More Than 3 Users

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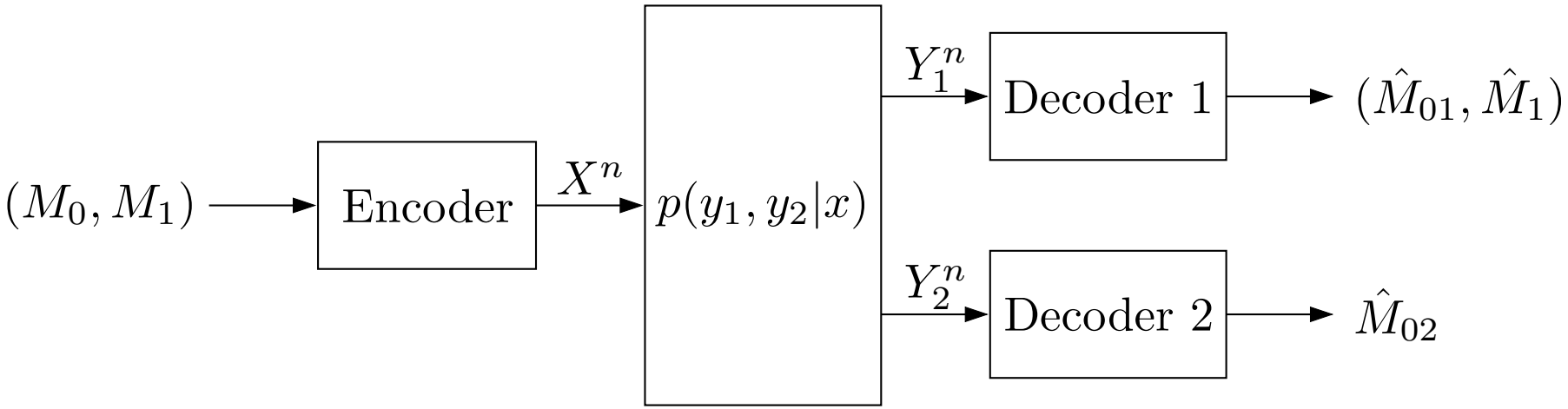
- For example, naive extension of Han-Kobayashi inner bound to > 2 user-pair IC can be improved using interference alignment [1]
- We present two other examples of such cases

DM-BC with Degraded Message Sets

- Consider a 2-receiver DM-BC $(\mathcal{X}, p(y_1, y_2|x), \mathcal{Y}_1 \times \mathcal{Y}_2)$
- Sender X wishes to send:

  common message $M_0 \in [1 : 2^{nR_0}]$ to both receivers, and

  private message $M_1 \in [1 : 2^{nR_1}]$ to receiver Y_1



- The capacity region is the closure of the set of achievable rate pairs (R_0, R_1)

Theorem (Körner–Marton [2])

The capacity region of the DM-BC with degraded message sets is the set of rate pairs (R_0, R_1) *such that*

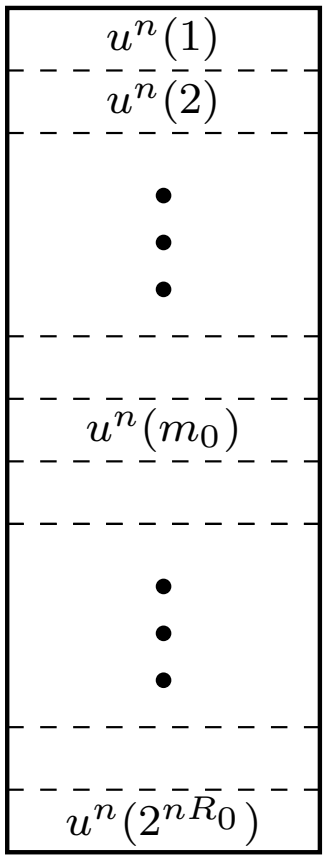
$$\begin{aligned} R_0 &\le I(U;Y_2), \\ R_1 &\le I(X;Y_1|U), \\ R_0 + R_1 &\le I(X;Y_1) \end{aligned}$$

for some $p(u,x)$

- Achievability follows by superposition coding

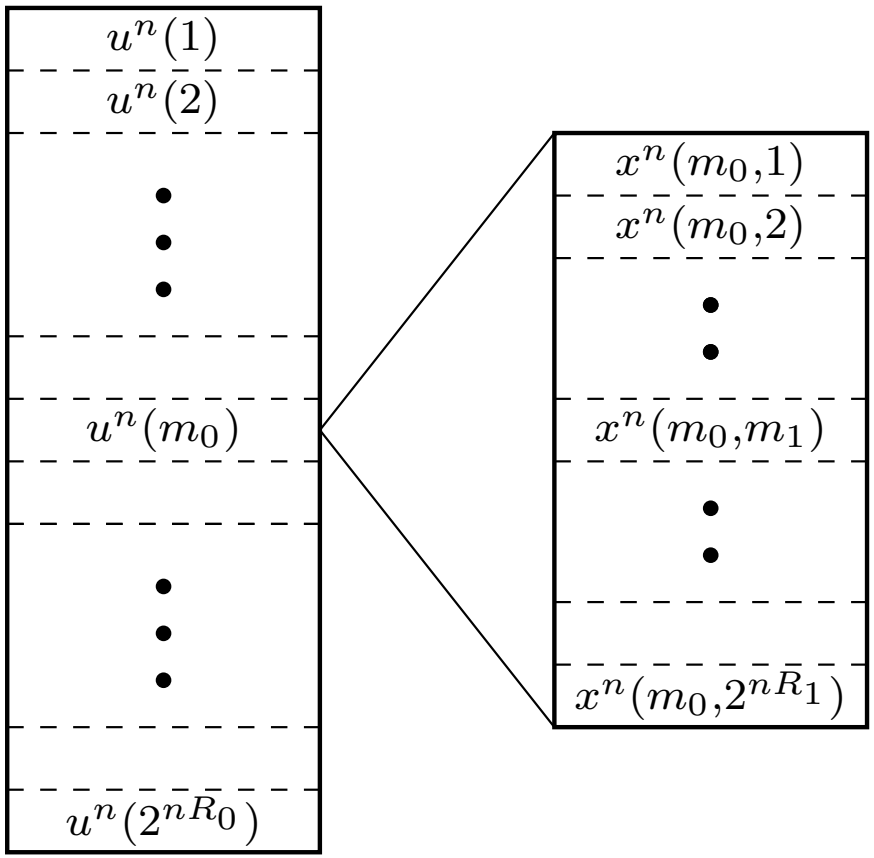
Outline of Achievability: Codebook Generation

- Fix $p(u, x)$. Generate 2^{nR_0} i.i.d. sequences (cloud centers) $u^n \sim \prod_{i=1}^{n} p(u_i)$



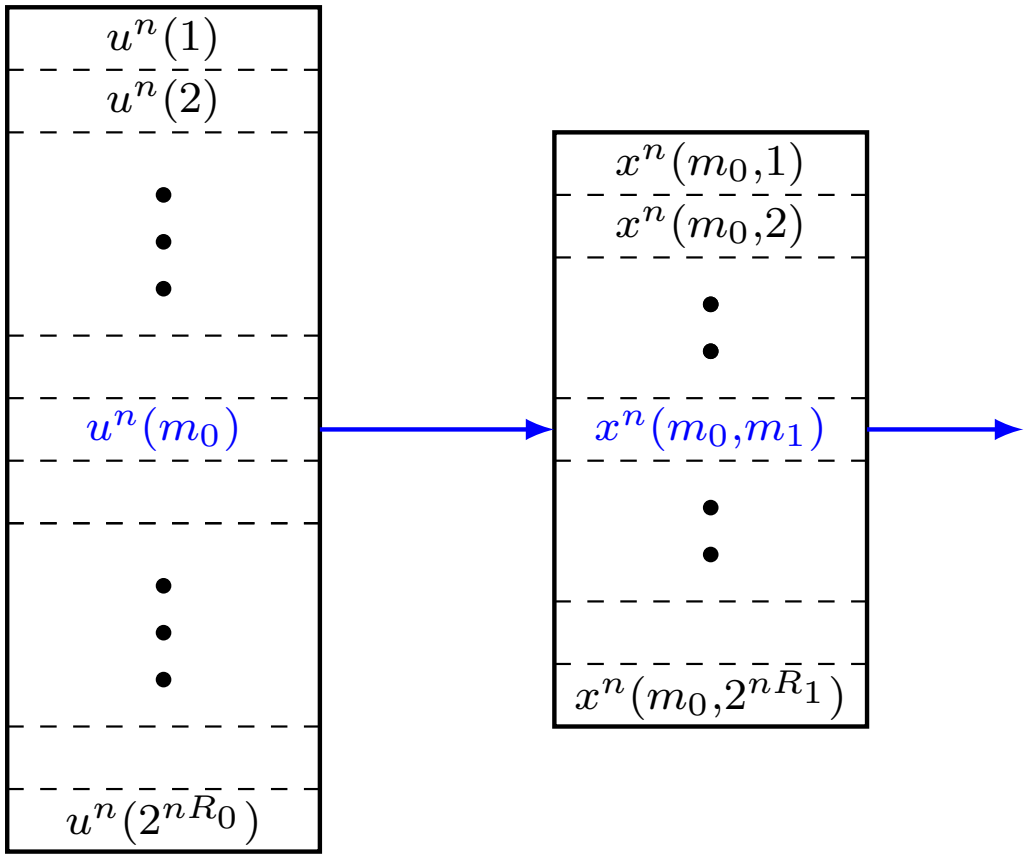
Outline of Achievability: Codebook Generation

- For each $u^n(m_0)$, generate 2^{nR_1} conditionally i.i.d. sequences (satellite codewords) $x^n \sim \prod_{i=1}^{n} p(x_i|u_i)$



Outline of Achievability: Encoding

- To send message pair (m_0, m_1), transmit satellite codeword $x^n(m_0, m_1)$



Outline of Achievability: Decoding

- Y_2 declares that a message $\hat{m}_{02}$ is sent if it is the unique message such that $(u^n(\hat{m}_{02}), y_2^n) \in \mathcal{T}_\epsilon^{(n)}$

  The probability of error $\to 0$ as $n \to \infty$ if

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$$\begin{aligned} R_1 &< I(X; Y_1|U), \\ R_0 + R_1 &< I(X; Y_1) \end{aligned}$$

- This completes the proof of achievability

Alternative Characterization

- We can prove the weak converse for the region consisting of rate pairs (R_0, R_1) satisfying

$$\begin{aligned} R_0 &\leq I(U;Y_2), \\ R_0 + R_1 &\leq I(U;Y_2) + I(X;Y_1|U), \\ R_0 + R_1 &\leq I(X;Y_1) \end{aligned}$$

  for some $p(u,x)$

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  We can also show that they have the same boundary points

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- Clearly this region includes the first characterization region
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- Can we show that the above region is achievable directly?
- The answer is yes, and the proof involves (unnecessary) rate splitting:

