

Collision Helps

Algebraic Collision Recovery for Wireless Erasure Networks

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Abstract—Current medium access control mechanisms are based on collision avoidance and collided packets are discarded. The recent work on ZigZag decoding departs from this approach by recovering the original packets from multiple collisions. In this paper, we present an algebraic representation of collisions which allows us to view each collision as a linear combination of the original packets. The transmitted, colliding packets may themselves be a coded version of the original packets.

We propose a new acknowledgment (ACK) mechanism for collisions based on the idea that if a set of packets collide, the receiver can afford to ACK exactly one of them and still decode all the packets eventually. We analytically compare delay performance of such collision recovery schemes with other collision avoidance approaches in the context of a single hop wireless erasure network. From the delay perspective, our scheme, without any coordination, outperforms not only ALOHA-type random access mechanisms, but also centralized scheduling.

I. INTRODUCTION

The nature of the wireless network is intrinsically different from the wired network because of the sharing of the medium among several transmitters. Such a restriction generally has been managed through forms of scheduling algorithms to coordinate access to the medium, usually in a distributed manner. The conventional approach to the Medium Access Control (MAC) problem is contention-based protocols in which multiple transmitters simultaneously attempt to access the wireless medium and operate under some rules that provide enough opportunities for the others to transmit. Examples of such protocols in packet radio networks include ALOHA, MACAW, CSMA/CA, etc[1].

However, in many contention-based protocols, it is possible that two or more transmitters transmit their packet simultaneously, resulting in a *collision*. The collided packets are considered useless in the conventional approaches. There is a considerable literature on extracting partial information from such collisions. Gollakota and Katabi [4] showed how to recover multiple collided packets in a 802.11 system using *ZigZag decoding* when there are enough transmissions involving those packets. In fact, they suggest that each collision can be treated as a linearly independent equation of the packets involved. ZigZag decoding is based on interference cancelation, and hence, requires a precise estimation of channel attenuation

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and phase shift for each packet involved in a collision. ZigZag decoding provides a fundamentally new approach to manage interference in a wireless setting that is essentially decentralized, and can recover losses due to collisions. In this work, we wish to understand the effects of this new approach to interference management in the high SNR regime, where interference, rather than noise, is the main limiting factor for system throughput.

We first provide an abstraction of a single-hop wireless network with erasures when a generalized form of ZigZag decoding is used at the receiver, and network coding is employed at the transmitters. We introduce an algebraic representation of the collisions at the receivers, and study conditions under which a collision can be treated as a linearly independent equation (degree of freedom) of the original packets at the senders. We use this abstract model to analyze the delay performance of the system in various scenarios.

Second, we analyze a single-hop wireless erasure network, when each sender has one packet to deliver to all of its neighbors. We characterize the expected time to deliver all of the packets to each receiver when collisions of arbitrary number of packets are recoverable. We observe that with collision recovery we can deliver n packets to a receiver in $n+O(1)$ time slots, where n is the degree (contention level) of that particular receiver. This is significantly smaller than the delivery time of centralized scheduling and contention-based mechanisms such as slotted ALOHA. In the case that collisions of only a limited number of packets can be recovered, we propose a random access mechanism in conjunction with collision recovery to limit the level of contention at the receiver. Our numerical results show that such a scheme provides a significant improvement upon contention-based mechanisms even if each recoverable collision is limited to only two packets.

The literature related to collision recovery methods include the works by Tsatsanis *et al.* [9], and Paek and Neely [7]. In this literature, once a collision of k packets occurs, all senders remain silent until those involved in the collision retransmit another $k - 1$ times. Our proposed scheme, however, does not require such coordination and consecutive collisions can consist of arbitrary collection of packets.

The rest of this paper is organized as follows. In Section II, we present an abstract model of a single-hop wireless network with erasures. Section II-A discusses an algebraic representation of the collisions at the receivers. Section III is dedicated to mean delivery time characterization of a single-receiver system

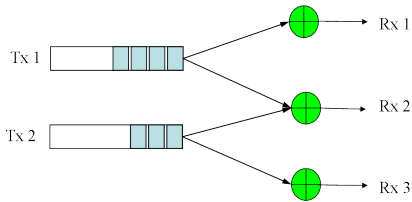


Fig. 1. Single-hop wireless network model with n senders and r receivers

for various interference management schemes. In Section IV, we generalize the results of the preceding section to the case of a single-hop wireless network with multiple receivers. Finally, concluding remarks and extensions are discussed in Section V.

