

On the Relationship between Transmission Power and Capacity of an Underwater Acoustic Communication Channel

Daniel E. Lucani
LIDS, MIT

Cambridge, Massachusetts, 02139
Email: dlucani@mit.edu

Milica Stojanovic
MIT

Cambridge, Massachusetts, 02139
Email: millista@mit.edu

Muriel Médard
LIDS, MIT

Cambridge, Massachusetts, 02139
Email: medard@mit.edu

Abstract—The underwater acoustic channel is characterized by a path loss that depends not only on the transmission distance, but also on the signal frequency. As a consequence, transmission bandwidth depends on the transmission distance, a feature that distinguishes an underwater acoustic system from a terrestrial radio system. The exact relationship between power, transmission band, distance and capacity for the Gaussian noise scenario is a complicated one. This work provides a closed-form approximate model for 1) power consumption, 2) band-edge frequency and 3) bandwidth as functions of distance and capacity required for a data link. This approximate model is obtained by numerical evaluation of analytical results which takes into account physical models of acoustic propagation loss and ambient noise. The closed-form approximations may become useful tools in the design and analysis of underwater acoustic networks.

I. INTRODUCTION

With the advances in acoustic communication technology, the interest in study and experimental deployment of underwater networks has been growing [1]. However, underwater acoustic channels impose many constraints that affect the design of wireless networks. They are characterized by a path loss that depends on both the transmission distance and the signal frequency, a feature that distinguishes an underwater acoustic system from a terrestrial radio system. Thus, not only the power consumption, but also the useful bandwidth depend on the transmission distance [2].

From an information theoretic perspective, both the distance between two nodes and the required capacity determine the power consumption for that link and the optimal transmission band. It is thus of interest to have a simple, closed-form expression that relates the transmission power to the desired capacity. This would enable an efficient design of both point to point links and underwater networks, eventually leading to a minimum cost overall network optimization. Thus, these expressions may be useful from both a theoretic and an engineering standpoint.

In this paper, simple closed-form approximations for the power consumption and operating frequency band as functions of distance and capacity are presented. This approximate model stems from an information theoretic analysis that takes into account a physical model of acoustic propagation loss,

and colored Gaussian ambient noise. It was shown in [2] that the transmission power as a function of the distance could be well approximated by $P(l) = pl^\gamma$. A similar relationship was shown to exist for the operating bandwidth. The coefficients in this model were determined as functions of the required signal to noise ratio.

The present work extends this idea of modeling the power and bandwidth as functions of distance, but the problem is cast into a slightly different framework. Namely, instead of using the SNR as a constraint, i.e. a fixed design parameter, the desired link capacity is used as a figure of merit. In few words, this work proposes approximate models for the parameters as functions of the capacity. This resulting model is useful for a broad range of capacities and distances.

The paper is organized as follows. In Section 2, a model of an underwater channel is outlined. In Section 3, a brief description of the numerical evaluation procedure is described. In Section 4, closed-form expressions for the parameters of interest are presented. Section 5 gives numerical results for different ranges of distance and capacity. Conclusions are summarized in the last section.

II. CHANNEL MODEL

An underwater acoustic channel is characterized by a path loss that depends on both distance l and signal frequency f as

$$A(l, f) = l^k a(f)^l \quad (1)$$

where k is the spreading factor and $a(f)$ is the absorption coefficient [2]. The spreading factor describes the geometry of propagation, e.g. $k = 2$ corresponds to spherical spreading, $k = 1$ to cylindrical spreading, and $k = 1.5$ to practical spreading. The absorption coefficient can be expressed in dB/km using Thorp's empirical formula for f in kHz:

$$10 \log a(f) = 0.11 \frac{f^2}{1 + f^2} + 44 \frac{f^2}{4100 + f^2} + 2.75 \cdot 10^{-4} f^2 + 0.003 \quad (2)$$

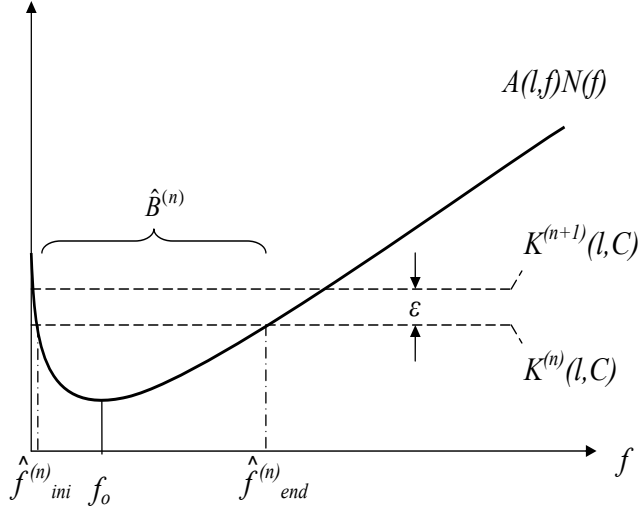


Fig. 1. Numerical procedure for computation of $P(l, C)$, $\hat{f}_{end}(l, C)$ and $\hat{B}(l, C)$, by incrementing $K(l, C)$ at each step by ϵ until a stopping condition is fulfilled