  Divide M_1 into two independent messages; M_{10} at rate R_{10} and M_{11} at rate R_{11}. Represent (M_0, M_{10}) by U and (M_0, M_{10}, M_{11}) by X

- We can show that (R_0, R_{10}, R_{11}) is achievable if

$$\begin{aligned} R_0 + R_{10} &< I(U; Y_2), \\ R_{11} &< I(X; Y_1|U), \\ R_0 + R_1 &< I(X; Y_1) \end{aligned}$$

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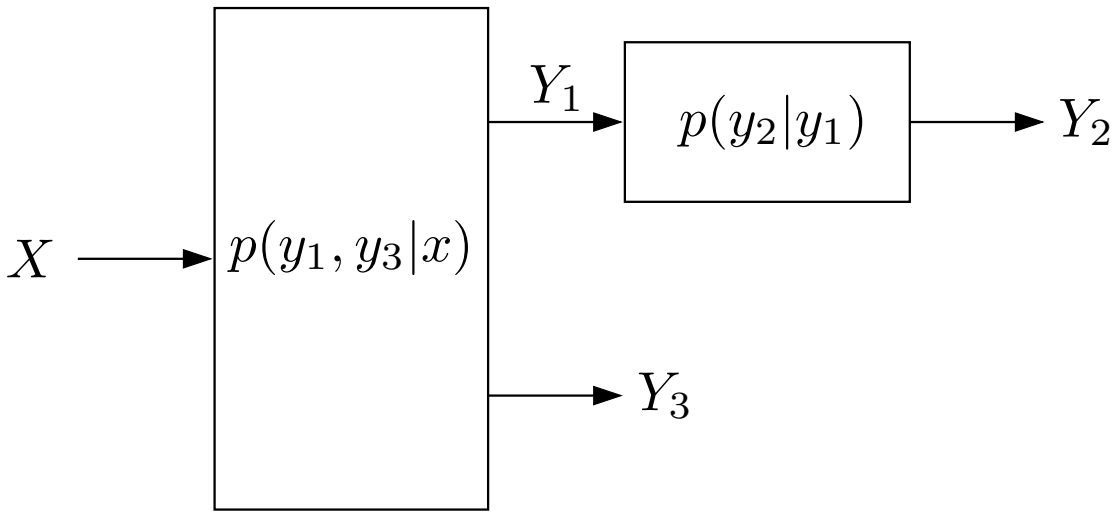
- The above rate splitting idea turns out to be crucial for 3-receiver DM-BC

Multi-level DM-BC with Degraded Message Sets

- A *multilevel* DM-BC [3] $(\mathcal{X}, p(y_1, y_3|x)p(y_2|y_1), \mathcal{Y}_1, \mathcal{Y}_2, \mathcal{Y}_3)$ is a 3-receiver DM-BC where Y_2 is a degraded version of Y_1
- Sender X wishes to send

  common message $M_0 \in [1 : 2^{nR_0}]$ to *all* receivers

  private message $M_1 \in [1 : 2^{nR_1}]$ only to receiver Y_1



- What is the capacity region?

- A straightforward extension of Körner–Marton capacity region for 2 receivers gives the inner bound consisting of (R_0, R_1) such that

$$R_0 < \min\{I(U;Y_2), I(U;Y_3)\},$$
$$R_1 < I(X;Y_1|U)$$

for some $p(u,x)$
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- This region turned out not to be optimal in general

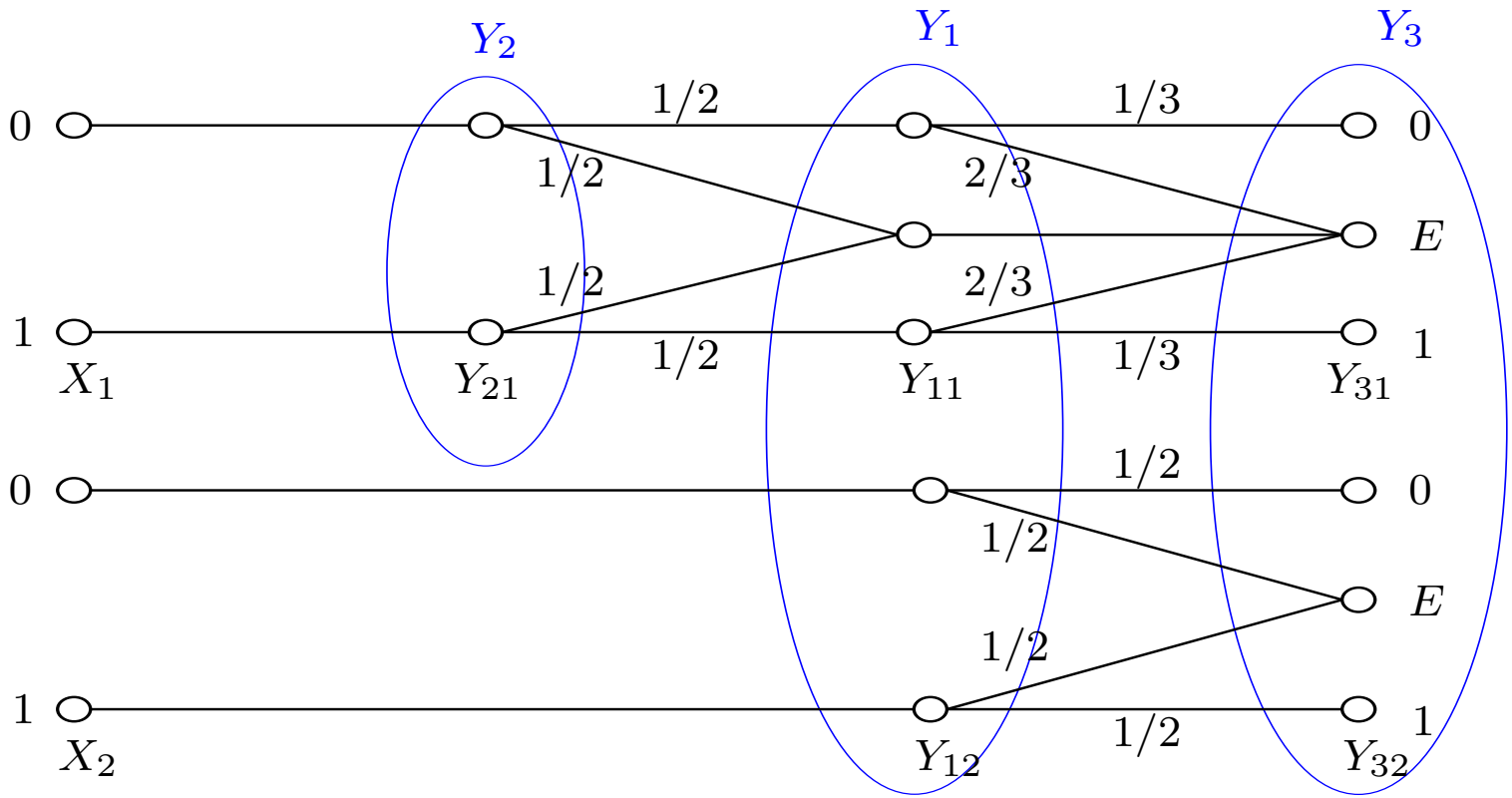
Theorem (Nair–El Gamal [4])

The capacity region of the 3-receiver multi-level DM-BC is the set of rate pairs (R_0, R_1) such that

$$R_0 \le \min\{I(U;Y_2), I(V;Y_3)\},$$
$$R_1 \le I(X;Y_1|U),$$
$$R_0 + R_1 \le I(V;Y_3) + I(X;Y_1|V)$$

for some $p(u,v)p(x|v)$

Reversely Degraded BEC Example [4]



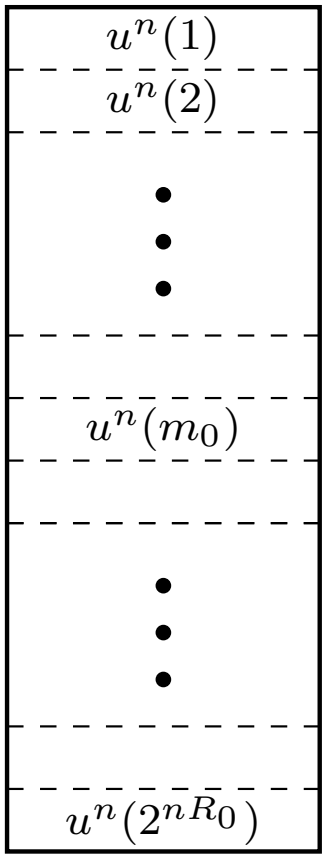
- Fix $R_0 = 1/2$:
  - With Körner-Marton extension: $R_1 = 5/12$
  - Capacity region: $R_1 = 1/2$

Outline of Achievability: Codebook Generation

- Split R_1 into R_{10} and R_{11}; $R_1 = R_{10} + R_{11}$

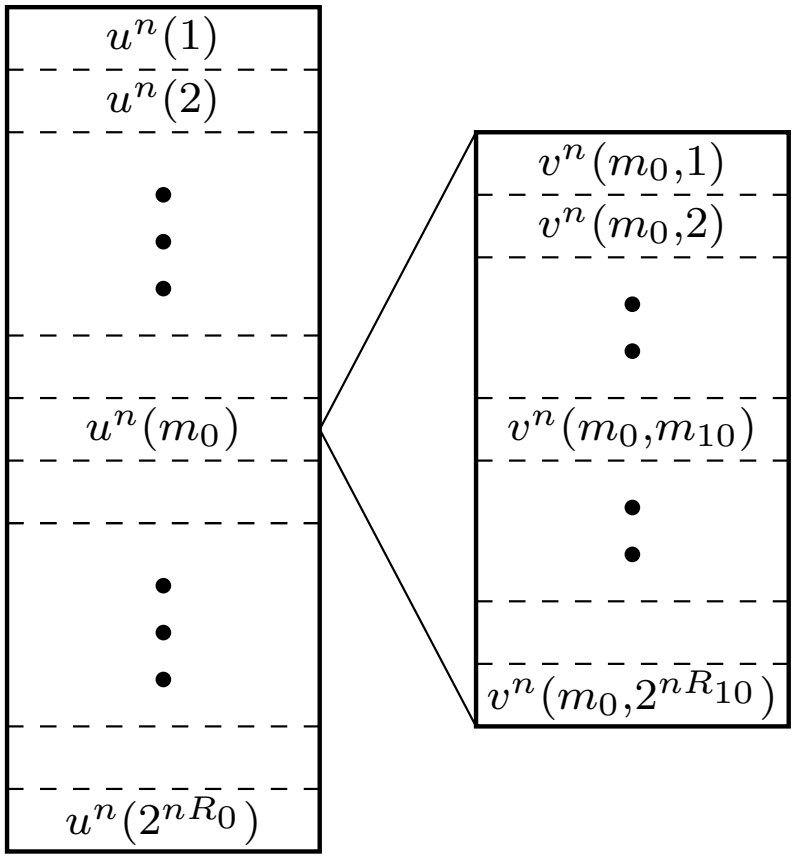
Outline of Achievability: Codebook Generation

