Sum capacity of a Class of Cyclically Symmetric Interference Channels

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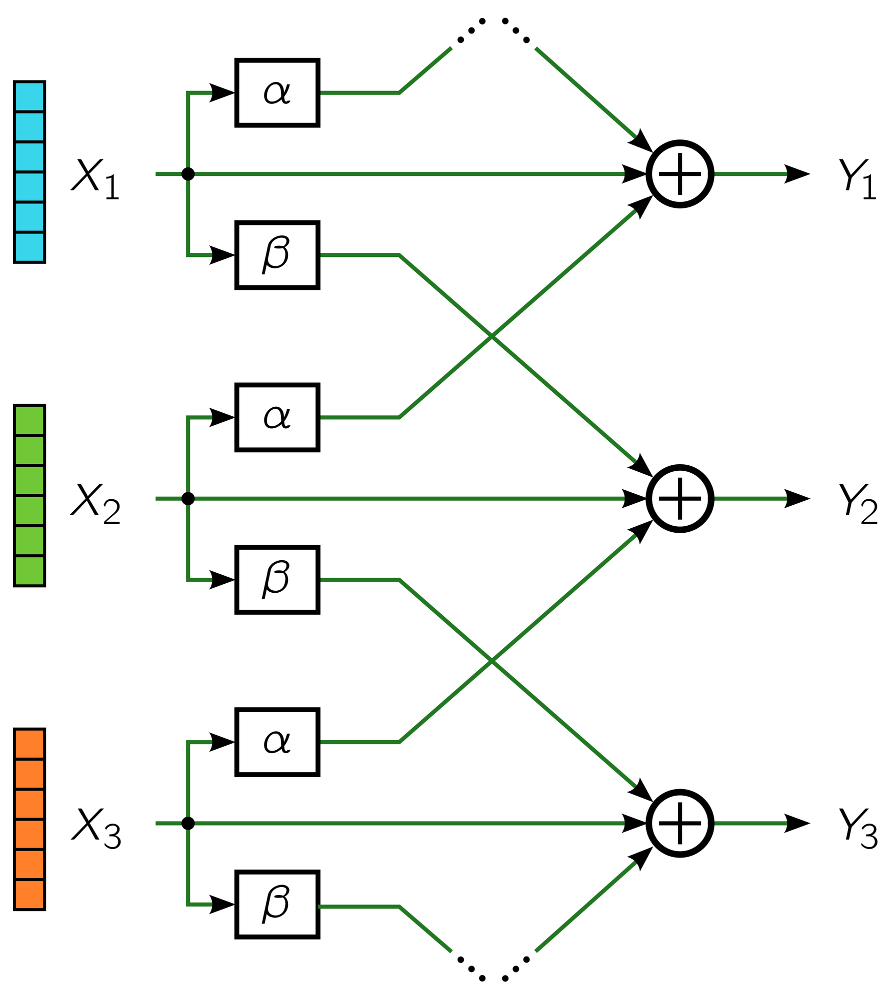
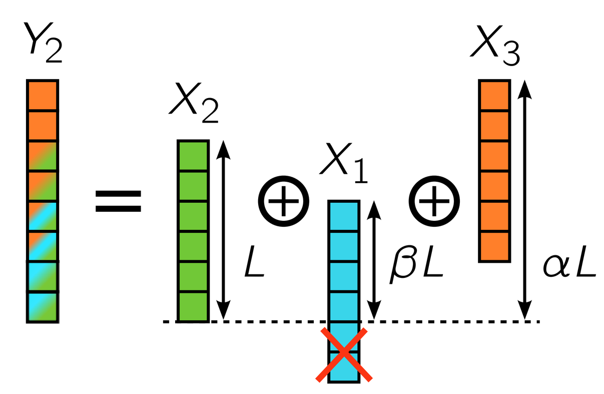
Abstract

- Consider a K-user deterministic interference channel
- Find the symmetric capacity (sum capacity) for a large set of parameters
- Capacity is achieved by intricate bit pipe assignment patterns