for frequencies above a few hundred Hz. For lower frequencies, the model is:

$$10 \log a(f) = 0.11 \frac{f^2}{1 + f^2} + 0.011f^2 + 0.002 \quad (3)$$

The noise in an acoustic channel can be modeled through four basic sources: turbulence, shipping, waves, and thermal noise[2]. The following formulas give the power spectral density (psd) of these noise components in dB re μ Pa per Hz as a function of frequency in kHz:

$$\log N_t(f) = 1.7 - 3 \log f \quad (4)$$

$$\log N_s(f) = 4 + 2(s - \frac{1}{2}) + 2.6 \log f - 6 \log(f + 0.03) \quad (5)$$

$$\log N_w(f) = 5 + 0.75w^{1/2} + 2 \log f - 4 \log(f + 0.4) \quad (6)$$

$$\log N_{th}(f) = -1.5 + 2 \log f \quad (7)$$

where the shipping activity s ranges from 0 to 1, for low and high activity, respectively, and w corresponds to the wind speed measured in m/s. The overall psd of the ambient noise is given by

$$N(f) = N_t(f) + N_s(f) + N_w(f) + N_{th}(f) \quad (8)$$

Let us assume that this is a Gaussian channel. Then, the capacity of this channel can be obtained using the waterfilling principle [2]. Also, assume that the power and band of operation can be adjusted to reach a certain capacity level. Thus, the capacity of a point-to-point link is

$$C = \int_{B(l, C)} \log_2 \left(\frac{K(l, C)}{A(l, f)N(f)} \right) df \quad (9)$$

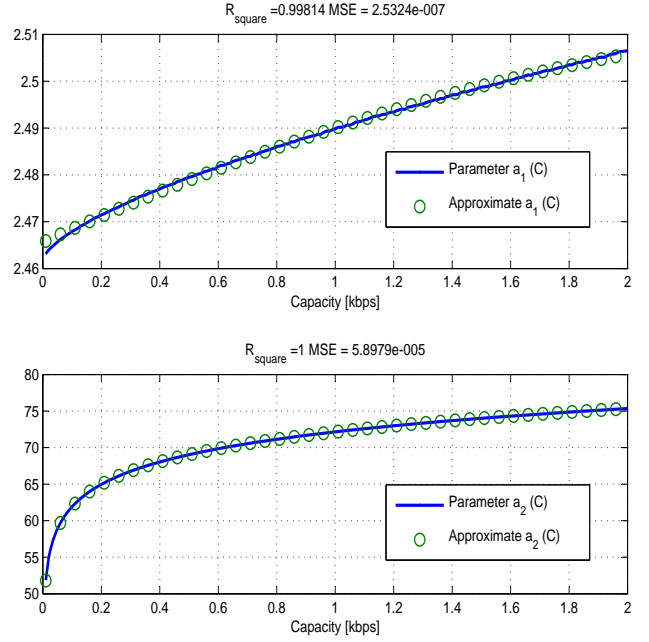


Fig. 2. Parameters a_1 and a_2 for $P(l, C)$ and approximate model $l \in [0, 10km]$, $C \in [0, 2kbps]$, $k = 1.5$, $s = 0.5$ and $w = 0m/s$

where $B(l, C)$ is the optimum band of operation. This band could be thought of as a union of non-overlapping intervals, $B(l, C) = \cup_i [f_{ini}^i(l, C), f_{end}^i(l, C)]$, where each non-overlapping band i has the lower-end frequency $f_{ini}^i(l, C)$ and the higher-end frequency $f_{end}^i(l, C)$ associated to it. In its simplest form $B(l, C) = [f_{ini}(l, C), f_{end}(l, C)]$.

The power consumption associated with a particular choice of (l, C) is given by

$$P(l, C) = \int_{B(l, C)} S(l, C, f) df \quad (10)$$

where $S(l, C, f) = K(l, C) - A(l, f)N(f)$, $f \in B(l, C)$.