- Fix $p(u,v)p(x|v)$. Generate 2^{nR_0} i.i.d. $u^n \sim \prod_{i=1}^{n} p(u_i)$



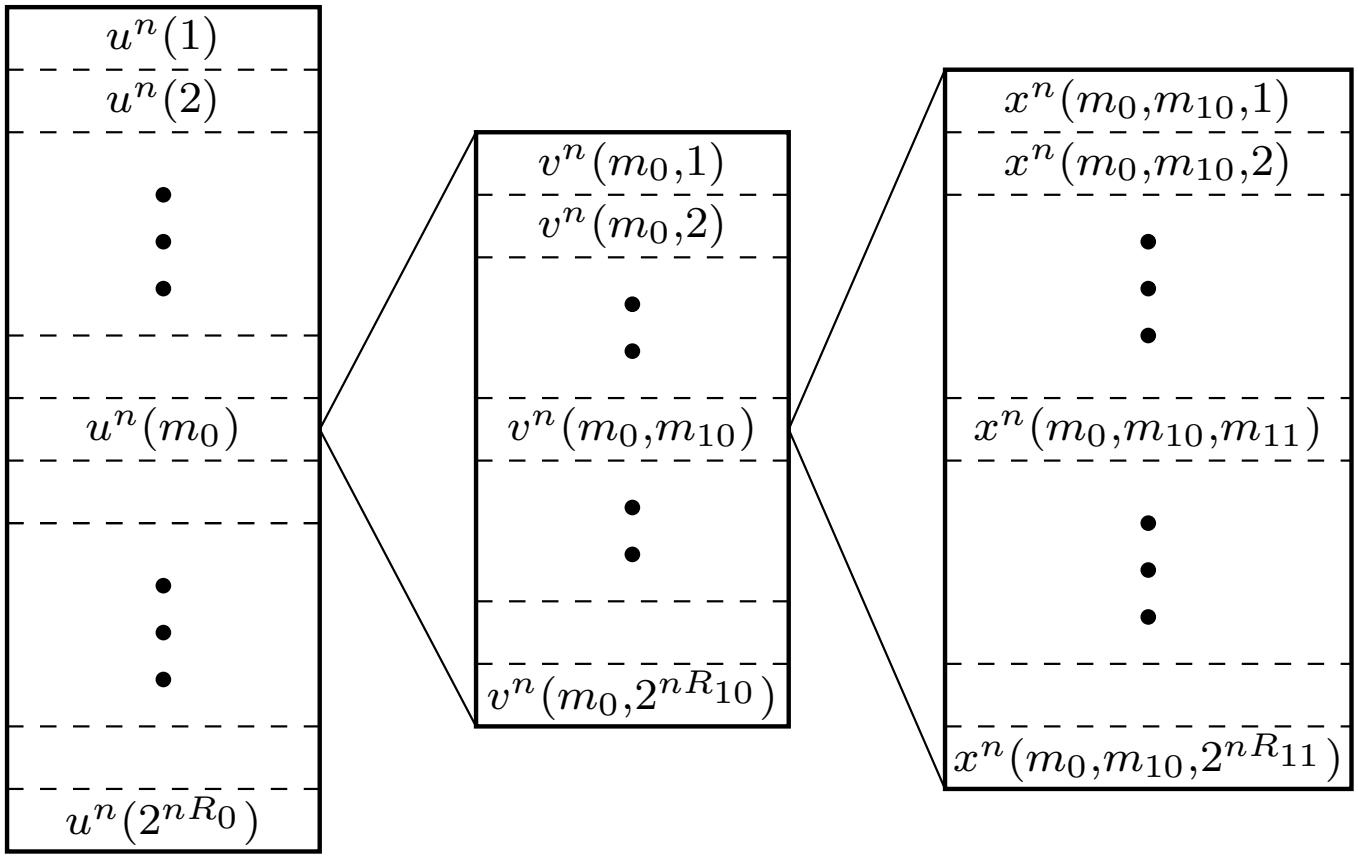
Outline of Achievability: Codebook Generation

- For each u^n, generate $2^{nR_{10}}$ conditionally i.i.d. $v^n \sim \prod_{i=1}^{n} p(v_i|u_i)$



Outline of Achievability: Codebook Generation

- For each v^n, generate $2^{nR_{11}}$ conditionally i.i.d. $x^n \sim \prod_{i=1}^{n} p(x_i|v_i)$



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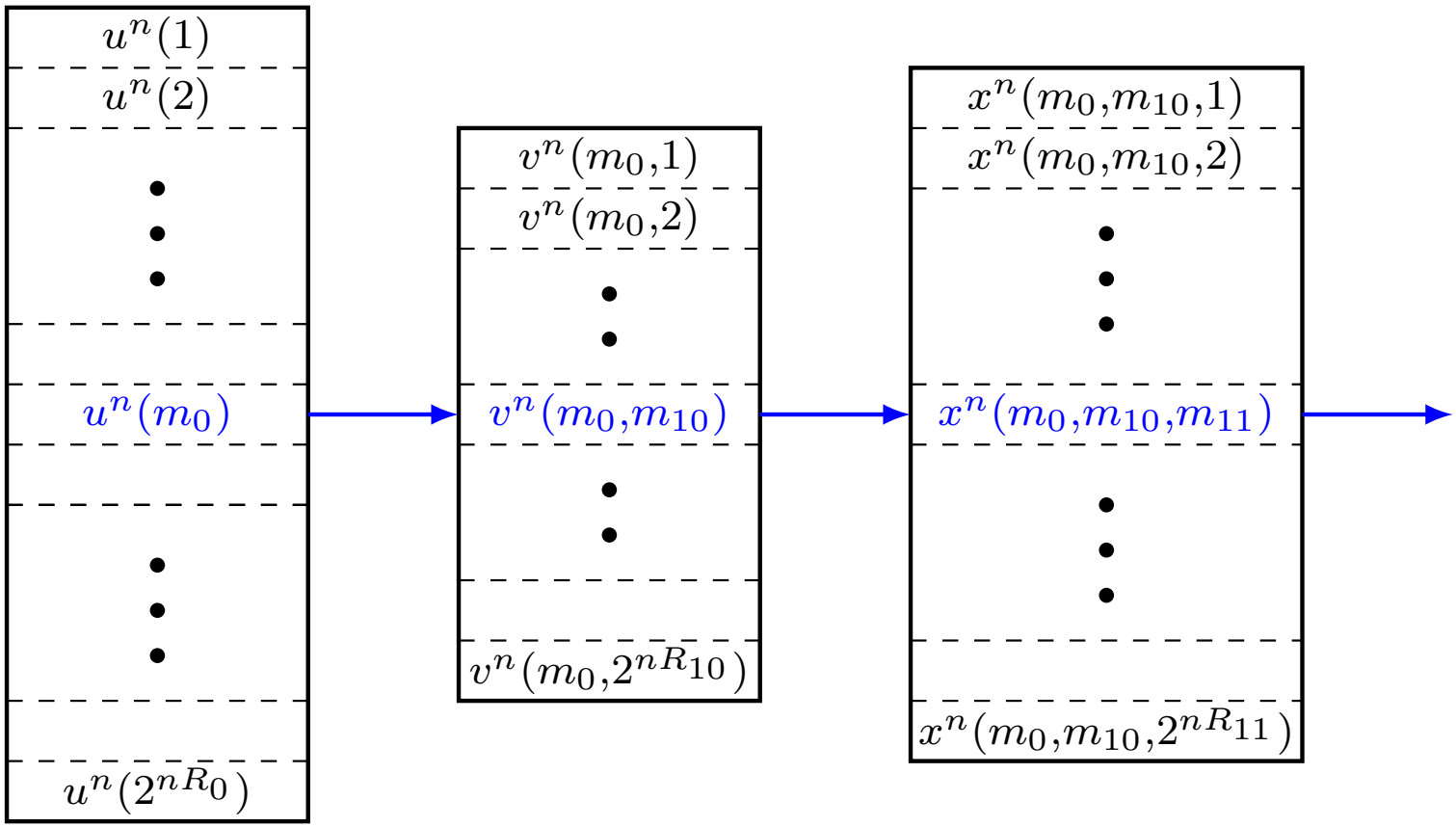

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$$\begin{aligned} R_{11} &< I(X; Y_1|V), \\ R_{10} + R_{11} &< I(X; Y_1|U), \\ R_0 + R_{10} + R_{11} &< I(X; Y_1) \end{aligned}$$

Outline of Achievability: Decoding

- If receiver Y_3 decodes m_0 directly by finding the unique $\hat{m}_{03}$ such that $(u^n(\hat{m}_{03}), y_3^n) \in \mathcal{T}_\epsilon^{(n)}$, we have $R_0 < I(U; Y_3)$, which together with previous conditions gives the extended Körner–Marton region

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- To achieve the larger region, receiver Y_3 decodes m_0 indirectly:

  It declares that $\hat{m}_{03}$ is sent if it is the unique index such that $(v^n(\hat{m}_{03}, m_{10}), y_3^n) \in \mathcal{T}_\epsilon^{(n)}$ *for some* $m_{10} \in [1 : 2^{nR_{10}}]$
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- Combining the bounds, substituting $R_{10} + R_{11} = R_1$, and performing F–M to eliminate R_{10} and R_{11} completes the proof of achievability

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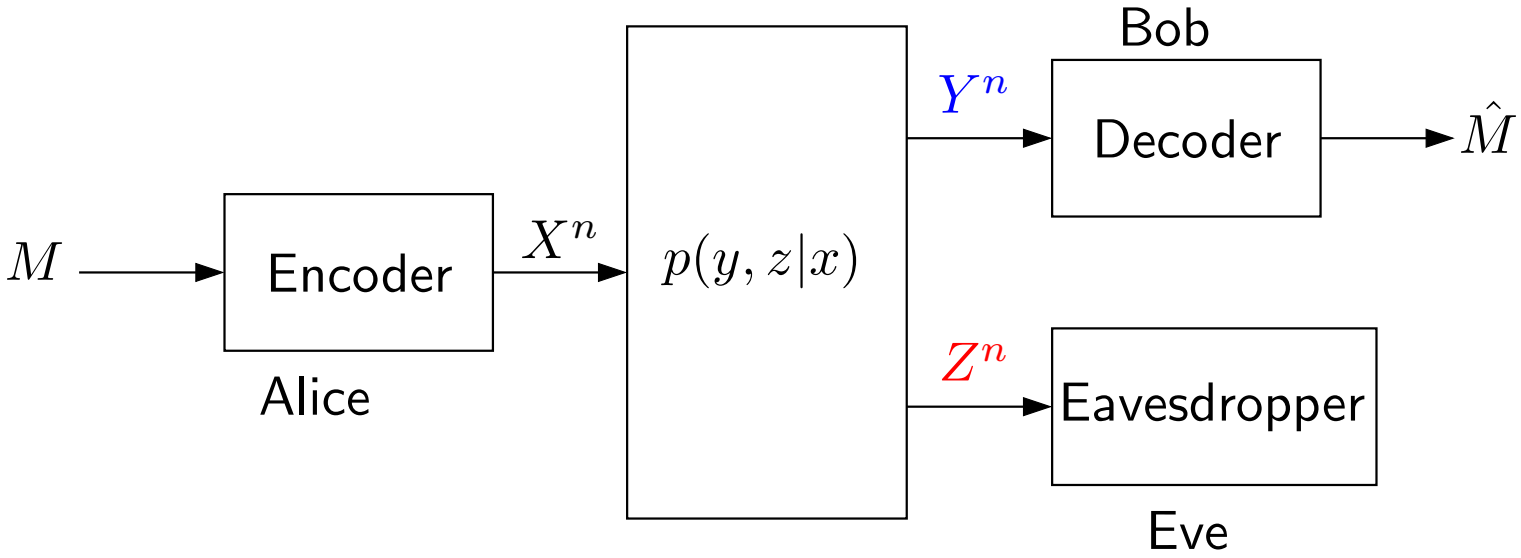
- This leads to a general method for constructing inner bounds for general k-receiver BCs with arbitrary messaging requirements, which is at least as large as existing inner bounds
- Optimality of these general results are not known

