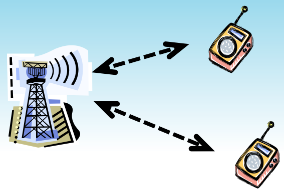
Efficient Codes using Channel Polarization

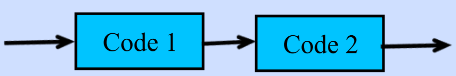
Bakshi, Jaggi, and Effros

STATUS QUO



- ***Practical capacity achieving schemes are not known for general multi-input multi-output channels***
- ***Codes based on channel polarization that achieve capacity for point-to-point, degraded broadcast and MAC have poor error performance***

NEW INSIGHTS

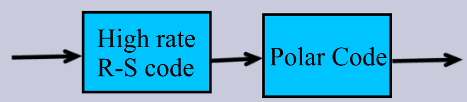


Concatenating Polar and R-S codes gives the best properties of both

- Use Polar codes as Code 2 as they achieve capacity
- Use R-S codes as Code 1 to reduce error probability
- Complexity

ACHIEVEMENT DESCRIPTION

At each encoder:



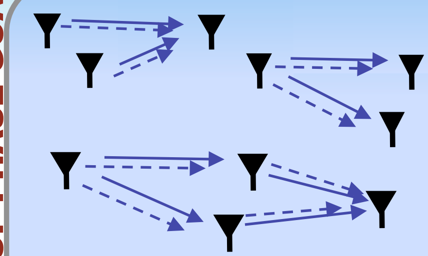
How it works:

- Divide input of blocklength N into N/f(N) sub -blocksof length f(N) each
- Apply high rate R-S code on the entire input followed by a polar code on each sub-block
- Decode the two stages one by one
- When the polar code fails on few of the sub-blocks, the R-S code can correct the error
- P(error) decays as exp(-o(N)); Complexity is O(N poly log N); excess rate goes to 0 asymptotically

Assumptions and limitations:

- Works for channels where capacity-achieving codes are known (e.g. point-to-point channels, degraded broadcast channels, multiple access channels)
- Dependence of error probability on excess rate unknown

END-OF-PHASE GOAL



- Joint decoding of the two stages may lead to a better error performance – we know this in special cases
- Use insight from concatenated coding scheme to design a better single stage coding scheme

COMMUNITY CHALLENGE

Find Polar Codes or a modification to achieve capacity for other types of channel.

Characterize the dependence on other parameters e.g., excess rate.

Concatenating Polar and R-S codes leads to more efficient codes for several different channels

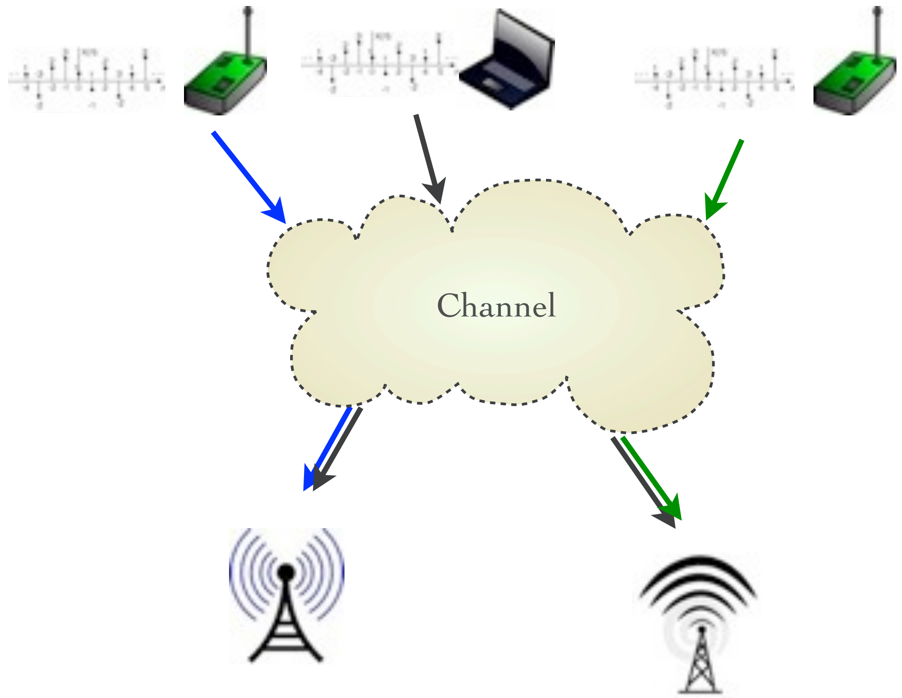
Efficient Codes based on Channel Polarization