Evidently, these expressions are quite complicated to be used in a computational network analysis. Also, they provide little insight into the relationship between power consumption, \hat{f}_{ini} and \hat{f}_{end} , in terms of the pair (l, C) . This motivates the need for an approximate model that will represent these relations for ranges of C and l that are of interest to acoustic communication systems. The model should also provide flexibility to changing other parameters, such as the spreading factor k , wind speed w and shipping activity s .

The dependence on the spreading factor k is quite simple. Let us assume that a model for $P(l, C)$ has been developed for a particular value of $k = k_i$, i.e. $P(l, C, k_i)$. To determine $P(l, C, k_j)$ for $k_j \neq k_i$, let us note that for a change in k , the product $A(l, f)N(f) = l^k a(f)^l N(f)$ constitutes a constant scaling factor with respect to f . Therefore, for a link of distance l the term $B(l, C)$ will remain unchanged. Thus, if the same capacity C is required for k_i and k_j , equation (9) shows that the only other term that can vary is $K(l, C)$,

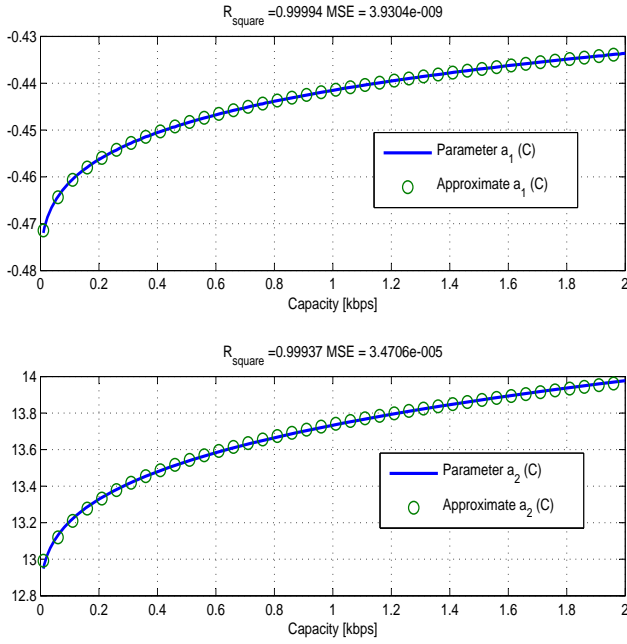


Fig. 3. Parameters a_1 and a_2 for $\hat{f}_{end}(l, C)$ and approximate model $l \in [0, 10km]$, $C \in [0, 2kbps]$, $k = 1.5$, $s = 0.5$ and $w = 0m/s$

i.e. $K(l, C, k)$. Then, $K(l, C, k_j) = l^{k_j - k_i} K(l, C, k_i)$. Finally, let us use the equation (10) to determine the relationship between $P(l, C, k_i)$ and $P(l, C, k_j)$. The dependence on the spreading factor k is quite simple. Let us assume that a model for $P(l, C)$ has been developed for a particular value of $k = k_i$, i.e. $P(l, C, k_i)$. To determine $P(l, C, k_j)$ for $k_j \neq k_i$. Note that for a change in k , the product $A(l, f)N(f) = l^k a(f)^l N(f)$ constitutes a constant scaling factor with respect to f . Therefore, for a link of distance l the term $B(l, C)$ will remain unchanged. Thus, if the same capacity C is required for k_i and k_j , equation (9), shows that the only other term that can vary is $K(l, C)$, i.e. $K(l, C, k)$. Then, $K(l, C, k_j) = l^{k_j - k_i} K(l, C, k_i)$. Finally, let us use equation (10) to determine the relation between $P(l, C, k_i)$ and $P(l, C, k_j)$.

$$P(l, C, k_j) = \int_{B(l, C)} (K(l, C, k_j) - l^{k_j} a(f)^l N(f)) df \quad (11)$$

$$= l^{k_j - k_i} \int_{B(l, C)} (K(l, C, k_i) - l^{k_i} a(f)^l N(f)) df \quad (12)$$

$$= l^{k_j - k_i} P(l, C, k_i) \quad (13)$$

Thus, any model for the transmission generated for some parameter k has a simple extension. Also, note that the transmission bandwidth remains the same for any value of k .