Wiretap channel

- Consider a 2-receiver DM-BC
- Sender X wishes to send a message $M \in [1 : 2^{nR}]$ to Y, while keeping it secret from Z



- Rate R is achievable if:

  Probability of error: $P_e^{(n)} = \mathsf{P}\{\hat{M} \neq M\} \to 0$ as $n \to \infty$

  Secrecy constraint: $I(M; Z^n)/n \to 0$ as $n \to \infty$
- The secrecy capacity C_{S} is the supremum of all achievable rates

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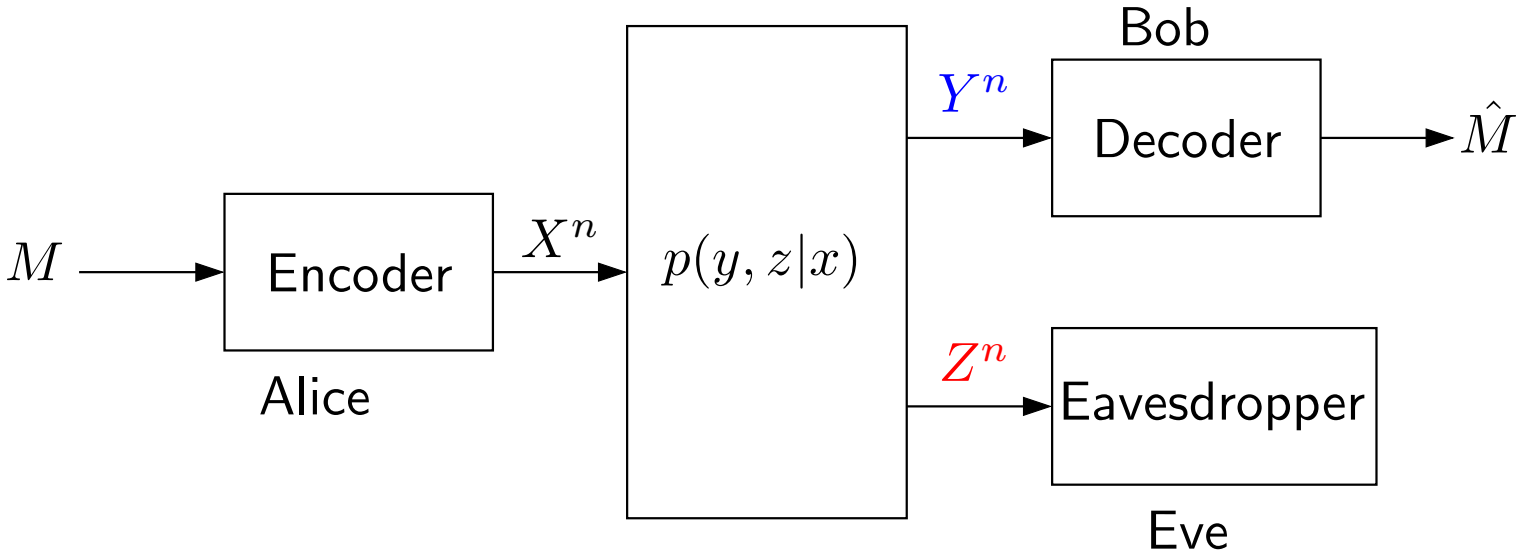
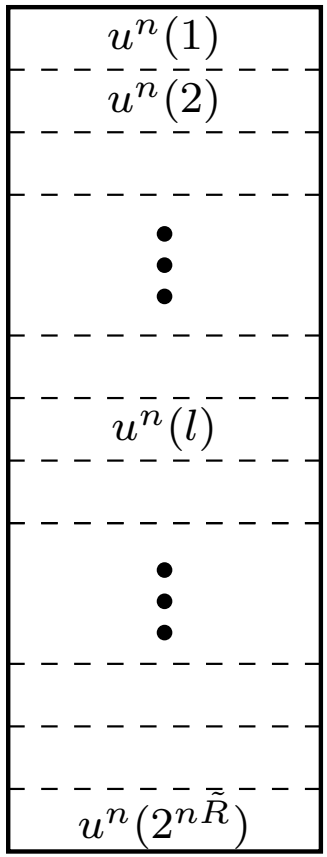


Theorem (Wyner [5], Csiszár–Körner [6])

$$C_{\mathrm{S}} = \max_{p(u,x)} \left(I(U;Y) - I(U;Z) \right)$$

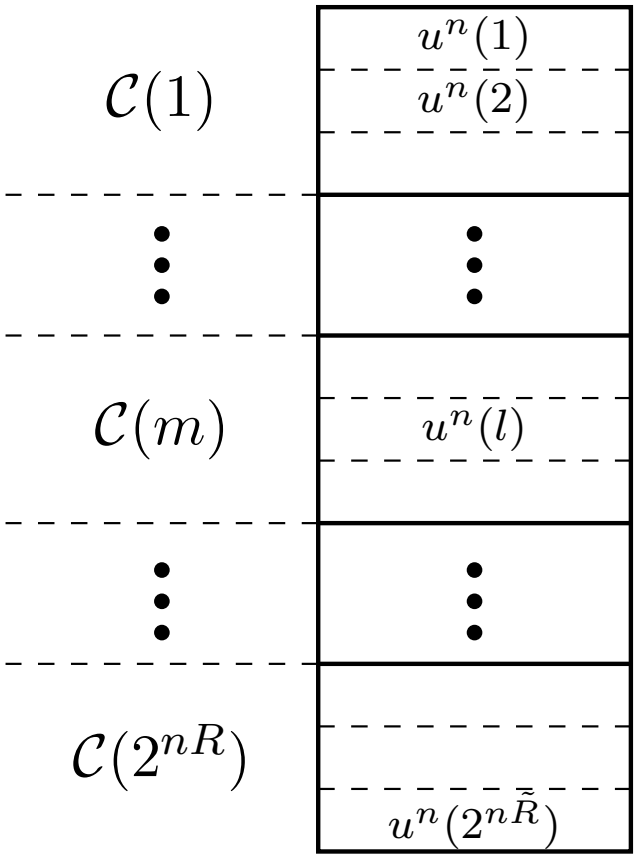
Outline of Achievability: Codebook Generation

- Generate $2^{n\tilde{R}}$ i.i.d. $u^n \sim \prod_{i=1}^{n} p(u_i)$.



Outline of Achievability: Codebook Generation

- Generate $2^{n\tilde{R}}$ i.i.d. $u^n \sim \prod_{i=1}^{n} p(u_i)$. Partition into 2^{nR} subcodes



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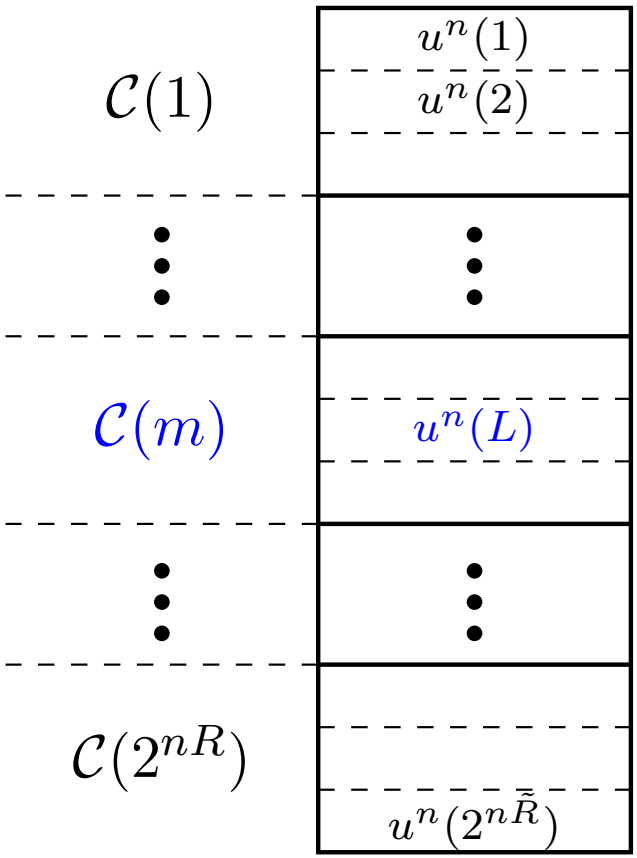

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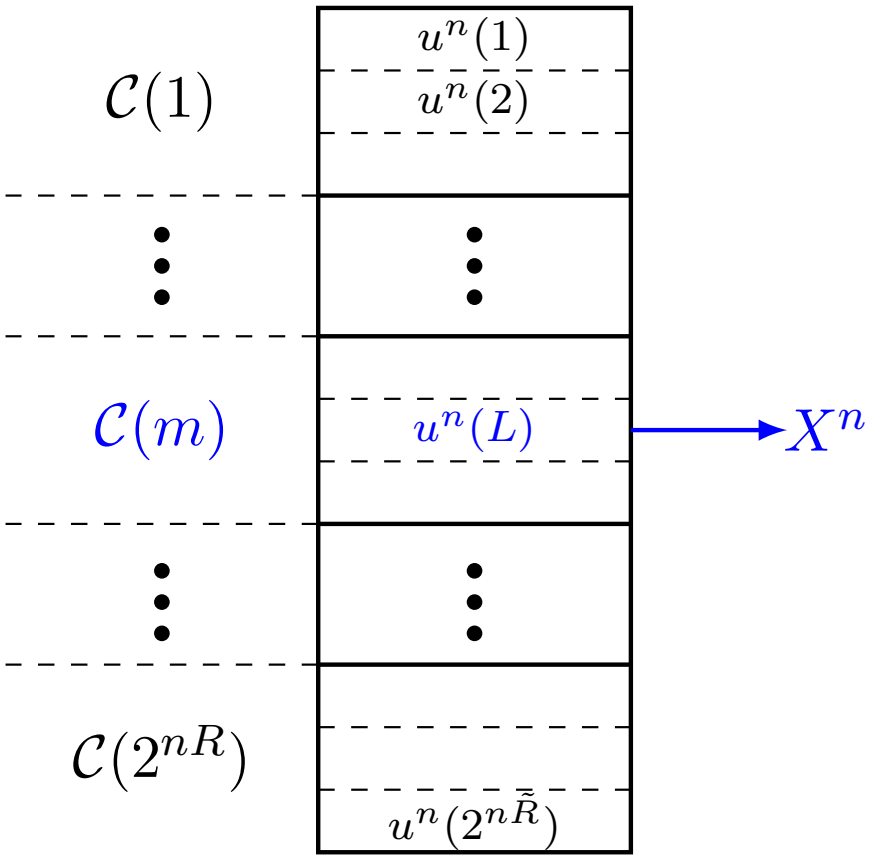
- To send m, choose random $u^n(L) \in \mathcal{C}(m)$. Transmit $X^n \sim \prod_{i=1}^n p(x_i|u_i)$