II. SYSTEM MODEL

The system consists of a single-hop wireless network with n senders and r receivers. We assume that a node cannot be both a sender and a receiver. The connectivity is thus specified by a bipartite graph. Fig. 1 shows an example of such a network.

We assume that time is slotted. Every sender is equipped with an infinite sized buffer. The goal of a sender is to deliver all of its packets to each of its neighbors, i.e., the set of receivers to which it is connected.

In every slot, a sender can *broadcast* a packet to its neighbors. Owing to the fading nature of the wireless channel, not all packet transmissions result in a successful reception at every neighbor. Each link between any sender i and any receiver j may experience packet erasures. These erasures occur with probability p , and are assumed to be independent across links and over time. This type of erasure is to model the effect of obstacles between the senders and the receivers. The channel state between i and j at time slot t is denoted by $c_{ij}(t)$.

At the end of every slot, each receiver is allowed to send an acknowledgment (ACK) to any one of the senders to which it is connected. A packet is retained in the sender's queue until it has been acknowledged by all the receivers. We ignore the overhead caused by the ACKs, and assume that the ACKs are delivered reliably without any delay.

Note that a collision of packets at a receiver does not immediately imply an erasure. It may be possible to extract useful information from collisions. In the following, we discuss how a collision could be thought of as a linear combination of the original packets at the sender.

A. An algebraic representation of collisions

In this section, we introduce an algebraic representation of collisions. The collision of two packets is essentially the superposition of the physical signal corresponding to the packets. A packet is a vector of bits that can be grouped into symbols over a finite field \mathbb{F}_l . For the rest of this section, we represent a packet as a polynomial over the delay variable D , with coefficients being the symbols of \mathbb{F}_l that form the packet. The mapping from the packet to the corresponding physical signal is a result of two operations – channel coding and modulation.

We abstract these two operations in the form of a map M from symbols over \mathbb{F}_l to the complex number field:

$$M : \mathbb{F}_l \rightarrow \mathbb{C}$$

We assume that the map M is such that given a complex number, there is a well-defined demodulation and channel decoding method that outputs the symbol from \mathbb{F}_l that is most likely to have been transmitted.

Remark 1: The above assumption essentially says that the channel coding occurs over blocks of $\log_2 l$ bits (corresponding to a single symbol of \mathbb{F}_l). Depending on l , this could mean a short code length, which would be effective only at high SNR.

Let $X(D)$ and $Y(D)$ be two packets at two different senders, represented as polynomials over \mathbb{F}_l . The coding and modulation results in a signal polynomial over the complex field: $S_X(D)$ and $S_Y(D)$. Suppose that these two packets collide with each other at a receiver twice, in two different time slots. We denote $h_j^{(t)}$ to be the channel coefficient in slot t from sender j .

When packets collide, they may not be perfectly aligned. Let $u_j^{(t)}$ denote the offset (in symbols) of the packet from sender j within slot t measured from the beginning of the slot. We assume that a packet is significantly longer than the offsets, so that the loss of throughput because of these offsets is negligible.

The channel gains, offsets and the identity of the packets that are involved in the collision are assumed to be known at the receiver. Then, the two collisions can be represented thus:

$$\begin{pmatrix} C_1(D) \\ C_2(D) \end{pmatrix} = \begin{pmatrix} h_1^{(1)} D^{u_1^{(1)}} & h_2^{(1)} D^{u_2^{(1)}} \\ h_1^{(2)} D^{u_1^{(2)}} & h_2^{(2)} D^{u_2^{(2)}} \end{pmatrix} \begin{pmatrix} S_1(D) \\ S_2(D) \end{pmatrix},$$

or alternately, $C = HS$.