Cyclically symmetric interference channel



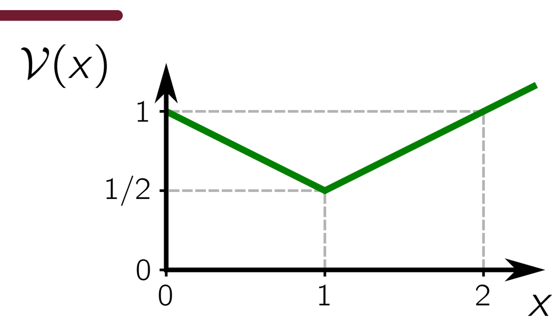
- Motivated by Gaussian interference channel
  - Capture asymptotic behavior (GDOF)
- Deterministic model
  - Inputs and outputs are bit vectors
  - Outputs are function of inputs
- K users with local interference
  - Wyner connectivity model
- Cyclically symmetric channel
  - Invariant to cyclic relabeling of users
  - Parameters L, α, β with $L, \alpha L, \beta L \in \mathbb{N}$



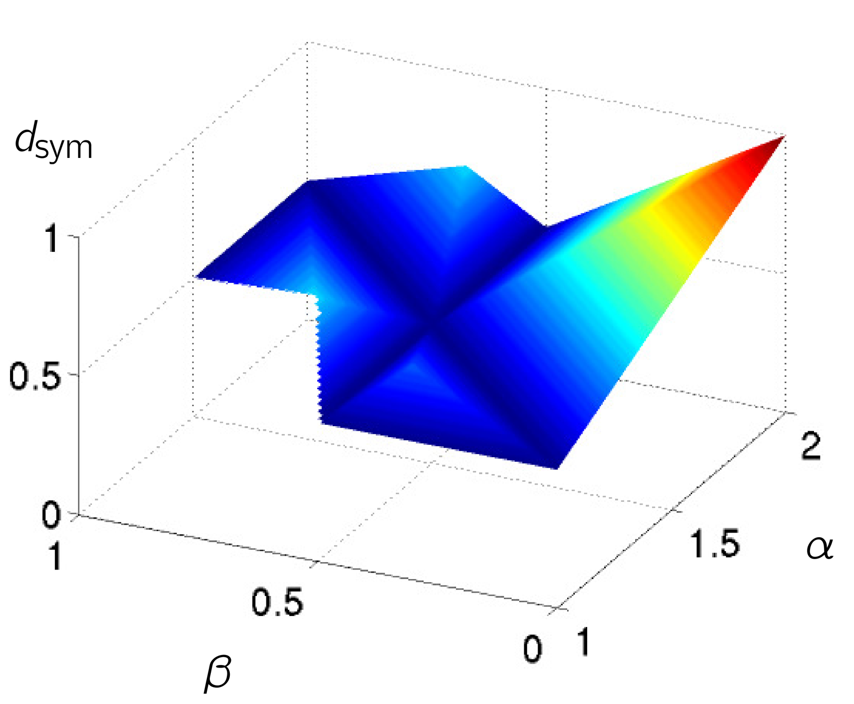
$$\mathbf{Z} = \begin{bmatrix} \mathbf{0}_{L\times L} \\ \mathbf{I}_L \end{bmatrix}, \quad \mathbf{D} = \mathbf{U}^T = \begin{bmatrix} \mathbf{0}^T & 0 \\ \mathbf{I}_{2L-1} & \mathbf{0} \end{bmatrix}, \quad Y_2 = \mathbf{Z}X_2 \oplus \mathbf{U}^{(\alpha-1)L}\mathbf{Z}X_3 \oplus \mathbf{D}^{(1-\beta)L}\mathbf{Z}X_1$$