III. NUMERICAL EVALUATION PROCEDURE

A numerical evaluation procedure similar to that in [2] is used to compute the value of $P(l, C)$, $\hat{f}_{ini}(l, C)$ and $\hat{f}_{end}(l, C)$, for a region of values of (l, C) . The procedure

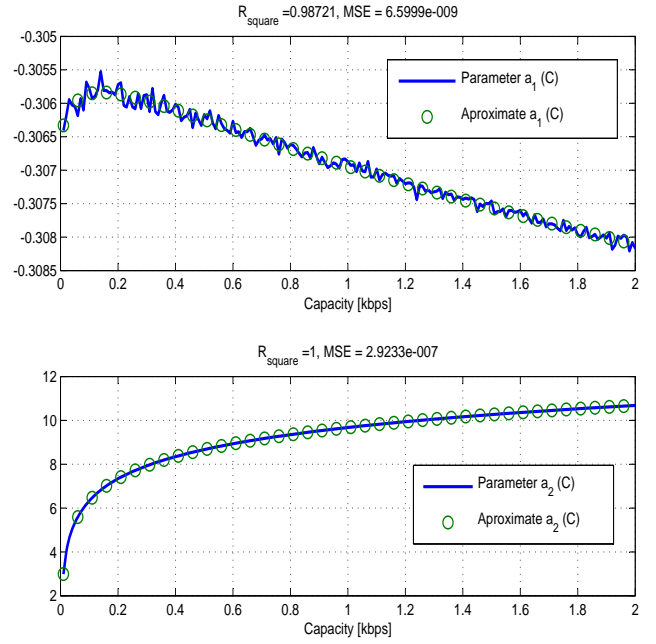


Fig. 4. Parameters a_1 and a_2 for $\hat{B}(l, C)$ and approximate model $l \in [0, 10km]$, $C \in [0, 2kbps]$, $k = 1.5$, $s = 0.5$ and $w = 0m/s$

starts by fixing a target value of the capacity C . Then, for each distance l , the initial value of $K(l, C)$ is set to the minimum value of the product $A(l, f)N(f)$, i.e. $K(l, C) = \min_f A(l, f)N(f)$. The frequency at which this occurs, i.e. $f_0 = \arg \min_f A(l, f)N(f)$, is called the optimal frequency.

After this, $K(l, C)$ is increased iteratively by a small amount (Figure 1), until the target capacity value C is met. Finally, this procedure is repeated for each value of C in a range of interest.

At the n -th step of the procedure, when $K^{(n)}(l, C)$ is increased by a small amount, the band $B^{(n)}(l, C)$ is determined for that iteration. This band is defined as the range of frequencies for which the condition $A(l, f)N(f) \leq K^{(n)}(l, C)$. Then, the capacity $C^{(n)}$ is numerically determined for the current $K^{(n)}(l, C)$ and $B^{(n)}(l, C)$, using the equation (9). If $C^{(n)} < C$, a new iteration is performed. Otherwise, the procedure stops.

IV. APPROXIMATE MODELS

By applying the above procedure for varying l and C , one arrived at the complete model for the power consumption

$$P(l, C) = 10^{\frac{a_1(C)}{10}} l^{a_2(C)} \quad (14)$$

where

$$a_1(C) = \beta_3 + \beta_2 10 \log_{10} C + \beta_1 (10 \log_{10}(C + 1))^2 \quad (15)$$

$$a_2(C) = \alpha_3 + \alpha_2 C + \alpha_1 C^2 \quad (16)$$

Below, two ranges of operation were studied. The first one is for $l \in [0, 10]km$, $C \in [0, 2]kbps$, and propagation factor

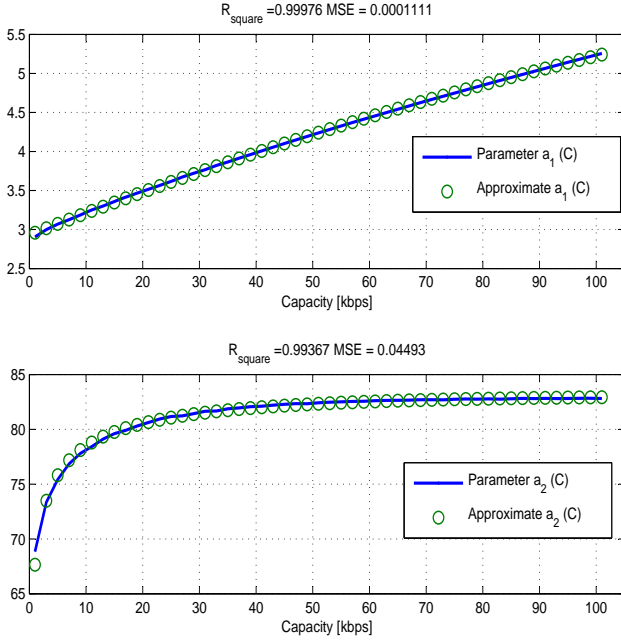


Fig. 5. Parameters a_1 and a_2 for $P(l, C)$ and approximate model $l \in [0, 100]km$, $C \in [0, 100]kbps$, $k = 1.5, s = 0.5$ and $w = 0m/s$