Decoding and Analysis of Secrecy Constraint

- Decoding: Y declares that $\hat{l}$ is sent if it is the unique index such that $(u^n(\hat{l}), y^n) \in \mathcal{T}_\epsilon^{(n)}$; declares message sent to be subcode index $\hat{m}$ of $u^n(\hat{l})$

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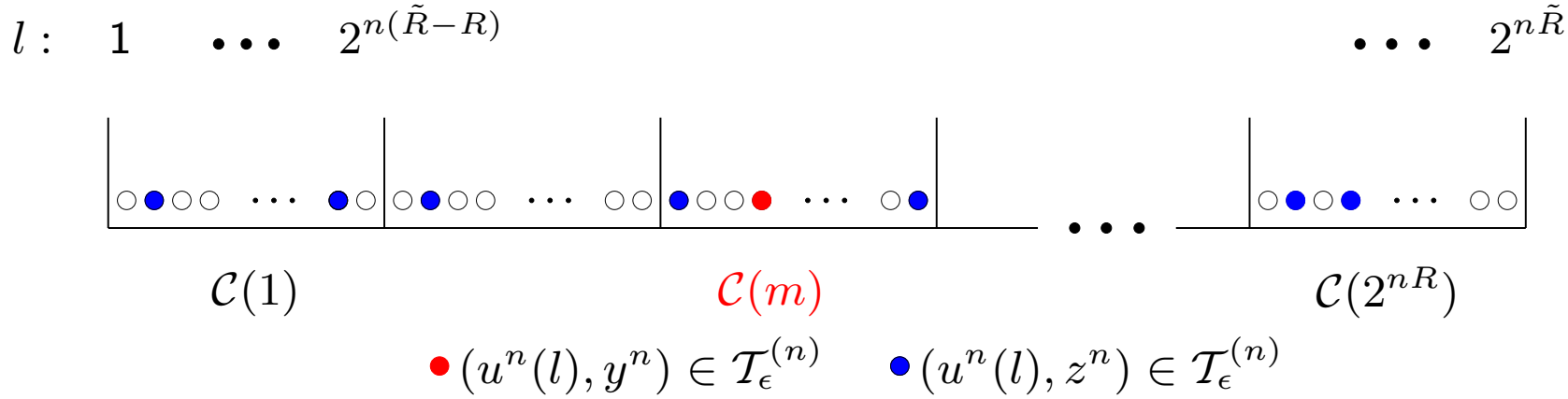
- Secrecy constraint: For each subcode $\mathcal{C}(m)$, Z has $\doteq 2^{n(\tilde{R}-R-I(U;Z))}$ jointly typical $u^n(l)$ sequences with z^n



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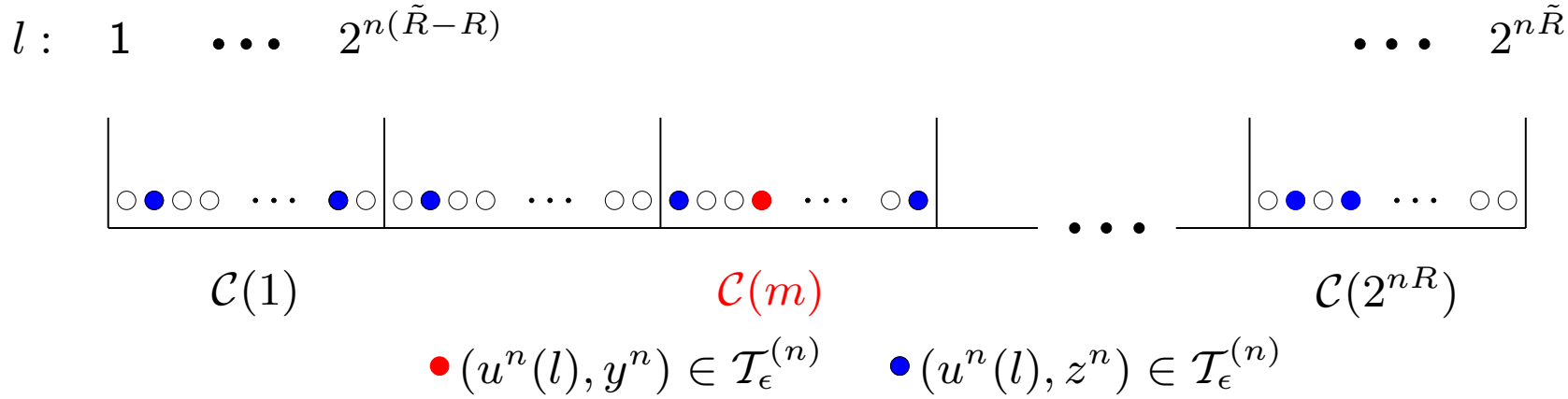
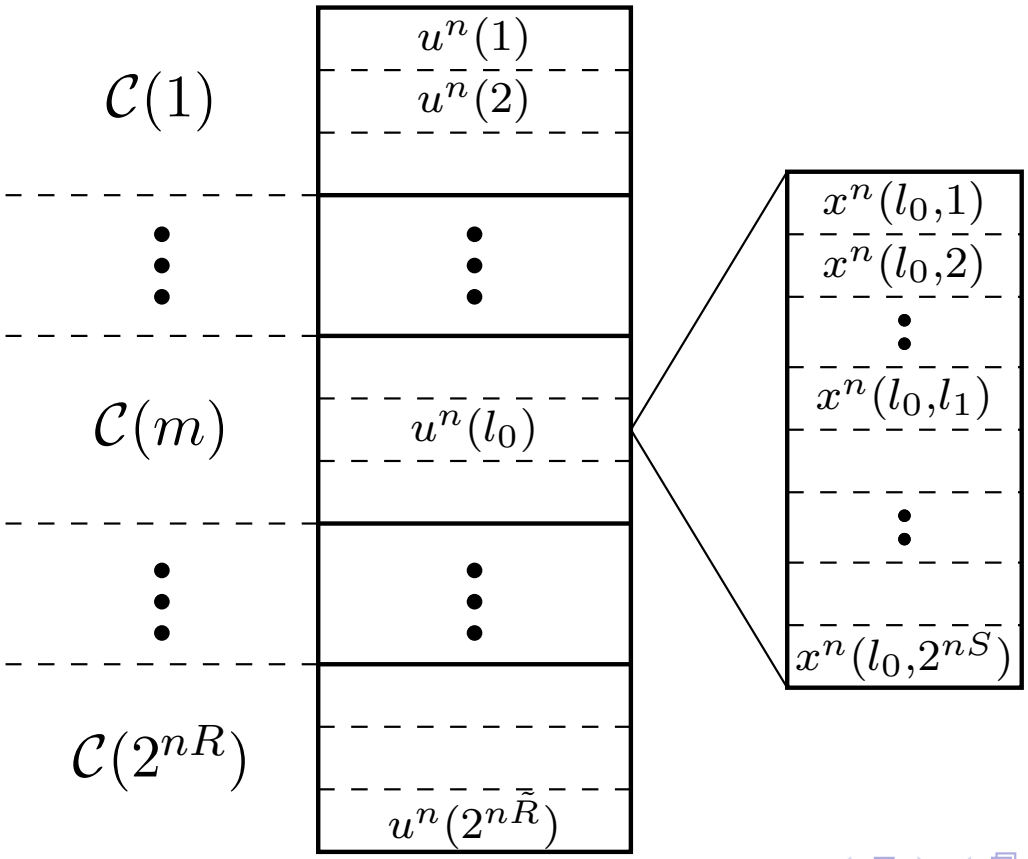


Thus if $\tilde{R} - R > I(U;Z)$, Z has roughly equal number of indices in each subcode, providing it with no information about the message

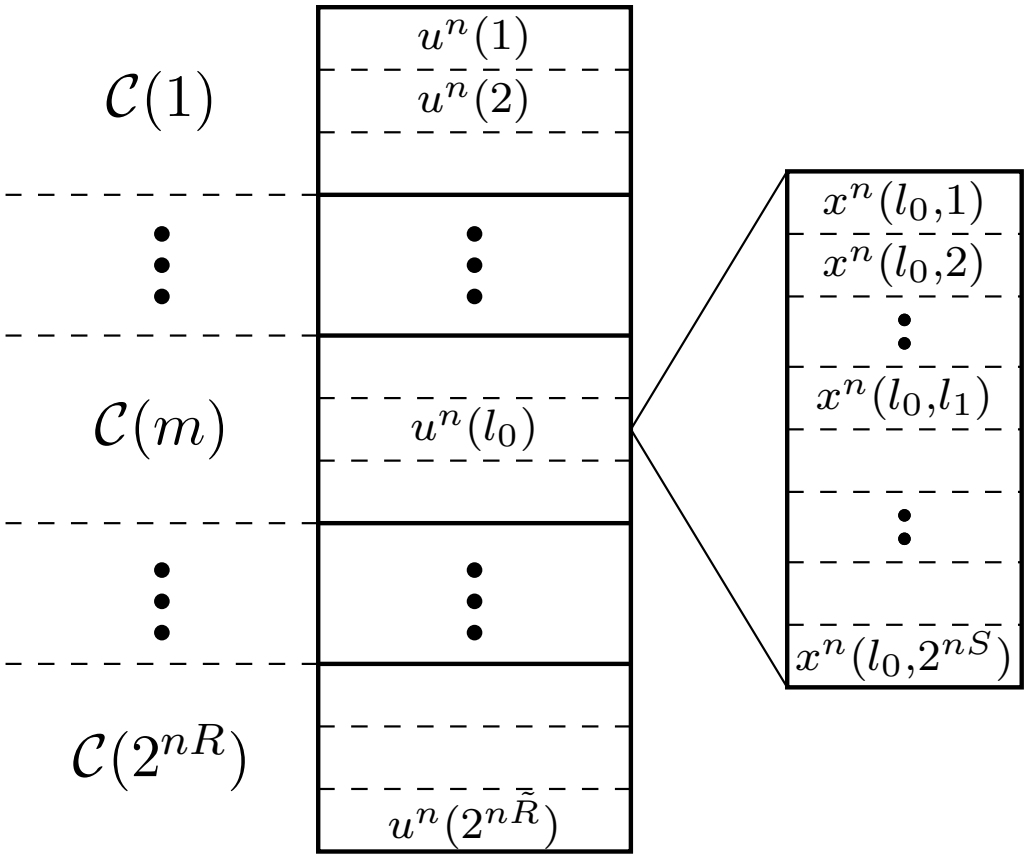
Observation

- Instead of choosing X^n at random, can generate for each u^n a subcode consisting of 2^{nS} conditionally i.i.d. codewords $x^n \sim \prod_{i=1}^n p(x_i|u_i)$; choose one at random for transmission



