Near-Optimal Power Control in Wireless Networks: A Potential Game Approach

Utku Ozan Candogan, Ishai Menache, Asuman Ozdaglar, Pablo Parrilo
Department of Electrical Engineering and Computer Science, MIT

Motivation

- Due to the shared nature of the wireless domain, the performance of each mobile depends on the resources allocated to others
- The power control problem, even as a centralized optimization problem with full information, is a fairly complex problem.
- An additional concern: self-interested behavior of mobiles.
- We consider the power allocation problem from the viewpoint of a central planner who wishes to impose a certain power-dependent objective in the network, using pricing.
- Using a potential-game approach, we provide a general distributed power control scheme that would (approximately) achieve any underlying system objective despite the selfishness of the mobiles.

The Model

- Set of mobiles $\mathcal{M} = \{1, \ldots, M\}$ share the same wireless spectrum (single channel).
- The power allocation of the mobiles $\mathbf{p} = (p_1, \ldots, p_M)$ (where $0 < \underline{P}_m \leq p_m \leq \bar{P}_m$.)
- The rate of user m is given by:

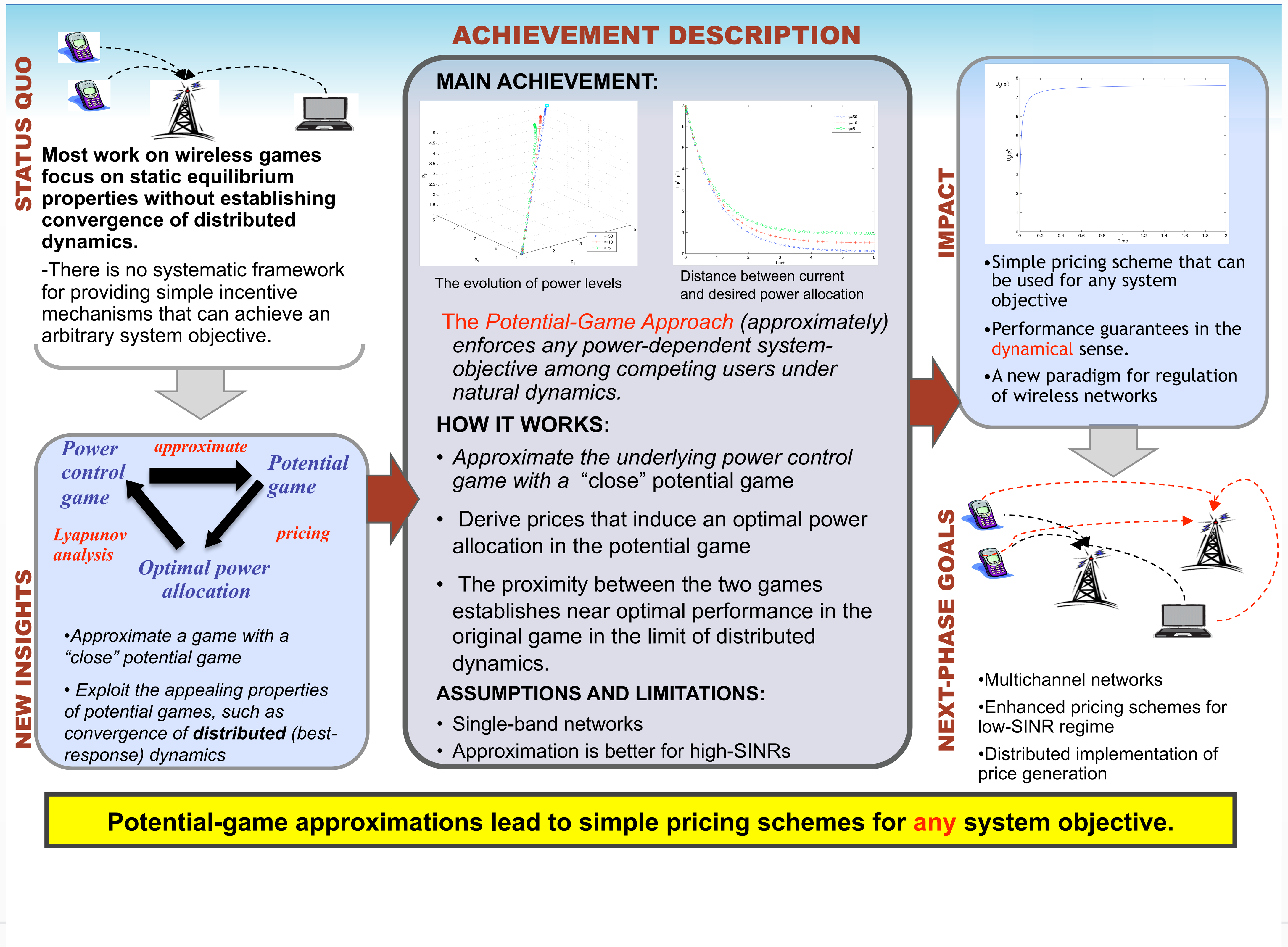
$$r_m(\mathbf{p}) = \log\left(1 + \gamma \mathrm{SINR}_m(\mathbf{p})\right),$$

where, $\gamma > 0$ is the spreading gain of the CDMA system and

$$\mathrm{SINR}_m(\mathbf{p}) = \frac{h_{mm} p_m}{N_0 + \sum_{k \neq m} h_{km} p_k},$$

- The payoff of player m: $u_m(\mathbf{p}) = r_m(\mathbf{p}) - \lambda_m p_m$.
- λ_m is set by the central planner, to maximize a system utility-function $U_0(\cdot)$.