of $k = 1.5$, which will be called case 1 from here on. The second one is for $l \in [0, 100]Kms$, $C \in [0, 100]kbps$, and propagation factor of $k = 1.5$, which will be called case 2. For both regions and different ranges of s and w , the power consumption $P(l, C)$ can be approximated by equations (14), (15) and (16).

Similar model are found to provide a good fit for the high/end frequency $\hat{f}_{end}(l, C)$ and for the bandwidth $\hat{B}(l, C) = \hat{f}_{end}(l, C) - \hat{f}_{ini}(l, C)$. These models are given by

$$\hat{f}_{end}(l, C) = 10^{\frac{\alpha_1(C)}{10}} l^{\alpha_2(C)} \quad (17)$$

where

$$a_1(C) = \beta_3 + \beta_2 10 \log_{10} C + \beta_1 (10 \log_{10} C)^2 \quad (18)$$

$$a_2(C) = \alpha_3 + \alpha_2 10 \log_{10} C + \alpha_1 (10 \log_{10} C)^2 \quad (19)$$

$$\hat{B}(l, C) = 10^{\frac{\alpha_1(C)}{10}} l^{\alpha_2(C)} \quad (20)$$

where

$$a_1(C) = \beta_3 + \beta_2 10 \log_{10} C + \beta_1 (10 \log_{10} C)^2 \quad (21)$$

$$a_2(C) = \alpha_4 + \alpha_3 10 \log_{10} C + \alpha_2 (10 \log_{10} C)^2 + \alpha_1 (10 \log_{10} C)^3 \quad (22)$$

V. NUMERICAL RESULTS

The transmission power, highest frequency and bandwidth of transmission band were computed for a variety of values of s , w and two ranges of interest of the pair (l, C) , i.e. $l \in [0, 10]Kms$, $C \in [0, 2]kbps$, and $l \in [0, 100]Kms$,

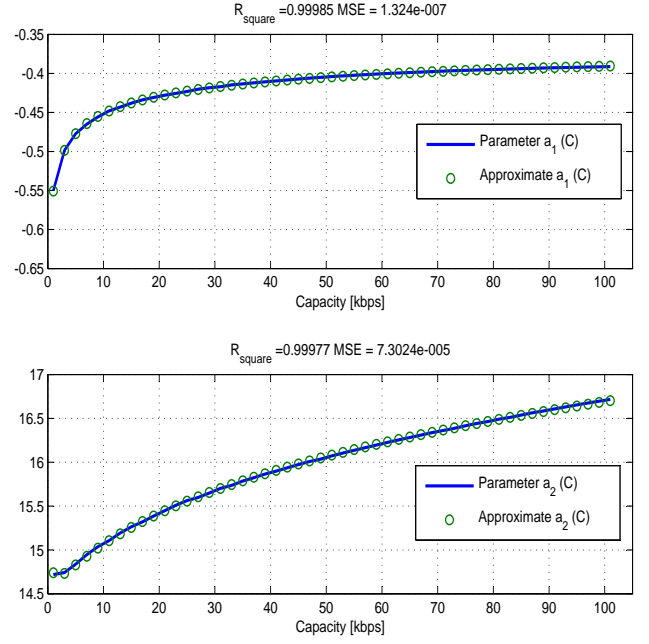


Fig. 6. Parameters a_1 and a_2 for $\hat{f}_{end}(l, C)$ and approximate model $l \in [0, 100]km$, $C \in [0, 100]kbps$, $k = 1.5, s = 0.5$ and $w = 0m/s$

TABLE I

a_1 APPROXIMATION PARAMETER VALUES FOR $P(l, C)$, $\hat{f}_{end}(l, C)$ AND $\hat{B}(l, C)$, WITH $l \in [0, 10]km$, $C \in [0, 2]kbps$, $k = 1.5, s = 0.5$ AND $w = 0m/s$

	α_1	α_2	α_3	α_4	MSE
$P(l, C)$	0	-0.00432	0.02873	2.46560	2.532e-7
$\hat{f}_{end}(l, C)$	0	4.795e-5	0.00246	-0.44149	3.930e-9
$\hat{B}(l, C)$	-5.958e-7	-2.563e-5	-0.000305	-0.30694	6.599e-9

$C \in [0, 100]kbps$. The models proposed fitted these cases quite well. Results are presented for the case of $k = 1.5$, $w = 0$ and $s = 0.5$, for both cases. Also for case 1, it will be seen that the α and β parameters show almost no dependence on the shipping activity factor s , especially if the wind speed is $w > 0$. Thus, the approximate model for this case could be simplified to only consider w as part of the model, instead of the pair (s, w) .

