Capacity of Coordinated Actions

Thomas Cover

Stanford University

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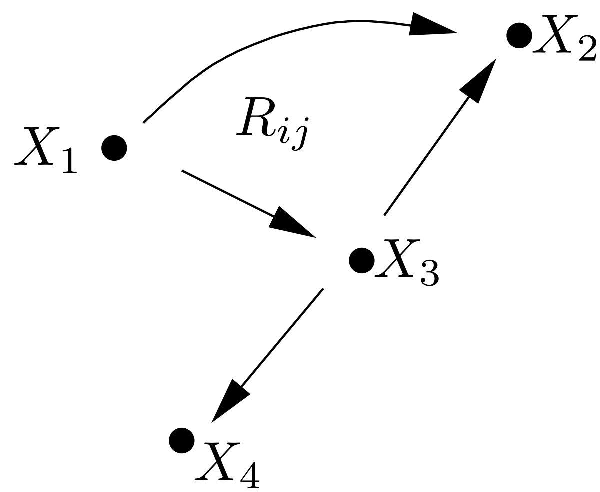
Joint work with Paul Cuff and Haim Permuter

Cooperation

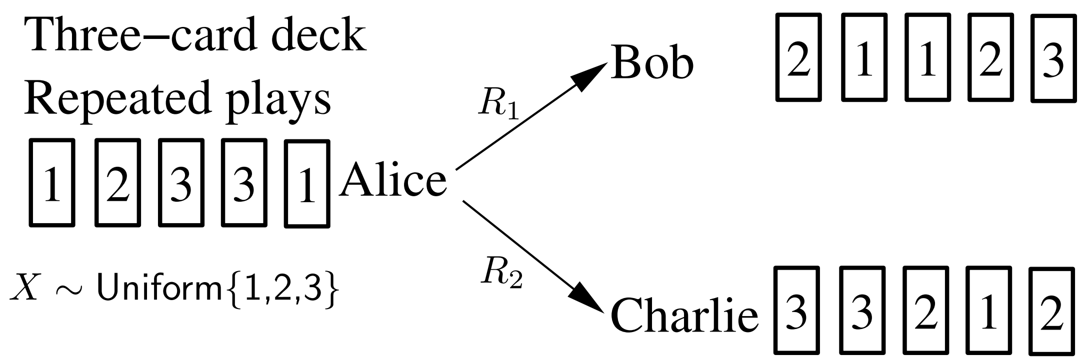


Purpose of network is to set up cooperative action.

Examples:
Basketball team
Armies
Distributed games
Stock market
Control
Civilization

What is the set of all joint distributions that can be set up in a communication network?

Example



The Goal

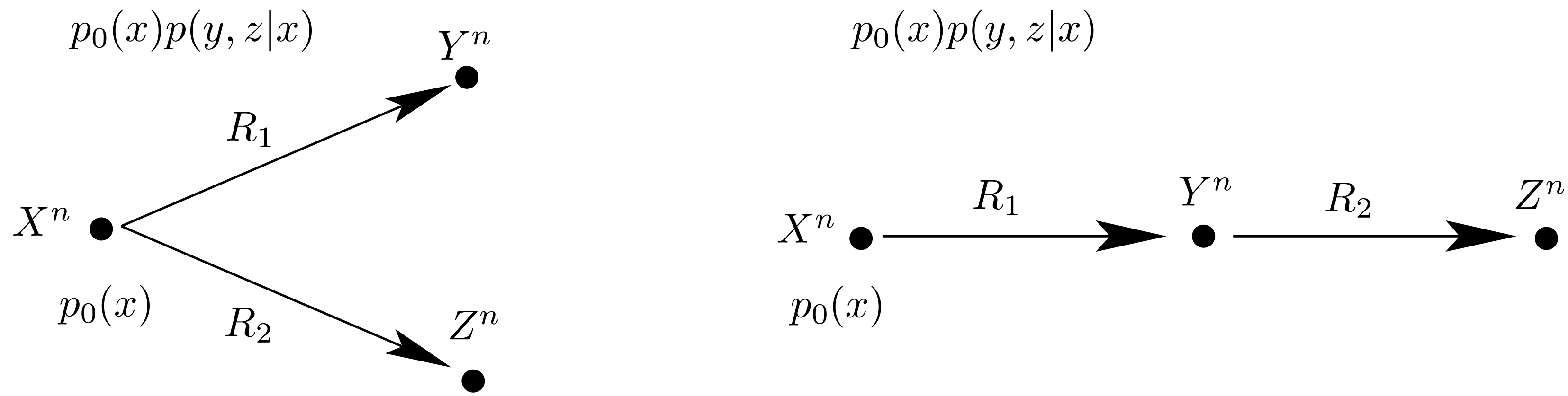
Bob and Charlie have to choose different cards from each other, and different from Alice, with equal probability.