Summary



The Incentive Mechanism

We use the properties of potential games in our approach. A game is a potential game if $\exists \Phi : E \to \mathbb{R}$ such that

$$\Phi(x_m, x_{-m}) - \Phi(y_m, x_{-m}) = u_m(x_m, x_{-m}) - u_m(y_m, x_{-m}),$$

Let $\tilde{r}_m(\mathbf{p}) = \log(\gamma \mathrm{SINR}_m(\mathbf{p}))$. Consider a related game $\tilde{\mathcal{G}}$ with utilities $\tilde{u}_m(\mathbf{p}) = \tilde{r}_m(\mathbf{p}) - \lambda_m p_m$. Note that $\tilde{u}_m(\mathbf{p}) \approx u_m(\mathbf{p})$. for $\gamma \gg 1$ or $h_{mm} \gg h_{km}$ for all $k \neq m$.

- $\tilde{\mathcal{G}}$ is a potential game with potential $\phi(\mathbf{p}) = \sum_m \log(p_m) - \lambda_m p_m$. Additionally, $\tilde{\mathcal{G}}$ has a unique equilibrium.
- A simple linear pricing scheme can be used to set a desired operating point as an equilibrium in $\tilde{\mathcal{G}}$.

Theorem 1. *Given a desired operating point $\mathbf{p}^*$, the unique equilibrium of $\tilde{\mathcal{G}}$ is $\mathbf{p}^*$ if the prices λ^* are chosen as*

$$\lambda_m^* = \frac{1}{p_m^*}, \quad \textit{for all } m \in \mathcal{M}$$

BR dynamics: $\dot{p}_m = \beta_m(\mathbf{p}_{-m}) - p_m$ for all $m \in \mathcal{M}$, where $\beta_m(\mathbf{p}_{-m}) = \arg\max_{p_m \in \mathcal{P}_m} u_m(p_m, \mathbf{p}_{-m})$.
Our idea: Use perturbed system analysis to study dynamics and equilibria in $\mathcal{G}$ using properties of $\tilde{\mathcal{G}}$.

Analysis of Dynamics

Let $SINR_m = \frac{P_m h_{mm}}{N_0 + \sum_{k \neq m} h_{km} \bar{P}_k}$ and denote by $\tilde{\mathcal{I}}_\epsilon$ the set of ϵ-equilibria of $\tilde{\mathcal{G}}$.

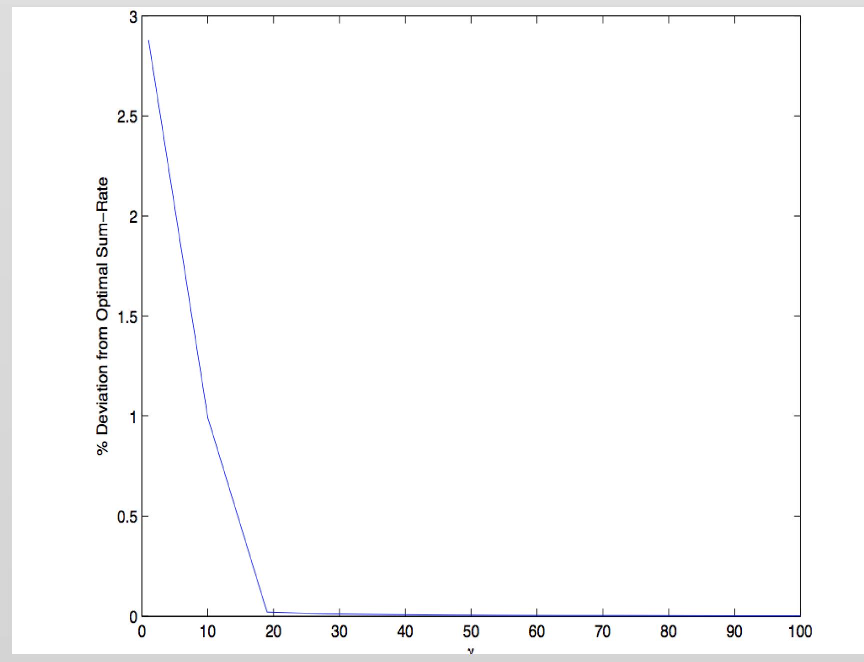
Theorem 2. *In the original game, best-response dynamics converges uniformly to the set $\tilde{\mathcal{I}}_\epsilon$, where*

$$\epsilon \leq \frac{1}{\gamma} \sum_{m \in \mathcal{M}} \frac{1}{SINR_m}.$$

The above characterization is used to bound the deviation from the optimal system utility. For example, for the sum-rate objective:

Theorem 3. *Let $\mathbf{p}^*$ be a maximizer of $U_0(\mathbf{p}) = \sum_m r_m(\mathbf{p})$. Then for every $\tilde{\mathbf{p}} \in \tilde{\mathcal{I}}_\epsilon$,*

$$|U_0(\mathbf{p}^*) - U_0(\tilde{\mathbf{p}})| \leq \sqrt{2\epsilon}(M-1) \sum_{m \in \mathcal{M}} \frac{\bar{P}_m}{\underline{P}_m}.$$



Percent loss in the system utility with respect to change in γ

Future Work

- Multichannel networks.
- Distributed methods for calculating the optimal prices.
- Applications of the potential-game approach to additional wireless network domains.