Figures 2, 3 and 4 show parameters a_1 and a_2 for $P(l, C)$, $\hat{f}_{end}(l, C)$, and $\hat{B}(l, C)$, respectively. This approximation was carried out for the first case with a propagation factor of $k = 1.5$, a shipping activity of $s = 0.5$ and a wind speed of $w = 0m/s$. The values of α 's and β 's are shown in Table I and II, for parameters a_1 and a_2 , respectively. These tables also show the mean square error (MSE) of the approximation with respect to the actual parameters. In Figure 4, there is a considerable variation in the values of parameter $a_1(C)$. However, note that the y-axis of the plot shows very little variation.

Figures 5, 6 and 7 show parameters a_1 and a_2 for $P(l, C)$, $\hat{f}_{end}(l, C)$, and $\hat{B}(l, C)$, respectively. This approxi-

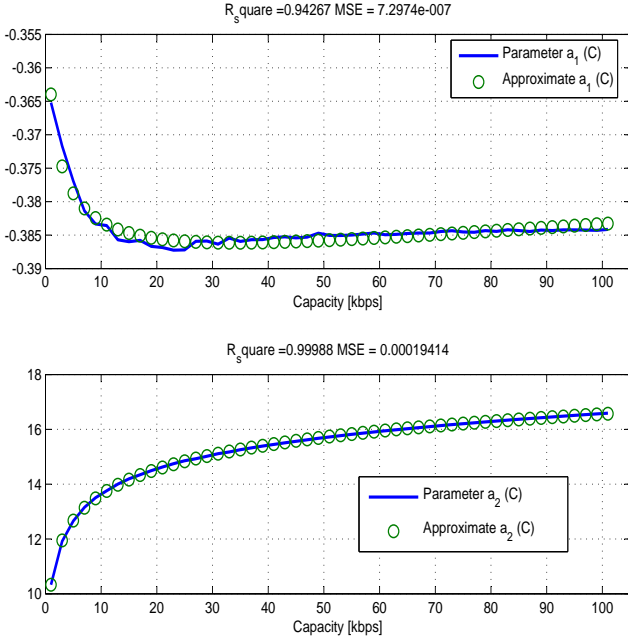


Fig. 7. Parameters a_1 and a_2 for $\hat{B}(l, C)$ and approximate model $l \in [0, 100km]$, $C \in [0, 100kbps]$, $k = 1.5$, $s = 0.5$ and $w = 0m/s$

TABLE II

a_2 APPROXIMATION PARAMETER VALUES FOR $P(l, C)$, $\hat{f}_{end}(l, C)$ AND $\hat{B}(l, C)$, WITH $l \in [0, 10km]$, $C \in [0, 2kbps]$, $k = 1.5$, $s = 0.5$ AND $w = 0m/s$

	β_1	β_2	β_3	MSE
$P(l, C)$	0.01166	1.0117	72.043	5.8979e-5
$\hat{f}_{end}(l, C)$	0.00171	0.07153	13.738	3.4706e-5
$\hat{B}(l, C)$	-5.163e-6	0.33427	9.6752	2.9233e-7

mation was carried out for the second case with a propagation factor of $k = 1.5$, a shipping activity of $s = 0.5$ and a wind speed of $w = 0$. The values of α 's and β 's are shown in Table III and IV, for parameters a_1 and a_2 , respectively. These tables also show the mean square error (MSE) of the approximation with respect to the actual parameters.

For both ranges, the proposed models give a very good approximation to the actual numerical values. Also note that for the $a_2(C)$ parameter of $P(l, C)$, it is possible to use a linear approximation, instead of a quadratic model.