Question

How much information must Alice send to Bob and to Charlie to achieve the goal?

Simple scheme requires $R_1 = R_2 = \log 3$. Is it optimal?

The formulation of coordinated actions problem



- The actions at node X are specified by nature: $p(x^n) = \prod_{i=1}^n p_0(x_i)$

- The actions at nodes Y, Z are chosen according to the information received at the nodes

- The goal is to find the rates (R_1, R_2) that can achieve the joint distribution $p_0(x)p(y,z|x)$

Each agent chooses actions

$$X^n \qquad\qquad Y^n$$

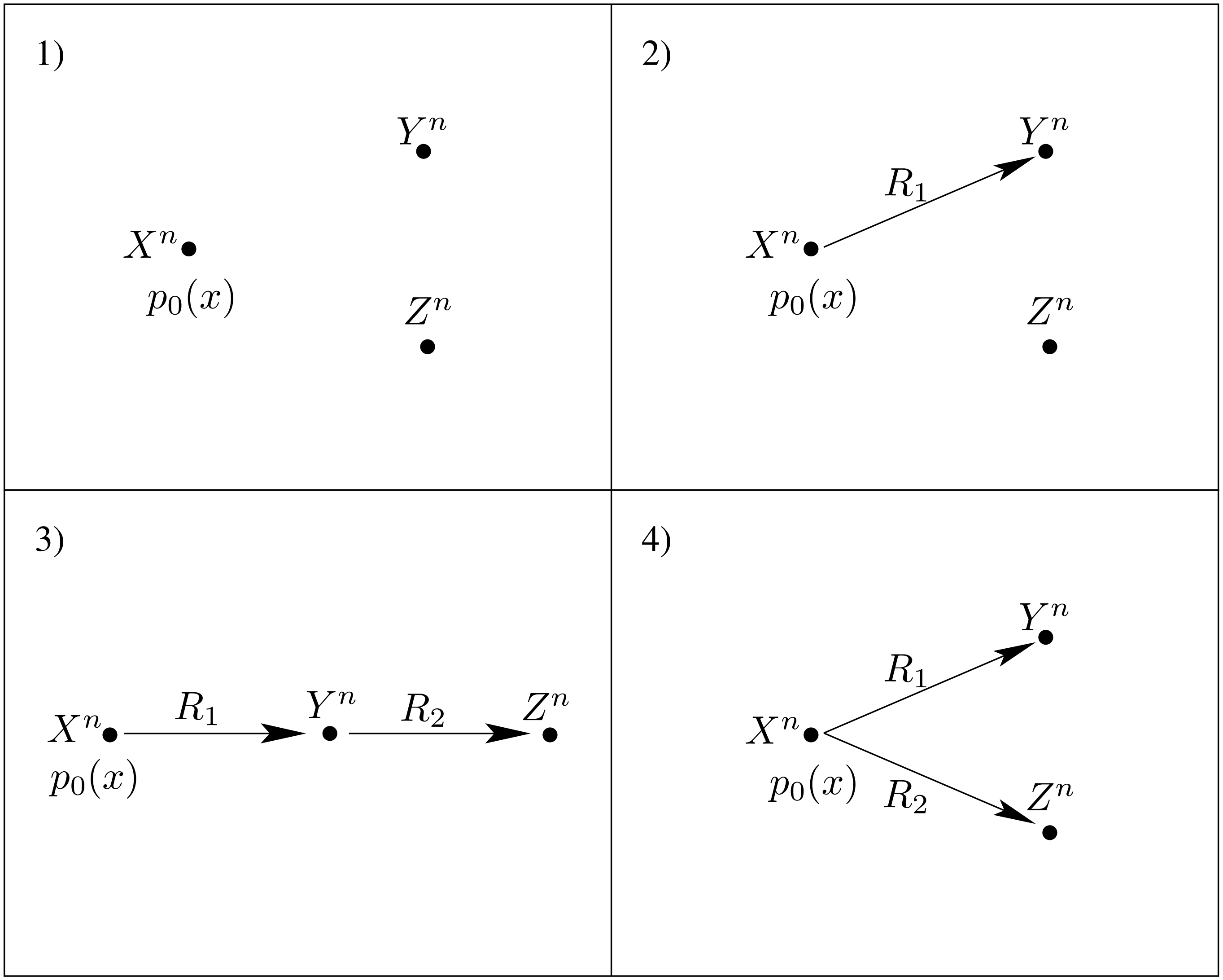
$$p(x, y)$$