Observation

- Z cannot decode U indirectly through X if $S + \tilde{R} - R \geq I(X;Z)$, or equivalently if $S \geq I(X;Z) - I(U;Z)$



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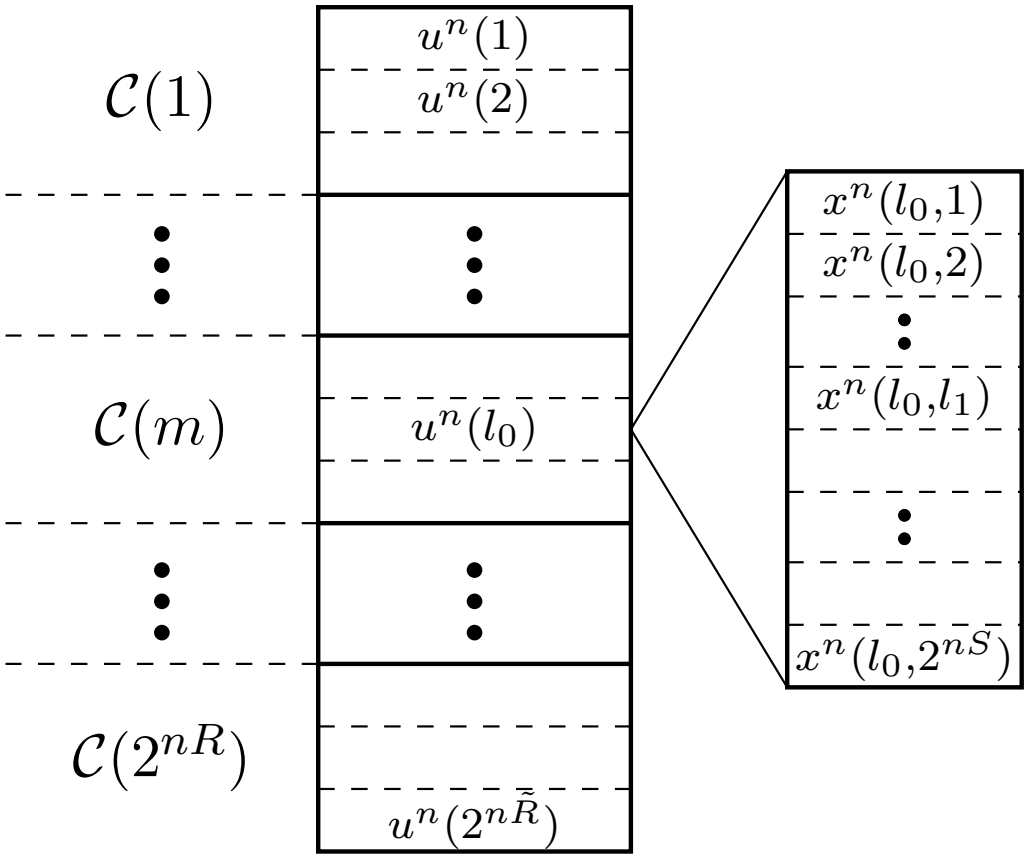
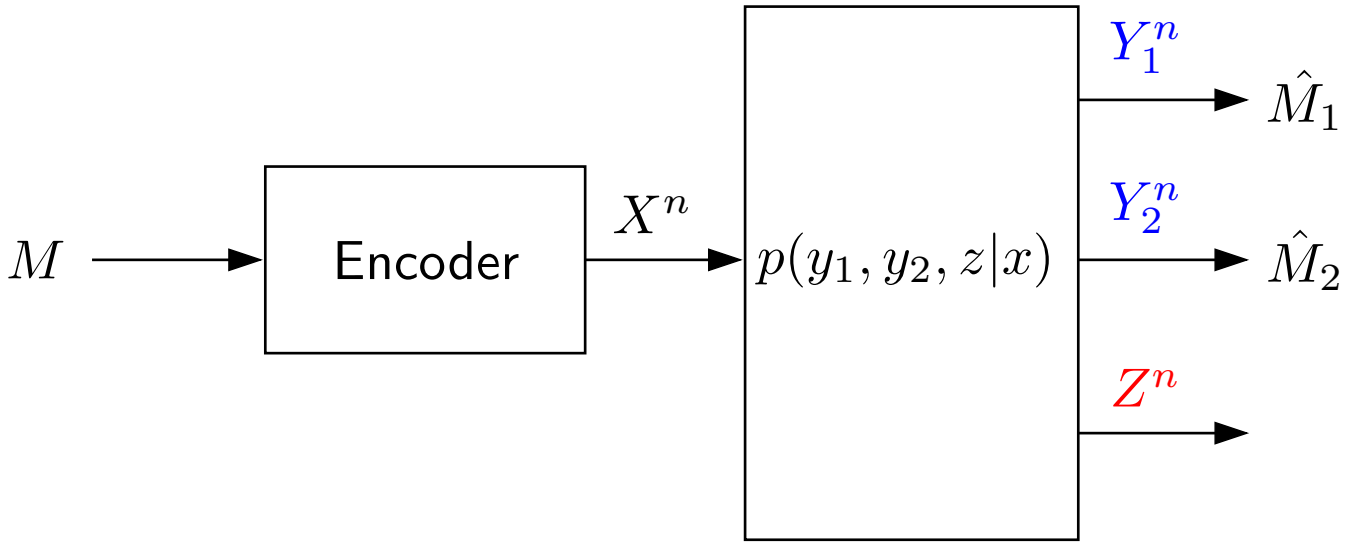


- This observation turned out to be crucial for ≥ 3 receivers

2 Receivers, 1 Eavesdropper Wiretap Channel

- Consider a 3-receiver DM-BC
- Sender X wishes to send a message $M \in [1 : 2^{nR}]$ to Y_1 and Y_2, while keeping it secret from eavesdropper Z



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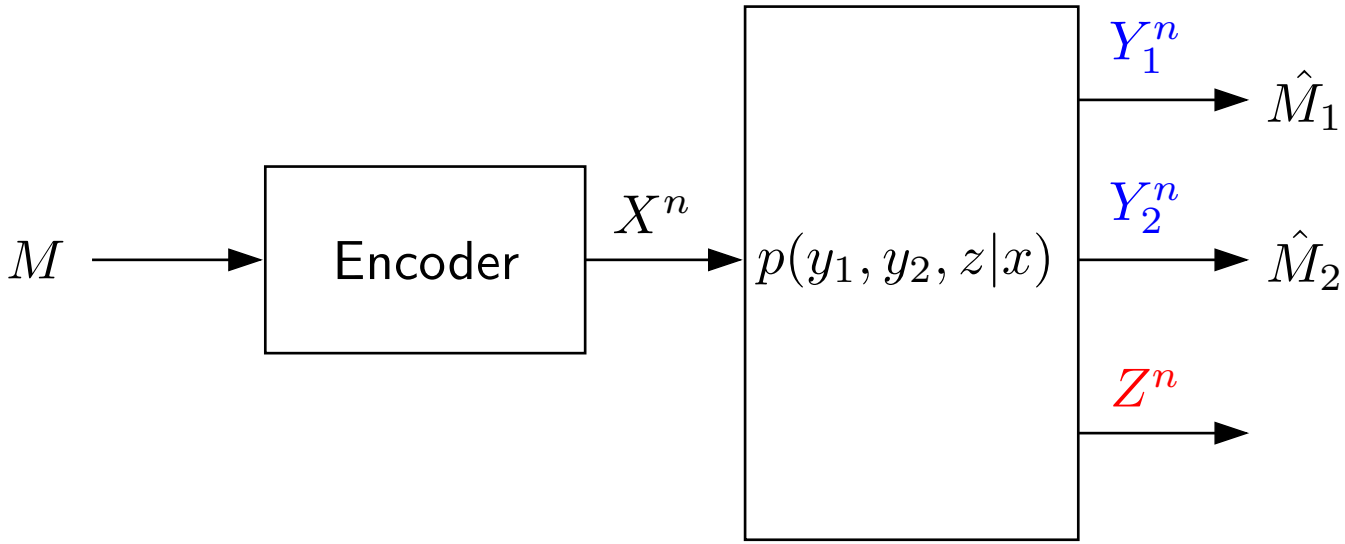

- Rate R is achievable if:

  Probability of error: $P_e^{(n)} = \mathsf{P}\{\hat{M}_1 \neq M \text{ or } \hat{M}_2 \neq M\} \to 0$ as $n \to \infty$

  Secrecy constraint: $I(M; Z^n)/n \to 0$ as $n \to \infty$

- Secrecy capacity C_{S} is the supremum of achievable rates

- Straightforward extension of Csiszár–Körner achievability scheme:

$$C_{\mathrm{S}} \geq \max_{p(u,x)} \min\{I(U;Y_1) - I(U;Z), I(U;Y_2) - I(U;Z)\}$$

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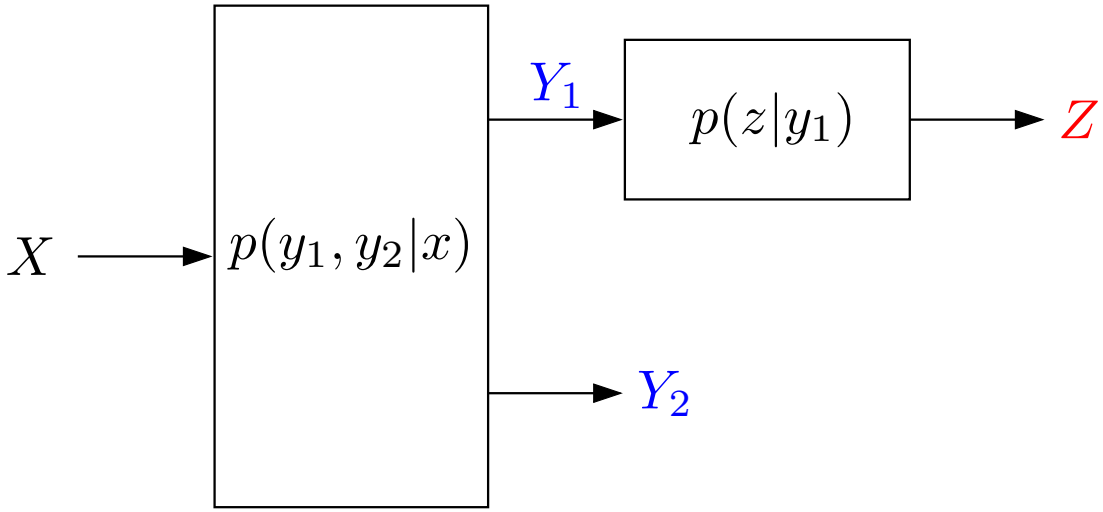

