/I_o requirement of 7 dB, roughly 200 users are shown for CDMA with $K = 1$. This is in contrast to roughly 360 users for the nine carriers using [3]. The difference is that the spreading gain used here is less than the processing gain used by [3]. Since the value of f is the same for FDMA and CDMA for each reuse number, the figure shows a roughly 4-dB cost associated with the ICI relative to using FDMA with $K = 1$. The figure also indicates that there would be a loss in E_b/I_o or a reduction in users if a channel assignment for a reuse number of 3 were used to reduce the OCI for CDMA. For high E_b/I_o design objectives, the figure shows interference benefits of conventional cellular frequency assignments for FDMA networks.

IV. Concluding Remarks

A formula that computes the number of users supported under peak load conditions was generalized and investigated under different spreading and interference conditions. This formula assumes an idealized hexagonal network of base stations, a uniform number of users in all base stations, and the same data rate requirement for all users. By considering an FDMA limit without ICI, a fundamental cost of CDMA was observed due to ICI. This cost should be offset by a reduction in the requirement as a result of using CDMA. The influence of multiuser detection on these results for CDMA is an area further research.

Frequency reuse through conventional frequency assignment with CDMA was observed to result in less users for the same E_b/I_o. When spread spectrum is combined with FDMA to eliminate ICI, higher E_b/I_o, and lower spreading gains result for the same number of users. Benefits of spread spectrum resulting in a reduction of the E_b/I_o requirement were not considered in the analysis but rather the costs associated with the received interference for a peak network load using uniform geographical assumptions. Spread spectrum and frequency reuse were considered jointly in this formulation as they both can significantly impact received interference and spectrum efficiency. The bandwidth effects of channel coding, modulation, and spread spectrum were considered as they impact the interference received by all users under peak network load conditions.

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