- If each agent can select their actions freely, then communication is not necessary!
- For joint distribution common randomness is needed

$$H_C(X; Y) = ...$$

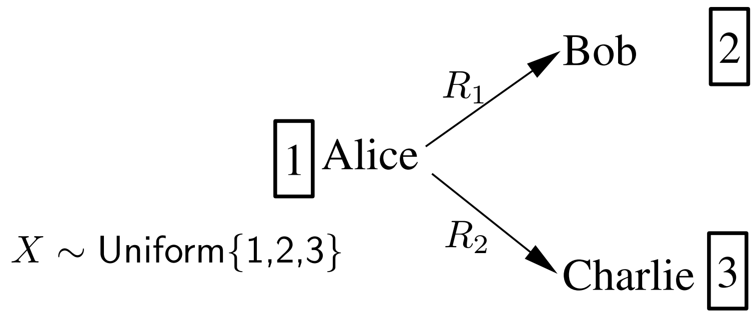
- For joint type no communication and no randomness is needed.

Building blocks of distributed action problems



Applications

- Task Assignments - agents must perform different jobs



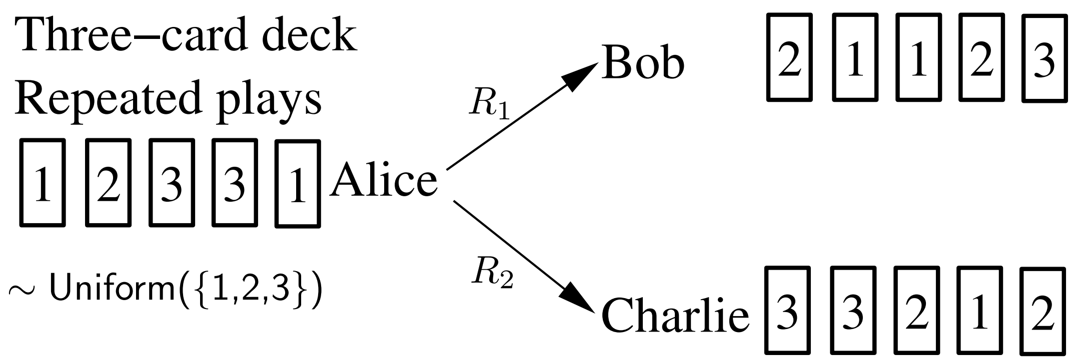
- Computation: parallelization and recombination

- Game theory - players must take actions according to an optimal distribution [Anantharam/Borkar05].