Therefore, with n collisions of the same n packets, it is possible to decode them all as long as the $n \times n$ transfer matrix H is invertible over the field of rational functions of D . The process of decoding by inverting this matrix is more general than the ZigZag procedure of [4]. The decoding process will result in the signals corresponding to the original packets. The signals will then have to be demodulated and decoded (channel coding) to obtain the original data. This algebraic representation formalizes the intuition introduced in [4] that every collision is like a linear equation in the original packets.

B. Combining packet coding with collision recovery

Due to the broadcast constraint of the wireless medium, a sender that wants to broadcast data to several receivers will have to code across packets over a finite field in order to achieve the maximum possible throughput. Random linear coding is known to achieve the multicast capacity over wireless erasure networks [6]. Suppose that the sender codes across packets over the field \mathbb{F}_l and that the coding coefficients are known at the receiver.

This can also be incorporated into the above formulation in the following sense. Suppose a receiver receives n collisions, where the colliding packets in each collision are themselves finite-field linear combinations of a collection of n original packets, then it is possible to decode all n packets.

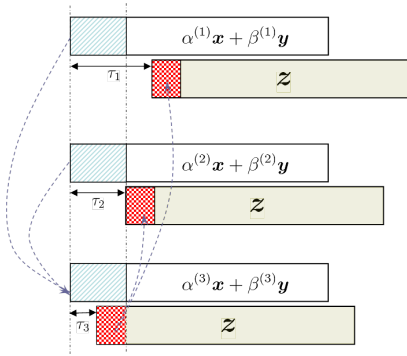


Fig. 2. Combining packet coding with collision recovery

This is immediately seen if we assume that the coding and modulation are linear operations, i.e., that M is a linear function with respect to the symbols of the original packets. In this case, the above matrix representation will still hold, and the invertibility condition for decoding will also be true. However, in general, the modulation operation may not be linear with respect to the original packets' symbols. Even in this case, we can still decode the n packets from n collisions.

We explain this using a simple example with two senders and one receiver. Suppose the first sender has two packets \mathbf{x} and \mathbf{y} and the second sender has a single packet \mathbf{z} . The first sender transmits a random linear combination of its two packets in every slot, while the second sender repeats packet \mathbf{z} in every slot. Figure 2 shows the collisions in three different time slots. Using the three collisions, the receiver can decode all three packets as follows. The offsets between the first and second senders' packets in the three collisions are τ_1 , τ_2 and τ_3 . From the figure, since the first τ_2 symbols of the first two collisions are interference-free, we can decode the first τ_2 symbols of \mathbf{x} and \mathbf{y} . Using this, we can compute the first τ_2 symbols of $\alpha^{(3)}\mathbf{x} + \beta^{(3)}\mathbf{y}$, and thereby obtain the first $(\tau_2 - \tau_3)$ symbols of \mathbf{z} . This process can be continued after subtracting these symbols from the other collisions.

We assume throughout this paper that the field size l is large enough such that every collision counts as a new degree-of-freedom (also called *innovative*) if and only if it involves at least one packet that has not yet been decoded. Every such collision counts as one step towards decoding the packets.

III. SINGLE RECEIVER CASE

In this section, we study a special case where there is only one receiver in the network. We shall show later in this paper that the results derived in this section generalize to the multiple receiver case. We study a scenario where every sender has a single packet that needs to be delivered to the receiver.

Definition 1: Consider a single-hop network with a single receiving node and n senders, each having one packet to transmit. Define the *delivery time*, $T_D(n)$, as the time to transmit all packets successfully to the receiver.

We can divide the delivery time into n portions, where the k^{th} portion corresponds to the additional time required for the

receiver to send the k^{th} ACK, starting from the time when the previous (i.e. $(k-1)^{\text{st}}$) ACK was sent. We define the following notation, for $k = 1, 2, \dots, n$:

$$\begin{aligned} T_k &= \text{Time when the receiver sends the } k^{\text{th}} \text{ ACK} \\ X_k &= T_k - T_{k-1} \quad (T_0 \text{ is assumed to be } 0). \end{aligned}$$

Note that $T_D(n)$ is then given by:

$$T_D(n) = T_n = \sum_{k=1}^n X_k \quad (1)$$

The goal of this section is to characterize the expectation of the delivery time for collision recovery, and to compare it with contention-based protocols and a central scheduling mechanism. First we study schemes that treat any collision as a loss. In this case, collisions have to be avoided either by centralized coordination among the senders, or in a distributed way by having senders access the channel in a probabilistic manner, as studied in the literature (see Chapter 4 of [1]).