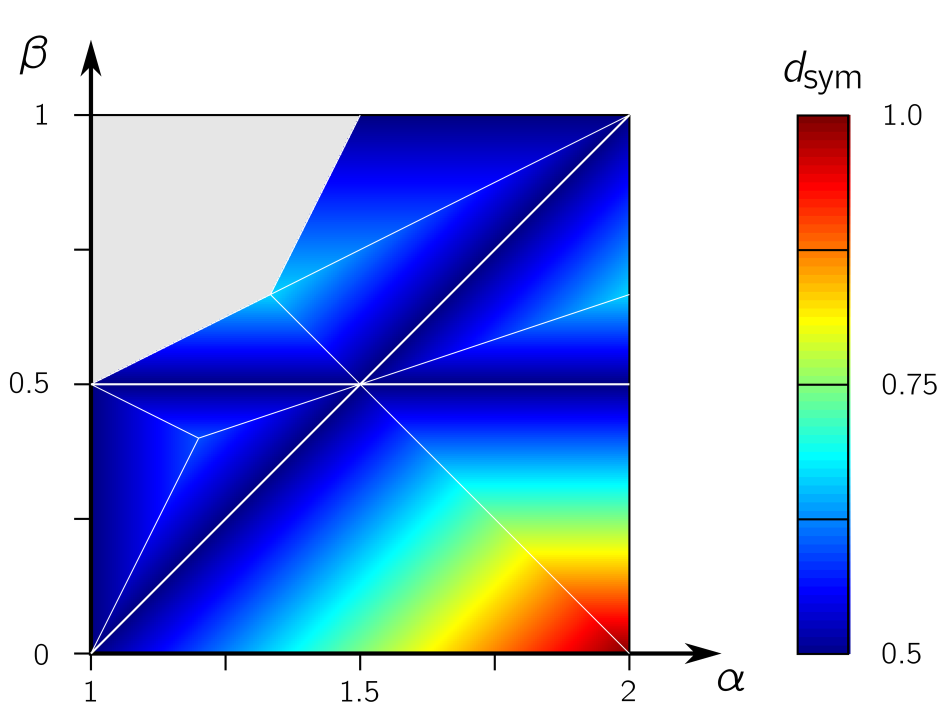
Main result

- **Normalized symmetric capacity** $d_{\text{sym}} = \sup R_{\text{sym}}/L$
- Define $\mathcal{V}(x) = \frac{1+|x-1|}{2}$



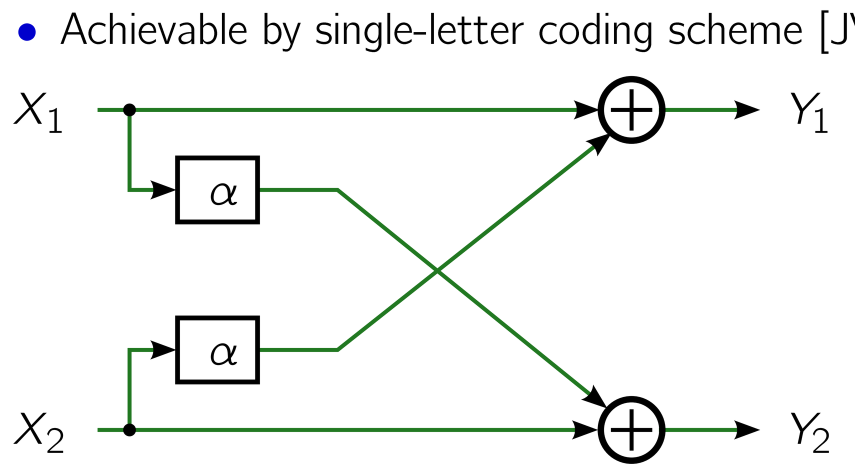
> The normalized symmetric capacity of the channel defined above with
> $$(\alpha, \beta) \in [1,2] \times [0,1] \quad \text{and} \quad \alpha \geq \min\{2\beta, \beta/2+1\}$$
> is $$d_{\text{sym}} = \min\{1, \mathcal{V}(\alpha), \mathcal{V}(\beta), \mathcal{V}(2\beta), \mathcal{V}(\alpha-\beta)\}$$