Let us analyze the low range low rate for different values of s and w . Table V shows the values for α and β parameters

TABLE III

a_1 APPROXIMATION PARAMETER VALUES FOR $P(l, C)$, $\hat{f}_{end}(l, C)$ AND $\hat{B}(l, C)$, WITH $l \in [0, 100km]$, $C \in [0, 100kbps]$, $k = 1.5$, $s = 0.5$ AND $w = 0m/s$

	α_1	α_2	α_3	α_4	MSE
$P(l, C)$	0	-5.617e-5	0.02855	2.9305	0.00011
$\hat{f}_{end}(l, C)$	0	-0.00019	0.01186	-0.55076	1.32e-7
$\hat{B}(l, C)$	1.696e-6	4.252e-5	-0.00249	-0.36397	7.29e-7

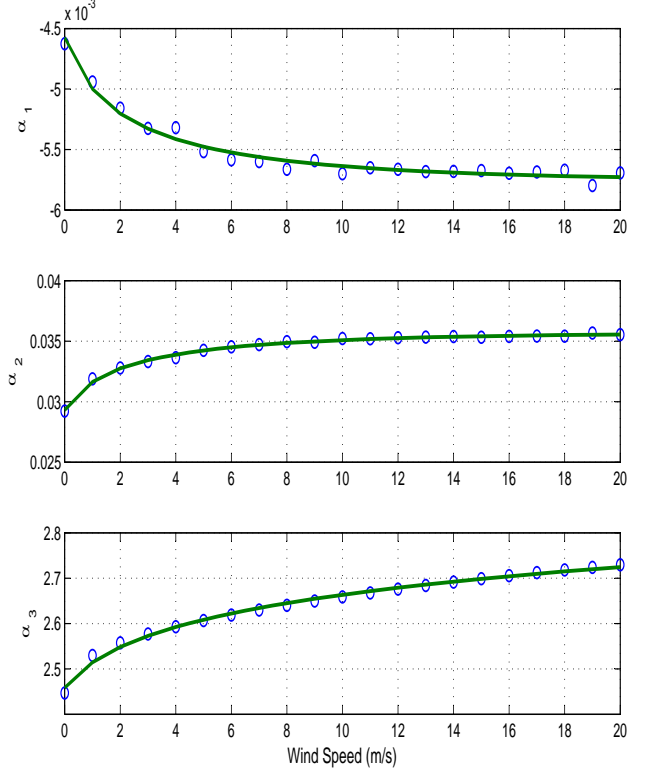


Fig. 8. Values of α as function of w . $l \in [0, 10km]$, $C \in [0, 2kbps]$, $k = 1.5$, $s = 0.5$

TABLE IV

a_2 APPROXIMATION PARAMETER VALUES FOR $P(l, C)$, $\hat{f}_{end}(l, C)$ AND $\hat{B}(l, C)$, WITH $l \in [0, 100km]$, $C \in [0, 100kbps]$, $k = 1.5$, $s = 0.5$ AND $w = 0m/s$

	β_1	β_2	β_3	MSE
$P(l, C)$	-0.032936	1.4104	67.946	0.04493
$\hat{f}_{end}(l, C)$	0.0065157	-0.032693	14.739	7.3024e-5
$\hat{B}(l, C)$	-0.0018252	0.34788	10.328	0.00019414

in the approximate $P(l, C)$ model for variations of s and w . It is interesting that the parameters change very little with respect to the shipping activity s while they show greater dependency on the wind speed factor. This is not unexpected. For low data rates and low ranges, the transmission band is at a high frequency (between 5 and 40 KHz) and from the noise equations of the model the shipping activity depends on the frequency as $O(f^{-3.4})$ while the wind speed factor has a dependency as $O(f^{-2})$. Thus, w should have much more effect upon the parameters.

Therefore, a further approximation is to discard s and consider parameters α and β to be functions of w only. Figure 9 shows these relations when computed for this region a $k = 1.5$ and $s = 0.5$, which have a very simple approximation. For example, the model in equation 23 gives a good approximation.