A. Centralized scheduling

We assume that the receiver, upon successfully receiving a packet, sends an acknowledgment to the corresponding sender. With centralized scheduling, we assume the following policy. The channel is initially reserved for sender 1, up to the point when its packet is acknowledged. At this point, the channel is reserved for sender 2, and so on. In this setting, the calculation of the expected delivery time is straightforward. For each sender, the delivery is complete in the first slot when the channel from that sender to the receiver is not under erasure. The time X_k between the $(k-1)^{\text{st}}$ and the k^{th} ACK, which is also the delivery time for the k^{th} sender, is thus a geometric random variable, with mean $\frac{1}{1-p}$. This implies that the total expected delivery time under centralized scheduling policy is given by:

$$\mathbb{E}[T_D(n)] = \sum_{k=1}^n \mathbb{E}[X_k] = \frac{n}{1-p}.$$

Note that the delivery time for centralized scheduling is normally a lower bound for the delivery time of other distributed probabilistic approaches because it ensures that there is no collision. However, even perfect collision avoidance does not result in full exploitation of the resources because of the empty time slots due to channel erasures.

B. Random access

In this case, we assume that in every slot, each sender transmits its packet with probability q until it is acknowledged. The choice of whether to transmit or not is made independently across senders and across time. Note that, by controlling the access probability q , the senders can control the level of contention and thereby prevent collisions.

Theorem 1: The expected delivery time for the random access scheme with an access probability q is given by:

$$\mathbb{E}[T_D(n)] = \sum_{k=1}^n \frac{1}{kq_e(1-q_e)^{k-1}}.$$

where $q_e = q(1-p)$ is the effective probability of access, after incorporating the erasures.

Proof: If a sender decides to transmit in a given slot, then it might still experience an erasure with probability p . Hence, the effective access probability is $q_e = q(1-p)$.

Consider the interval corresponding to X_{n-k+1} . In this interval, there are k unacknowledged senders. Therefore, at each time slot, the number of senders that the receiver can hear from follows a binomial distribution with parameters (k, q_e) . A successful reception occurs when exactly one sender is connected, which happens with probability $kq_e(1-q_e)^{k-1}$. Thus, X_{n-k+1} is a geometric random variable with mean $(kq_e(1-q_e)^{k-1})^{-1}$. The result follows from Eqn. (1). ■

Corollary 1: If the access probability $q = \frac{1}{n}$, then:

$$\mathbb{E}[T_D(n)] = O(n \log n).$$

C. Collision Recovery

Next, we consider the scenario where the receiver has collision recovery capability. In this scenario, every sender transmits its packet in every slot until acknowledged by the receiver.

With collision recovery, there are multiple ways to acknowledge a packet. The conventional method is to ACK a packet when it is decoded. However, we propose a new ACK mechanism that is not based on decoding. The key observation is that upon receiving an equation (collision), the receiver can afford to ACK *any one* of the senders involved in that collision.

In the following theorem, we show that this form of acknowledgments will still ensure that every packet is correctly decoded by the receiver eventually.

Theorem 2: Consider a single-hop network with n senders and one receiver capable of performing collision recovery. Suppose the receiver, upon a reception, acknowledges an arbitrary sender among those involved in the collision. When the receiver sends the n^{th} ACK, it can successfully decode all n packets.

Proof: Let W_k be the set of packets that have been decoded at time T_k , i.e., immediately after sending the k^{th} ACK. Also, let A_k be the set of packets that have been ACKed at time T_k including the k^{th} ACK. We shall show that $W_k \subseteq A_k$ for all $k = 1, 2, \dots, n$.

For any $k = 1, 2, \dots, n$, let $|W_k| = m$. This means, among the first k receptions, there are at least m linearly independent equations involving only these m packets (from Section II-A). For every reception, the receiver always ACKs exactly one of the senders involved in the collision. This means, corresponding to these m equations, m ACKs were sent by the receiver to a set of senders within W_k .