- Quantum information - quantum coding of mixed states

  [Barbum/Caves/Fuchs/Schumacher01], [Kramer/Savari07]

An Achievable Scheme for the Question

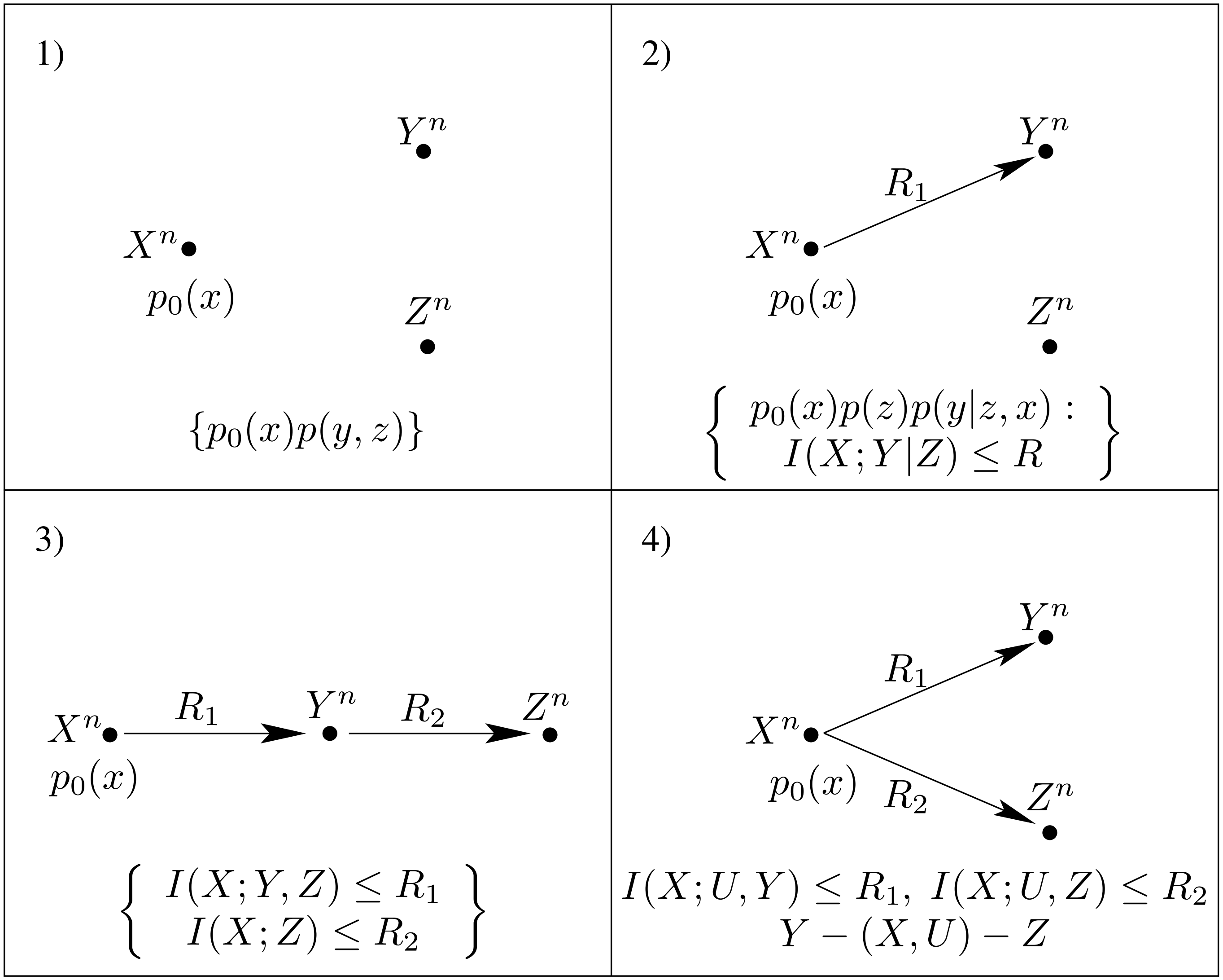


We restrict $Y = \{1, 2\}$ and $Z = \{2, 3\}$. Y will choose 1 and Z will choose 3 as default, unless X tells them to choose 2.

$$R_1 \quad \geq \quad I(X; U, Y) = H(\frac{1}{3}) - 0 = 0.918$$

Showed that $R_1 = R_2 = \log 3 - \log \phi = 0.890...$ is achievable, where ϕ is the golden ration, $\phi = \frac{1+\sqrt{5}}{2}$.

Building blocks of distributed action problems



Summary

- What joint distributions are achievable?

- Useful in task assignment, game theory, communication, social planning and quantum information theory.