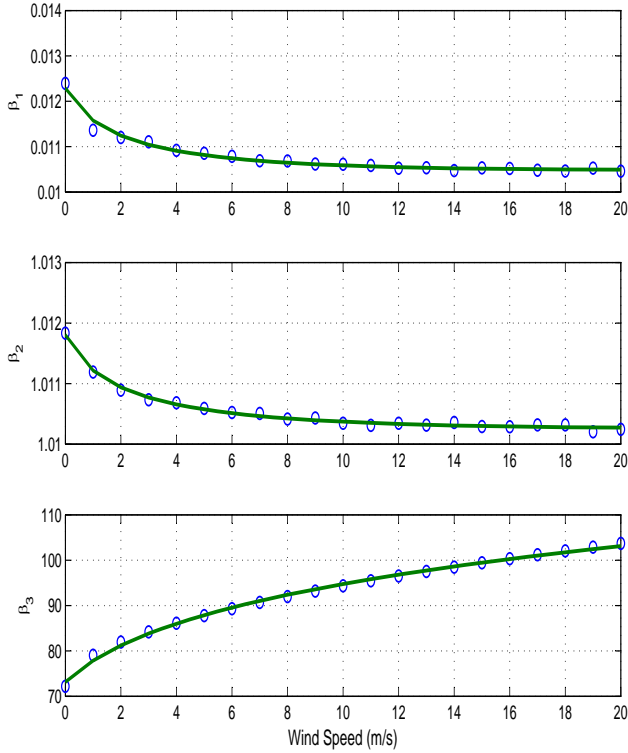


Fig. 9. Values of β as function of w . $l \in [0, 10km]$, $C \in [0, 2kbps]$, $k = 1.5, s = 0.5$

$$\psi_i(w) = \gamma_3 + \gamma_2 10 \log_{10}(w + 1) + \gamma_1 (10 \log_{10}(w + 1))^2 \quad (23)$$

where $\psi_i(w) = \beta_i, \forall i$ and $\psi_i(w) = \alpha_i, \forall i$. Table VI shows γ parameters for the different α 's and β 's.

VI. CONCLUSION

This paper offers an insight into the dependence of the transmission power, bandwidth, and the band-edge frequency of an underwater acoustic link on the capacity and distance. It provides closed-form approximate models for the time-invariant acoustic channel, taking into account a physical model of acoustic path loss and the ambient noise, assuming that the channel is Gaussian. These approximate models were shown to provide a good fit to the actual empirical values by numerical evaluation for different ranges of distance l and capacity C , as well as noise profiles corresponding to different shipping activity factor and wind speed.

The band-edge frequency $f_{end}(l, C)$ and the bandwidth $B(l, C)$ were also shown to be invariant to the spreading factor k , while the power scales as $P(l, C, k') = l^{k' - k} P(l, C, k)$.

For a certain range of values (l, C) , the approximate model of $P(l, C)$ was shown to be almost independent of the shipping activity factor s while having a marked dependency on the wind speed w . This dependence, however, is quite smooth and could be approximated by a simple model, thus resulting in a

TABLE V
DEPENDENCY ON s AND w OF α AND β PARAMETERS, WITH
 $l \in [0, 10km]$, $C \in [0, 2kbps]$, $k = 1.5$

w	s	α_1	α_2	α_3	β_1	β_2	β_3
0	0	-0.0050	0.0299	2.444	0.01237	1.0118	72.182
	0.5	-0.0046	0.0292	2.447	0.01239	1.0118	72.190
	1	-0.0049	0.0304	2.469	0.01236	1.0119	72.271
2	0	-0.0053	0.0328	2.5572	0.01117	1.0109	81.960
	0.5	-0.0052	0.0327	2.5574	0.01120	1.0109	81.961
	1	-0.0056	0.0336	2.5594	0.01133	1.0108	81.966
5	0	-0.0056	0.0342	2.6065	0.01083	1.0106	87.777
	0.5	-0.0055	0.0342	2.6066	0.01085	1.0106	87.777
	1	-0.0053	0.0339	2.6071	0.01088	1.0105	87.778
10	0	-0.0057	0.0351	2.6588	0.01061	1.0103	94.360
	0.5	-0.0057	0.0352	2.6588	0.01061	1.0104	94.360
	1	-0.0056	0.0351	2.6589	0.01059	1.0104	94.360
20	0	-0.0083	0.0420	2.7245	0.01079	1.0094	103.69
	0.5	-0.0057	0.0355	2.7295	0.01047	1.0102	103.70
	1	-0.0057	0.0355	2.7294	0.01046	1.0102	103.70

TABLE VI
APPROXIMATION PARAMETERS OF α AND β FOR $P(l, C)$, WITH
 $l \in [0, 10km]$, $C \in [0, 2kbps]$, $k = 1.5, s = 0.5$

	γ_1	γ_2	γ_3
α_1	5.2669e-6	-0.000157	-0.004575
α_2	-2.971e-5	0.000865	0.029306
α_3	0.000152	0.01809	2.4586
β_1	9.924e-6	-0.00027	0.012288
β_2	7.799e-6	-0.000219	1.0118
β_3	0.068091	1.3659	73.144

complete model for the $P(l, C)$ for a range of values (l, C) that is of interest to a typical underwater communication system. Hence, these models can be used in network optimization problems to determine the optimal power consumption for some required data rate.

Future work will focus on studying convexity properties of the $P(l, C)$ model and using it in network optimization problems.

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