An ACKed sender never transmits again. Since the receiver always ACKs one of the senders involved in a collision, no sender will be ACKed more than once. Hence, these m ACKs are sent to m distinct senders in W_k . This means all senders in W_k have been ACKed.

We have shown that $W_k \subseteq A_k$ for all $k = 1, 2, \dots, n$. A sender that has been ACKed will not transmit again. Hence, every reception will only involve senders whose packet has not

been decoded. This implies that every reception is innovative (see Section II-A).

Therefore, at the point of sending the n^{th} ACK, the receiver has n linearly independent equations in n unknowns, and hence can decode all the packets. ■

We shall now derive the expected delivery time for a receiver with collision recovery capability.

Theorem 3: For collision recovery approach, the expected delivery time is given by:

$$\mathbb{E}[T_D(n)] = \sum_{k=1}^n \frac{1}{1-p^k} = n + O(1).$$

Proof: At time T_k , k distinct senders have been ACKed, and only $(n-k)$ senders will attempt transmission. From the proof of Theorem 2, every collision at the receiver will result in an innovative linear combination. Hence, an innovative reception occurs if and only if not all of the $(n-k)$ senders experience an erasure. The time to receive the next innovative packet, X_{k+1} , is thus a geometric random variable with mean $1/(1-p^{n-k})$. Now, by Eqn. 1, we obtain the following:

$$\begin{aligned} \mathbb{E}[T_D(n)] &= \sum_{k=1}^n \frac{1}{1-p^k} = n + \sum_{k=1}^n \frac{p^k}{1-p^k} \\ &\leq n + \frac{1}{1-p} \sum_{k=1}^n p^k \leq n + \frac{p}{(1-p)^2} = n + O(1). \end{aligned}$$

■

Let us now compare this scheme with a centralized scheduling mechanism. Centralized scheduling requires a central controller that assigns every time slot to a single sender, and achieves a delivery time of $n/(1-p)$. In contrast, in the collision recovery approach, no coordination is necessary among the senders, and yet, the delivery time is $n + O(1)$, that is close to the lower bound of n slots for delivering n packets.

Such an improvement in performance can be explained as follows. For centralized scheduling, since only one user is scheduled to transmit in a time-slot, the time-slot will be wasted from the receiver's point of view, with probability p . In contrast, with collision recovery, since all the unacknowledged senders attempt to access the channel in a given slot, we obtain a *diversity* benefit – if even one of the attempting senders does not experience an erasure, the slot is useful to the receiver.

D. Collision recovery with random access

The earlier subsection assumed that a collision of any number of packets can be treated as a linear equation involving those packets. The largest number of packets that can be allowed to collide for collision recovery to still work depends on the range of the received Signal-to-Noise Ratio (SNR). In practice, if a collision involves more than 3 or 4 packets, then a collision recovery method such as ZigZag decoding process is likely to fail, owing to error propagation.

Hence, in a more realistic setup, we need to limit the level of contention in order to ensure that more collisions at the receiver are useful. In this part of the paper, we explore

the possibility of combining collision recovery with random access. Instead of allowing every unacknowledged sender to transmit, each sender opportunistically transmits its packet with some probability q . Thus, the expected number of transmitting senders is reduced, which in turns limits the expected number of colliding packets in one time slot. We assume that any collision involving more than C packets is not useful. This scheme is expected to perform better than conventional random access with no collision recovery, since a collision of C or fewer packets is not useless, but is treated as one received linear equation. Under this assumption, we can derive the expected delivery time in a manner similar to the analysis of simple random access.

Theorem 4: The expected delivery time for the random access scheme with an access probability q is given by:

$$\mathbb{E}[T_D(n)] = \sum_{k=1}^n \frac{1}{\sum_{m=1}^{\min(C,k)} \binom{k}{m} q_e^m (1-q_e)^{k-m}},$$

where $q_e = q(1-p)$ is the effective probability of access, after incorporating the erasures.

Proof: Consider the interval corresponding to X_{n-k+1} . In this interval, there are k unacknowledged senders. Therefore, as in Theorem 1, at each time slot, the number of senders that the receiver can hear from follows a binomial distribution with parameters (k, q_e) , where q_e is the effective access probability of a sender, given by $q_e = q(1-p)$.