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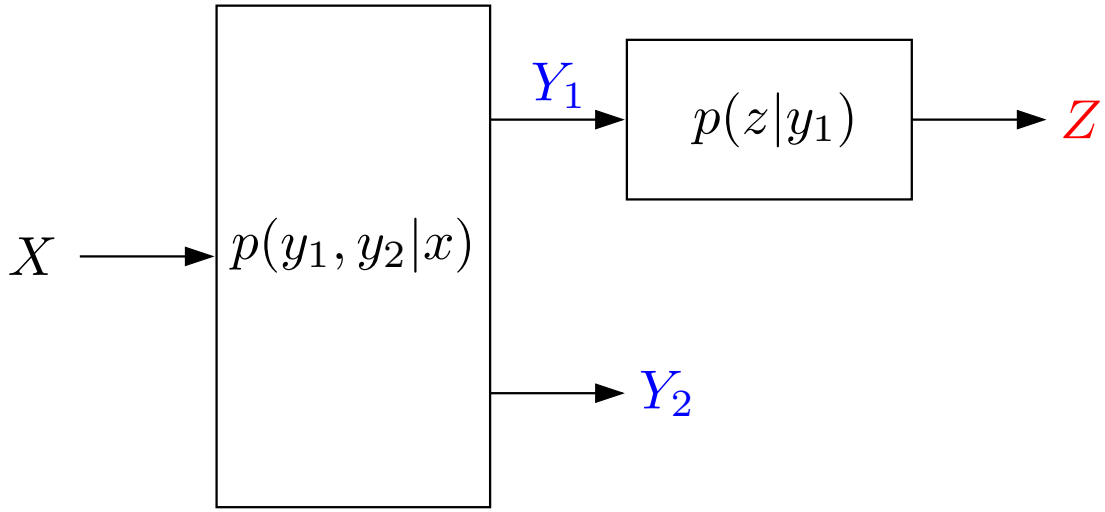

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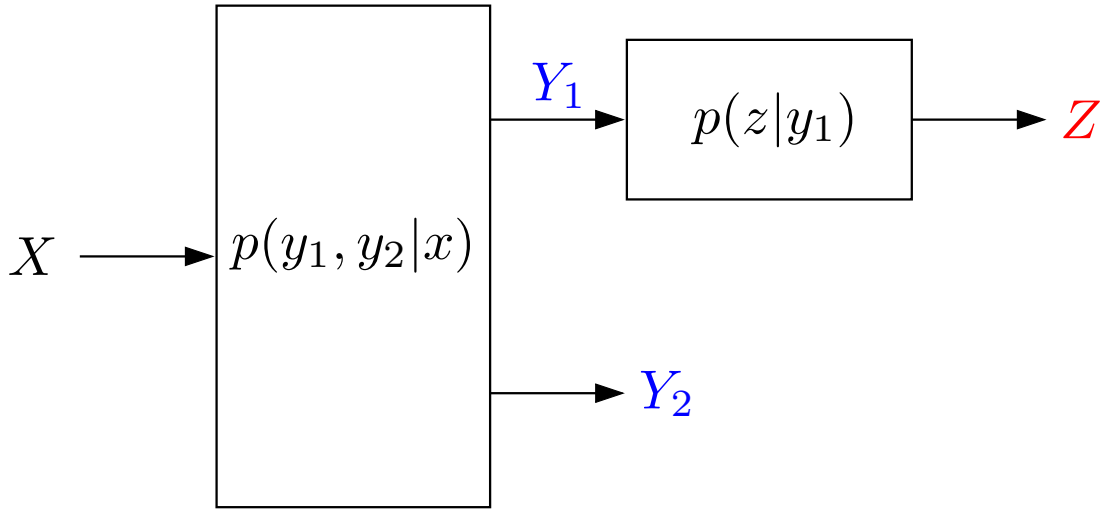

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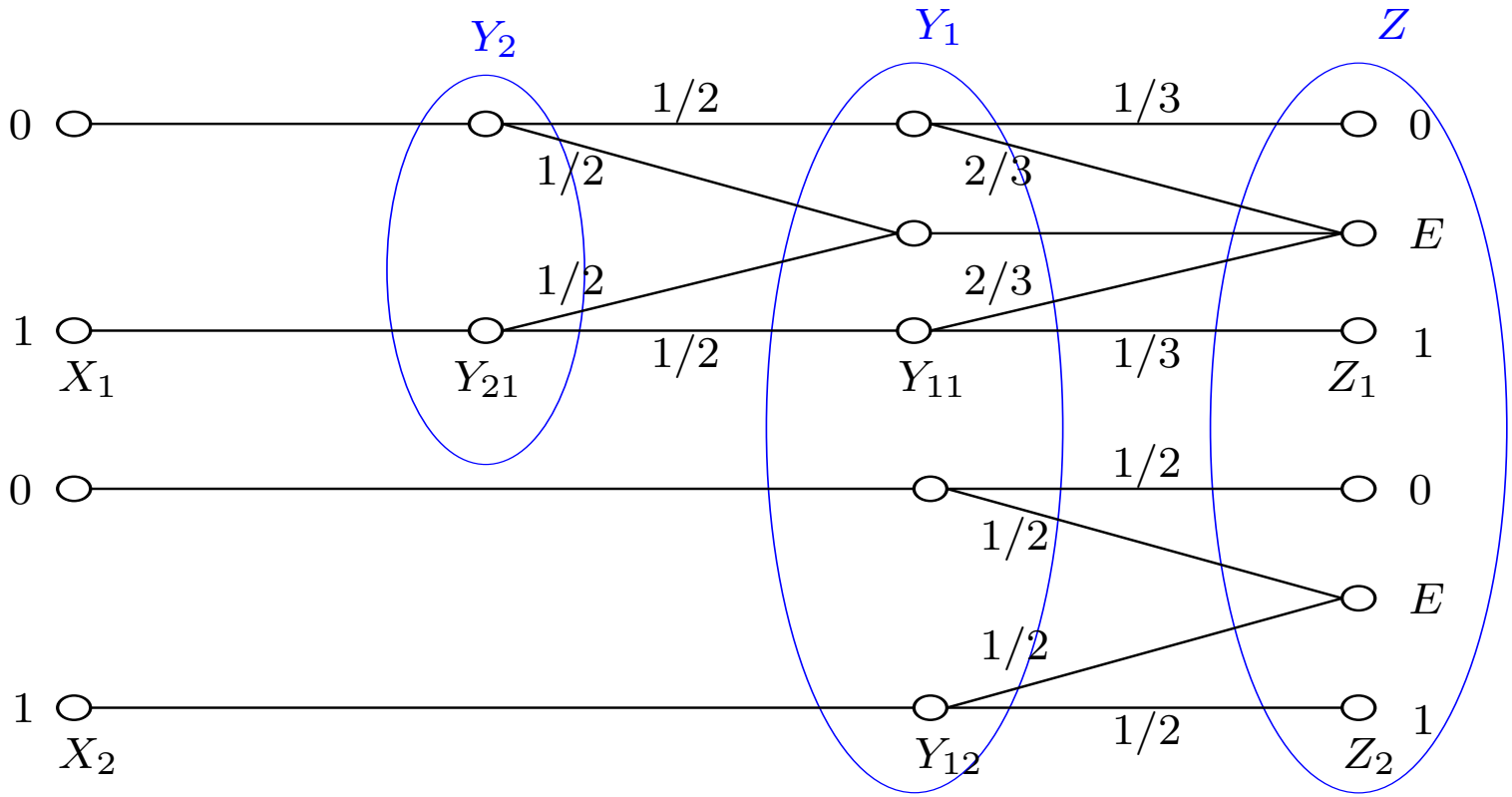
New lower bound (Chia–El Gamal [7])

$$C_{\mathrm{S}} \geq \max_{p(u,x)} \min\{I(X;Y_1) - I(X;Z), I(U;Y_2) - I(U;Z)\}$$

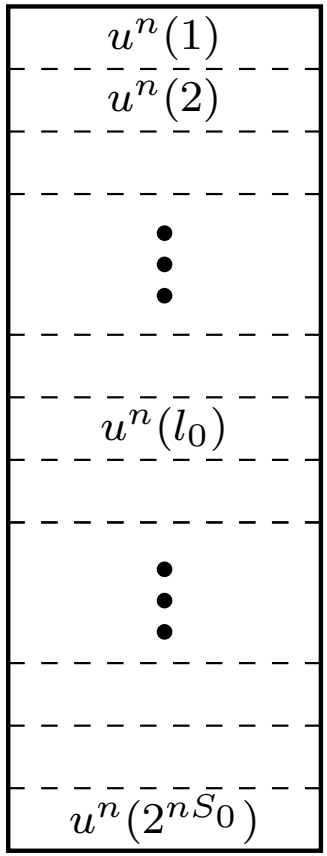
Reversely Degraded BEC Example [4]



- With Csiszár–Körner extension: Optimal secrecy rate $R < 5/6$
- With new scheme, set $U = X_1$; X_1 and X_2 independent $\mathrm{Bern}(1/2)$ $\Rightarrow C_{\mathrm{S}} = 5/6$ [8]

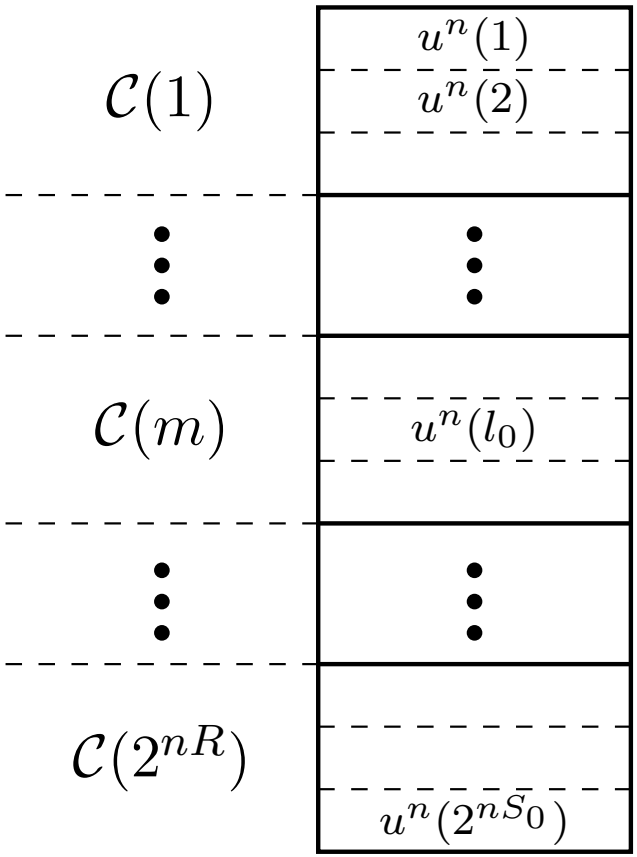
Outline of Achievability: Codebook Generation

- Generate 2^{nS_0} i.i.d. $u^n \sim \prod_{i=1}^{n} p_U(u_i)$.