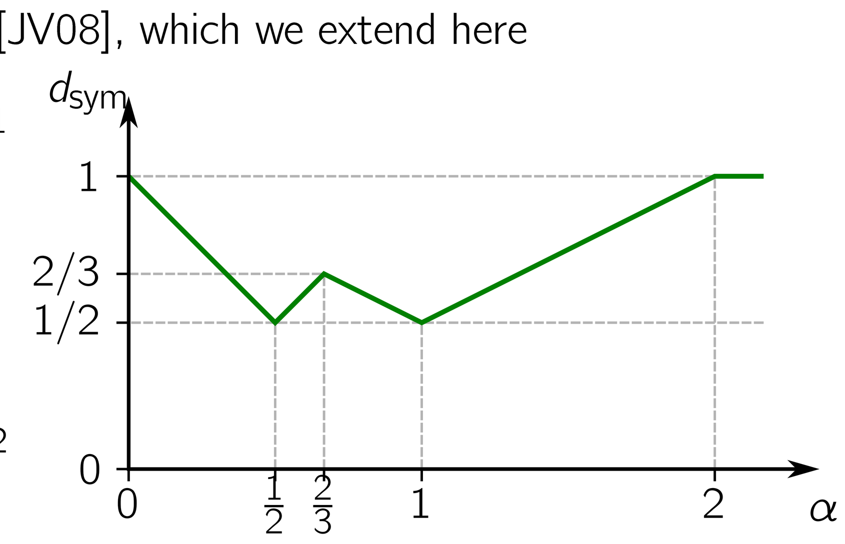




Comparison to the two-user case

- Symmetric capacity is known [EC82, ADT07, BT08]
- Achievable by single-letter coding scheme [JV08], which we extend here





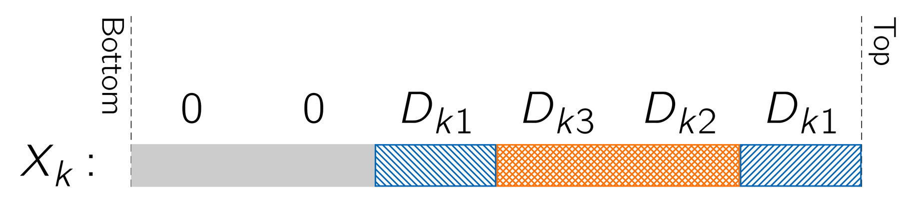
Capacity-achieving codes

- Characteristics of the proposed coding scheme:
  - Single-letter
  - Linear encoding and decoding
  - All transmitters use the same code
- For given (α, β), encode as follows:

Let D_k: independent message bits
$\mathbf{G}(\alpha,\beta)$: assignment matrix of size $L \times L\, d_{\text{sym}}(\alpha,\beta)$
Use $X_k = \mathbf{G}(\alpha,\beta) D_k, \quad k = 1,2,3$

- Example:

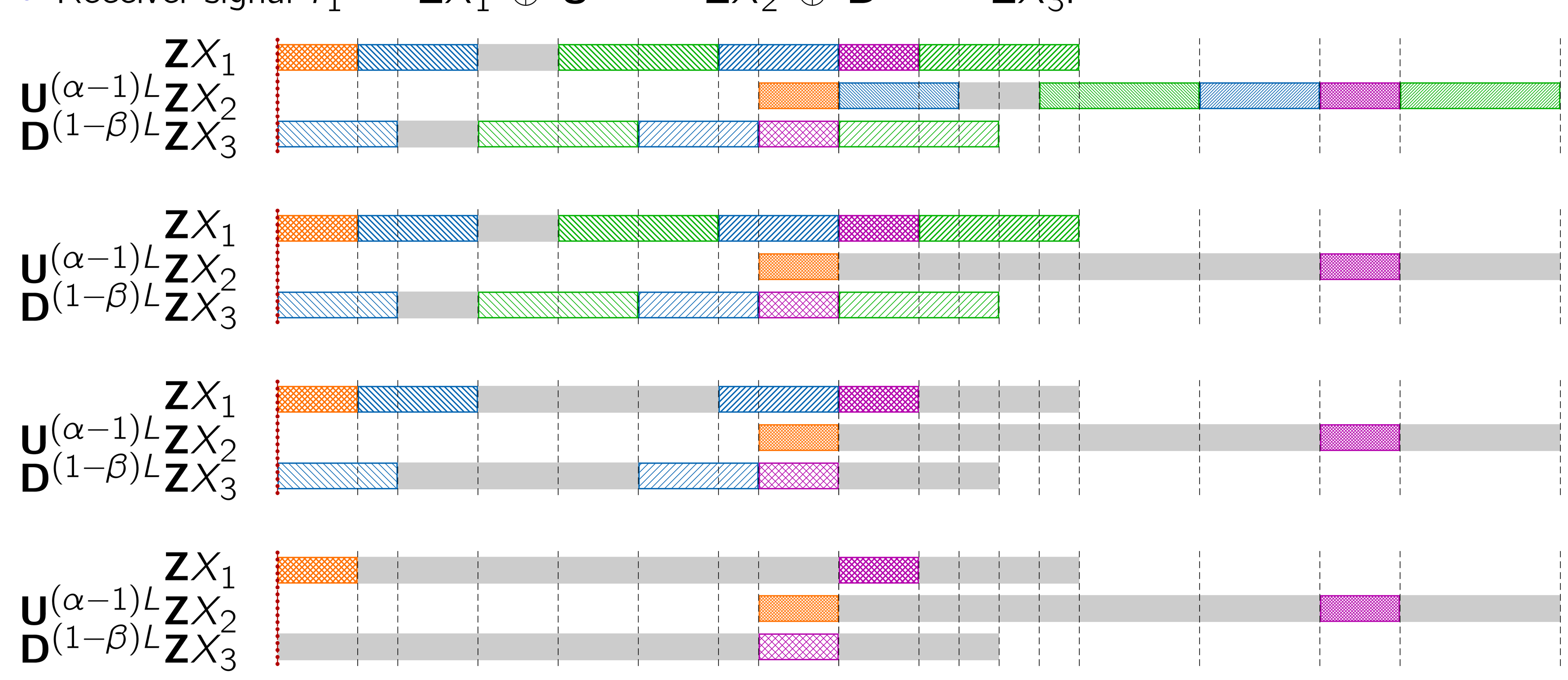
$$X_k = \begin{bmatrix} 1&0&0 \\ 0&1&0 \\ 0&0&1 \\ 1&0&0 \\ 0&0&0 \\ 0&0&0 \end{bmatrix} \begin{bmatrix} D_{k1} \\ D_{k2} \\ D_{k3} \end{bmatrix}$$



Example

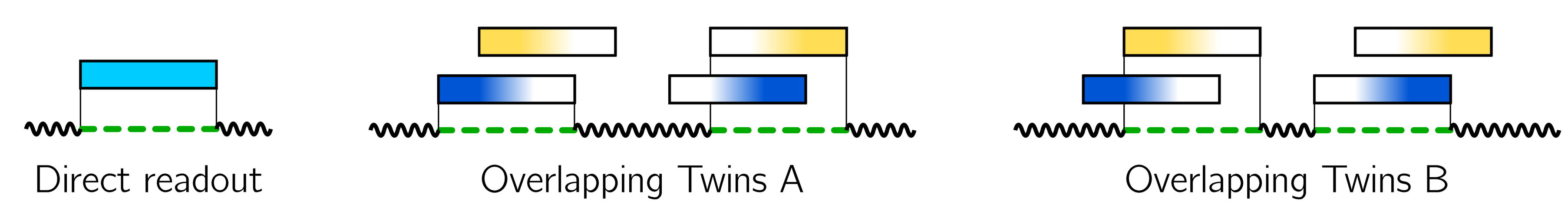
- Consider $\alpha = 1.6$, $\beta = 0.9$, with optimum $d_{\text{sym}} = 0.55$

- Receiver signal $Y_1 = \mathbf{Z}X_1 \oplus \mathbf{U}^{(\alpha-1)L}\mathbf{Z}X_2 \oplus \mathbf{D}^{(1-\beta)L}\mathbf{Z}X_3$:

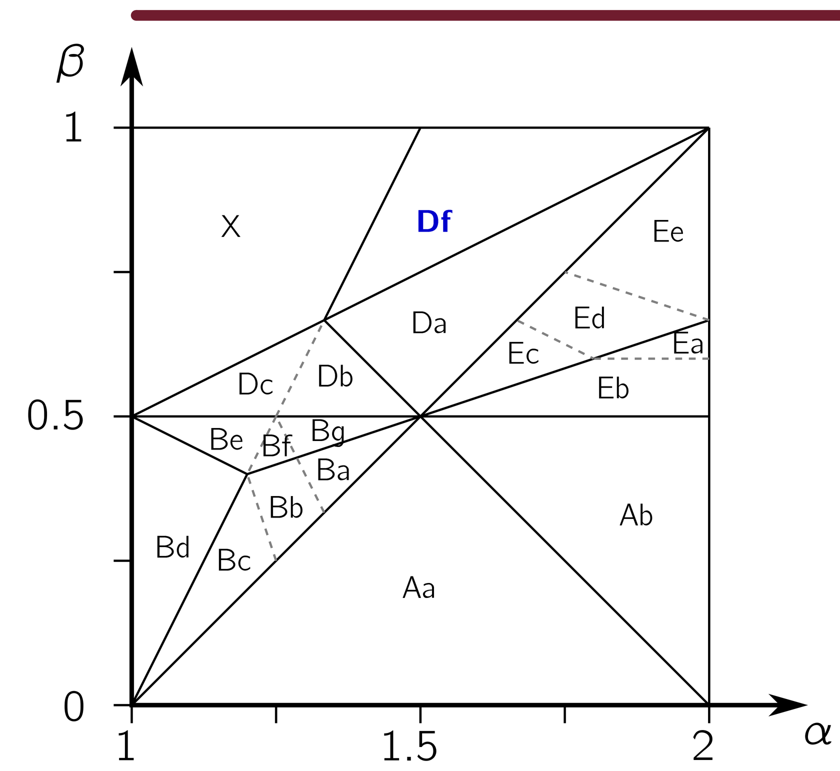


Decoding rules

- Each step above matches one of three basic rules

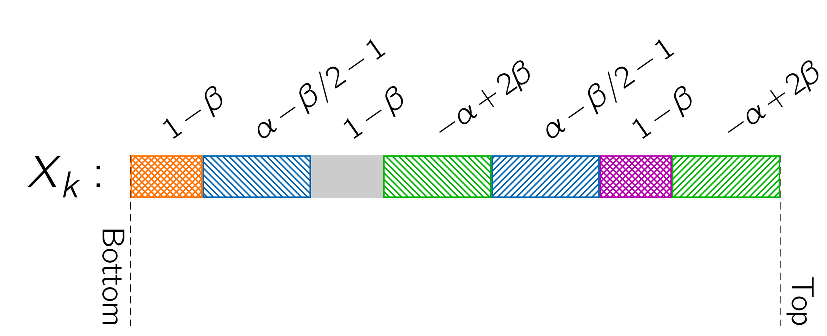


Different patterns for different regions



Example: Region Df

Rate $d_{\text{sym}} = 1 - \beta/2$ is achieved by



Such assignments exist for all regions.

References

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[BT08] G. Bresler and D. Tse. "The two-user Gaussian interference channel: a deterministic view". *Euro. Trans. Telecomm.*, vol. 19, no. 4, pp. 333–354, Jun. 2008. ArXiv:0807.3222.

[EC82] A. A. El Gamal and M. H. M. Costa. "The capacity region of a class of deterministic interference channels". *IEEE Transactions on Information Theory*, vol. 29, no. 2, pp. 343–346, Mar. 1982.

[JV08] S. A. Jafar and S. Vishwanath. "Generalized degrees of freedom of the symmetric K user Gaussian interference channel". 2008. arXiv:0804.4489.

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