A successful reception occurs when C or fewer senders is connected, which happens with probability

$$p_k = \sum_{m=1}^{\min(C,k)} \binom{k}{m} q_e^m (1-q_e)^{k-m}.$$

Thus, X_{n-k+1} is a geometric random variable with mean $1/p_k$. Using Eqn. 1, we obtain the desired result. ■

The design parameter q should be chosen so as to minimize the delivery time. Unfortunately, the exact characterization of the optimal q in closed form seems difficult to obtain. In the following section, we compare the expected delivery time for the above schemes, with the optimal values of q computed numerically.

E. Numerical results

Fig. 3 shows the expected delivery time for the different schemes discussed above, as a function of the number of senders n . The plot compares conventional random access approach with collision recovery approaches for different values of the contention limit C , which is the maximum number of packets that can be allowed to collide for the collision to be considered useful.

The contention level is controlled by adjusting the access probability q . In the unlimited collision recovery case, i.e., when we have no contention limit, there is no need to reduce contention through random access, and hence q is set to 1. For the other cases, for each n , the value of q is chosen so as to minimize the delivery time.

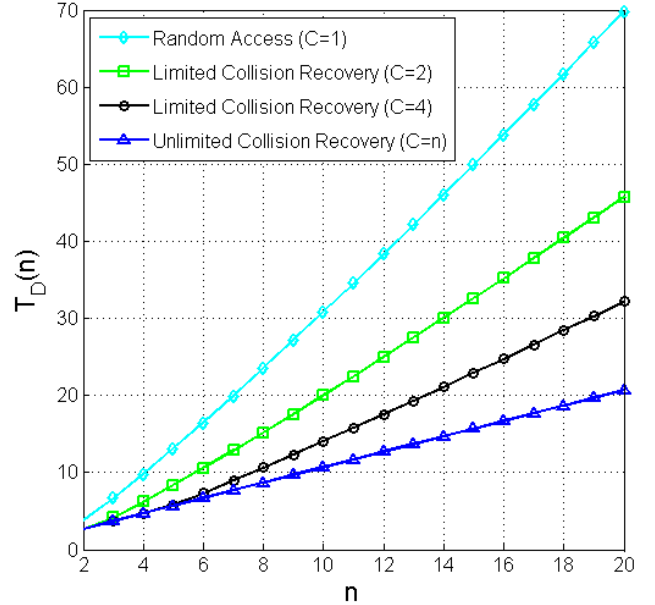


Fig. 3. The delivery time for $p = 1/3$ for different schemes

The main observation is that by allowing collision recovery, the expected delivery time is significantly reduced, as compared to conventional random access where any collision is treated as being useless. We also observe that the delivery time drops with an increase in the contention limit C . In the unlimited collision recovery case, we can see that the delivery time is very close to the best possible time of n slots.

The value of the erasure probability p , is fixed at $1/3$. However, we found that varying the value of p does not significantly affect the delivery time for the other schemes. In contrast, the plot for the centralized scheduling case (not shown in the figure), would be a straight line with slope $1/(1-p)$. In other words, the delivery time for centralized scheduling is sensitive to p .

Intuitively, the reason is, the random access approaches are allowed to change the access probability to reach a certain level of contention at the receiver. As the erasure probability p increases, the senders can compensate by increasing their access probability q to achieve the same contention level.

IV. MULTIPLE RECEIVER CASE

In this section, we generalize the results of the preceding parts to the case of a single-hop wireless erasure channel with multiple senders and receivers. Denote by $\Gamma_O(i)$ the set of receivers that can potentially receive a packet from sender i , and write $\Gamma_I(j)$ for the set of senders that can reach receiver j . Recall that the senders are constrained to *broadcast* the packets on all outgoing links. The goal of each sender is to deliver all the packets in its queue to each of its neighbors. In the following we characterize the delivery time of the network for the collision recovery approach.

Define the *delivery time* of receiver j , $T_D^{(j)}$, as the time taken by receiver j to successfully decode all packets transmitted from all senders in $\Gamma_I(j)$.