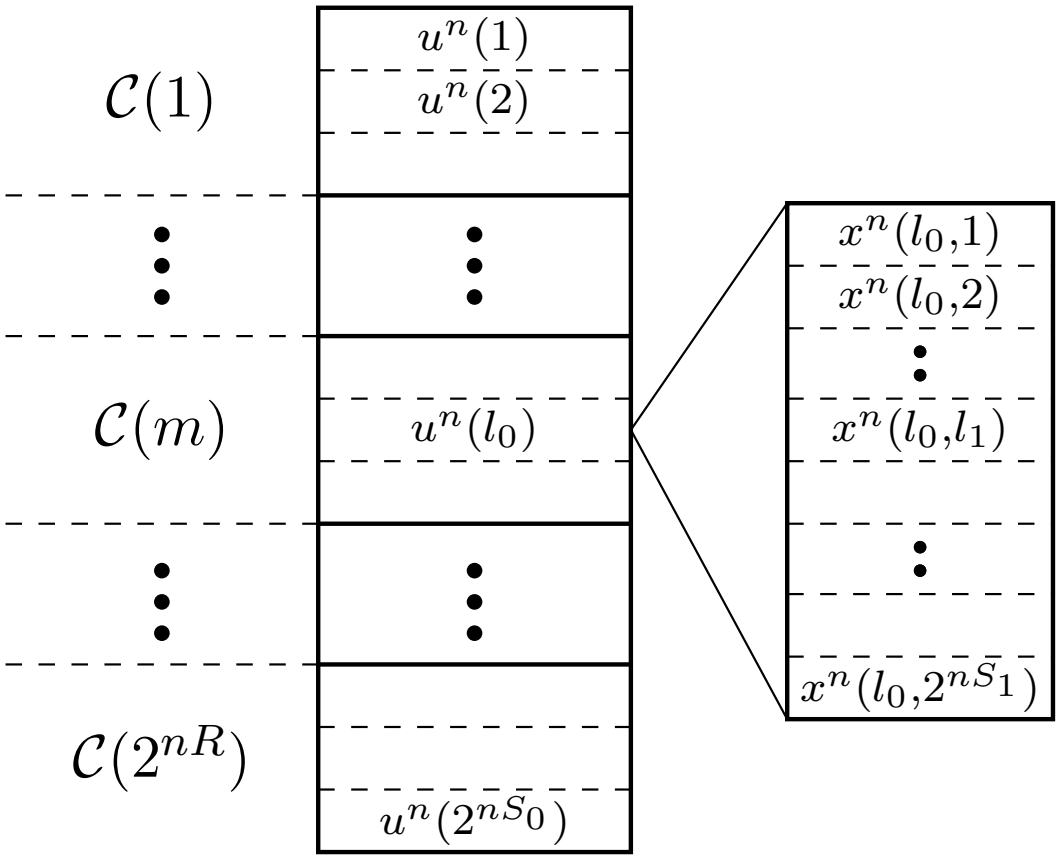
Outline of Achievability: Codebook Generation

- Generate 2^{nS_0} i.i.d. $u^n \sim \prod_{i=1}^{n} p_U(u_i)$. Partition into 2^{nR} subcodes



Outline of Achievability: Codebook Generation

- For each u^n, generate subcode of 2^{nS_1} conditionally i.i.d. sequences $x^n \sim \prod_{i=1}^{n} p(x_i|u_i)$



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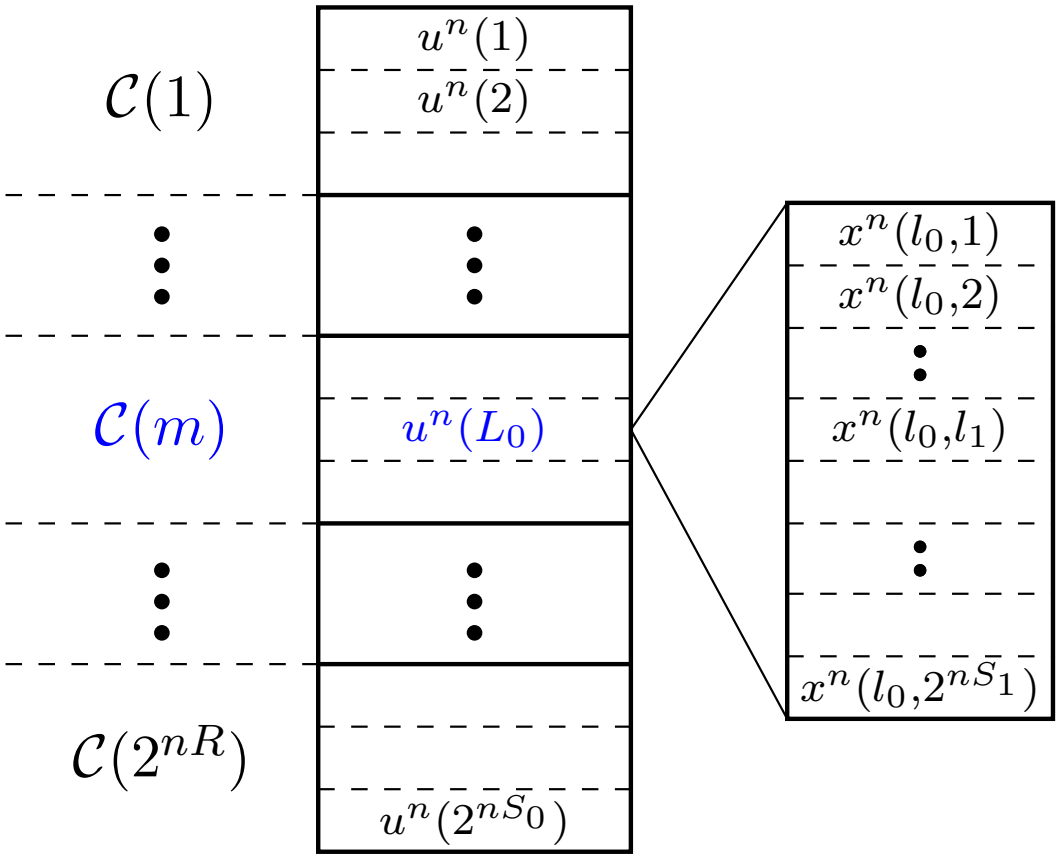
- To send m, choose random $u^n(L_0) \in \mathcal{C}(m)$.



Outline of Achievability: Encoding

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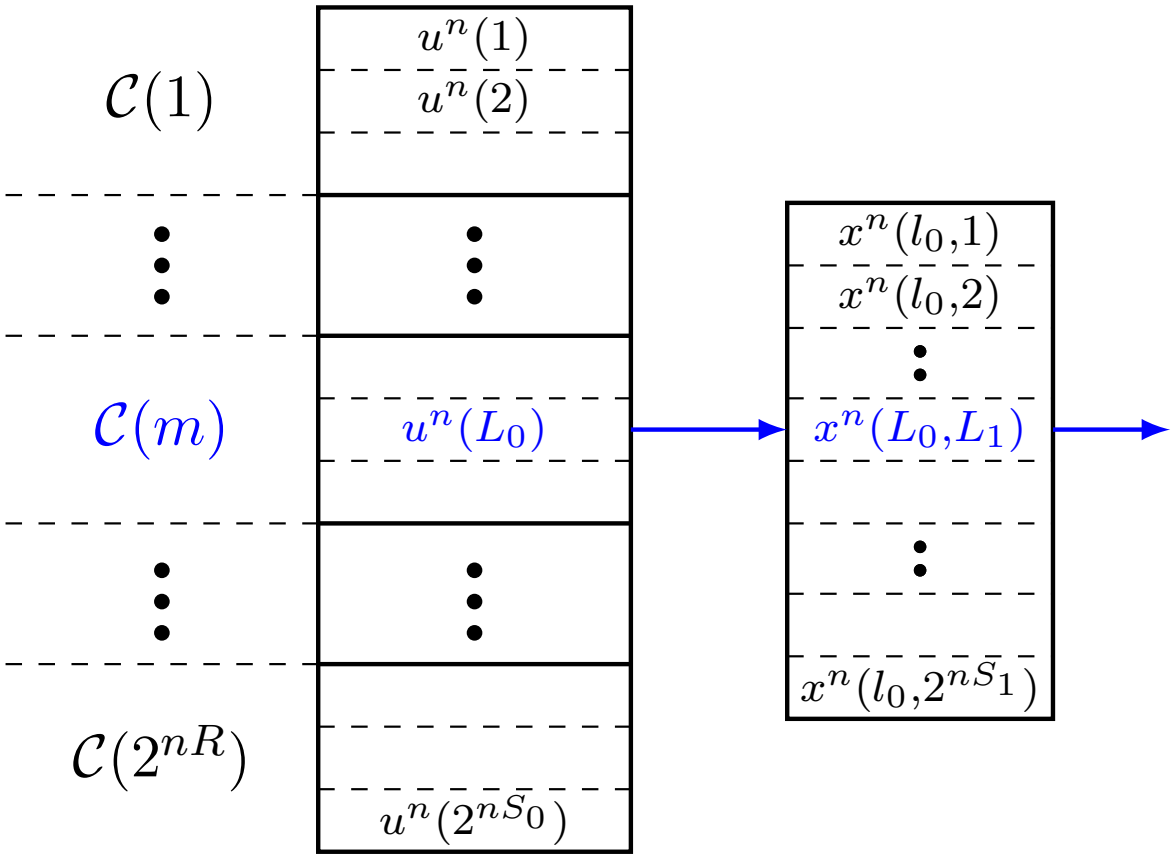

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- Combining bounds and performing Fourier–Motzkin elimination completes the proof

- By using Marton coding in addition, we obtain the following:

Generalized Lower Bound (Chia–El Gamal [7])

A rate R is achievable for sending a confidential message to receivers Y_1 and Y_2, while keeping it secret from eavesdropper Z if

$$R < \min\{I(U_1;Y_1|Q) - I(U_1;Z|Q),\ I(U_2;Y_2|Q) - I(U_2;Z|Q)\},$$
$$2R < I(U_1;Y_1|Q) + I(U_2;Y_2|Q) - 2I(U_0;Z|Q) - I(U_1;U_2|U_0)$$

for some $p(q,u_0,u_1,u_2,x) = p(q)p(u_0|q)p(u_1|u_0)p(x,u_2|u_0,u_1) = p(q)p(u_0|q)p(u_2|u_0)p(x,u_1|u_0,u_2)$ such that

$$I(U_1,U_2;Z|U_0) \le I(U_1;Z|U_0) + I(U_2;Z|U_0) - I(U_1;U_2|U_0)$$

- Optimal for:
  - Both Y_1 and Y_2 are less noisy than Z
  - Reversely degraded product channels [8]
  - MIMO?

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  - Compress–forward for networks
- Important research direction because it leads to new coding techniques for networks

Acknowledgments

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