A centralized scheduling scheme involves assigning at most one sender to each receiver so that collisions are avoided. However, unlike the single receiver case, it is not always feasible to assign exactly one sender to each receiver owing to interference. For example, in the configuration depicted in Fig. 1, we cannot allow both of the senders to transmit simultaneously. Hence, the delivery time for a particular receiver is larger than the case where other receivers are absent. Therefore, we have

$$T_D^{(j)} \geq \frac{|\Gamma_I(j)|}{1-p}.$$

With a collision recovery method such as ZigZag decoding in place, every sender keeps transmitting its packet until an acknowledgement is received from all of its neighbor receivers. If we use the acknowledgement mechanism as in the single receiver case, i.e., ACK any of the packets involved in a collision, then sending an acknowledgement does not necessarily correspond to receiving a degree of freedom. Moreover, multiple ACKs may be sent to the same sender while the other senders are not acknowledged even after decoding their packets. This is so since a sender does not stop broadcasting its packet unless receiving ACKs from all of its neighbors. Here, we slightly modify the acknowledgement mechanism as follows. Upon a reception at each receiver, the receiver acknowledges any of the packets involved in the reception (collision) that have not already been acknowledged.

Theorem 5: Consider a single-hop wireless erasure network with collision recovery at the receivers. The expected delivery time for each receiver j is bounded from above as

$$\mathbb{E}[T_D^{(j)}] \leq \sum_{k=1}^{|\Gamma_I(j)|} \frac{1}{1-p^k} = |\Gamma_I(j)| + O(1).$$

Proof: Fix a particular receiver j . Suppose each sender in $\Gamma_I(j)$ stops transmitting after receiving an ACK from j . By Theorem 2 all of the packets at the neighbors of j are decodable, once all of the senders in $\Gamma_I(j)$ are acknowledged, i.e., the system of $|\Gamma_I(j)|$ equation at receiver j is full rank. Therefore, even if the acknowledged packet gets retransmitted, the receiver j will have a full rank system after sending $|\Gamma_I(j)|$ ACKs. Now we can divide the delivery time into intervals corresponding to ACK instances, i.e.,

$$T_D^{(j)}(n) \leq \sum_{k=1}^{|\Gamma_I(j)|} X_k^{(j)}, \quad (2)$$

where $X_k^{(j)}$ is the duration between sending the $(k-1)^{st}$ ACK and k^{th} ACK. The inequality could be strict if the system of equations becomes full rank before sending the last ACK.

Note that, at a give time slot, a new ACK is sent by receiver j if and only if a collision is received that involves at least one unacknowledged packet. Therefore, $X_{k+1}^{(j)}$ is a geometric random variable with mean $(1-p^{|\Gamma_I(j)|-k})^{-1}$. Similarly to

the proof of Theorem 3, the desired result is followed from plugging this into (2). ■

The exact characterization of the expected delivery time requires characterizing the exact decoding process that is beyond the scope of this paper. Note that the upper bound on the expected deliver time given by Theorem 5 differs from the lower bound, $|\Gamma_I(j)|$, by only a small constant.

V. CONCLUSIONS AND FUTURE WORK

In this paper, we have studied the delay and throughput performance of collision recovery methods, e.g. ZigZag decoding [4], for a single-hop wireless erasure network. Using an algebraic representation of the collisions allowed us to view receptions at a receiver as linear combinations of the packets at the senders. This algebraic framework provides alternative collision recovery methods to ZigZag decoding and generalizations for the case when the transmitted packets are themselves coded versions of the original packets.

We have focused on the completion time for all of the senders to deliver a single packet to their neighbor receivers. We show that the completion time at a receiver with collision recovery is at most a constant away from the degree of that receiver which is the ultimate lower bound in this setup. Moreover, for the case that collisions of only a limited number of packets can be recovered, we propose a random access mechanism in conjunction with collision recovery to limit the level of contention at the receiver. Our numerical results show that such a scheme provides a significant improvement upon contention-based mechanisms even if each recoverable collision is limited to only two packets.

In our future work, we study the streaming case where packets arrive at each sender according to some arrival process and characterize the stability region of the system when a collision recovery mechanism is available at each receiver. Further, we present an acknowledgement mechanism similar to the one in [8] that could serve as an ARQ-type mechanism for achieving the capacity of a wireless erasure network when both broadcast and interference constraints are present.

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