Mayank Bakshi

Department of Electrical Engineering,
California Institute of Technology

(joint work with Sidharth Jaggi, CUHK and Michelle Effros, Caltech)

Motivation



Typical multiuser system

- Capacity bounds known in many cases

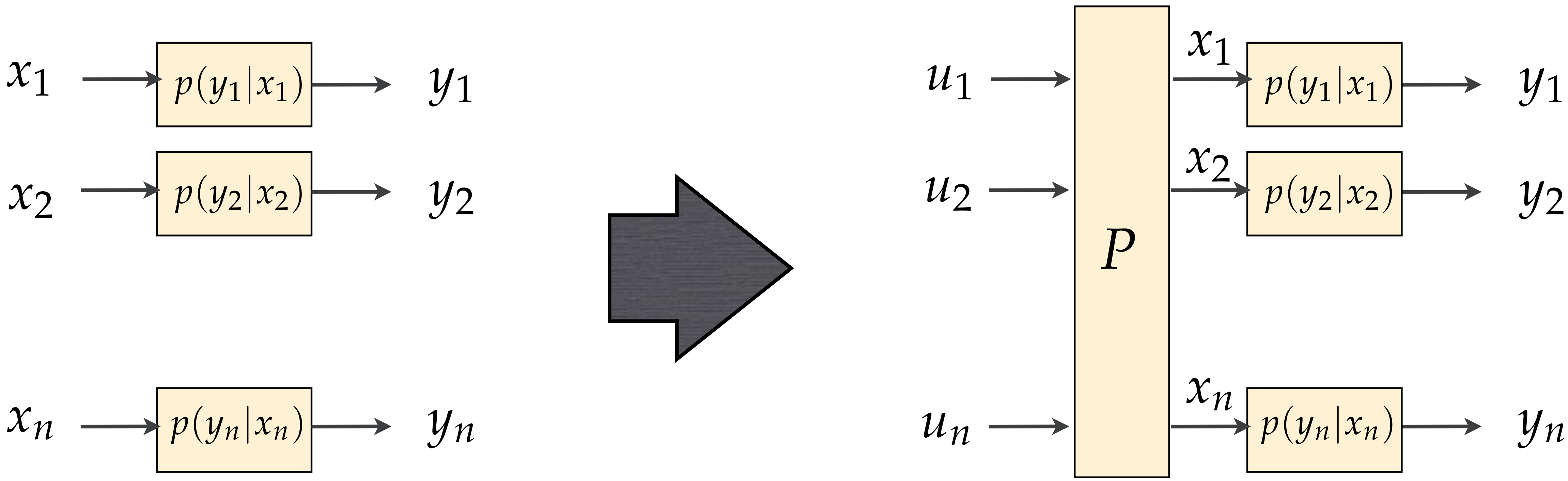
- Practical coding schemes unknown for most channels

Key Challenges:

- Encoding/Decoding Complexity
- Blocklength required to achieve desired error probability

Channel Polarization

e.g. Point-to-point channel



$x_i \longrightarrow y_i$

$u_i \longrightarrow (y^n, u^{i-1})$

Channel seen by each x_i is same (statistically)

Different u_i see different channels

Channel polarization:

Choose matrix P s.t. each u_i either sees a channel of capacity either close to 1 or close to 0 (depending on the value of *i*)

- Systematic procedure to construct P

- Successive cancellation based decoding rule

Main features:

Achieve capacity for arbitrary point-to-point channels

Encoding Complexity: $O(n \log n)$ (Close to linear)

Decoding Complexity: $O(n \log n)$ (Close to linear)

Error probability: $2^{-\sqrt{n}}$ (long block length required to get a desired error probability)

Can be applied to several multi-user channels as well

- Multiple access channel, degraded broadcast channel, Gelfand-Pinsker channel

Reed-Solomon Codes

$$(u_1, u_2, \ldots, u_k) \longrightarrow f(x) = u_1 + u_2 x + \ldots + u_k x^{k-1} \longrightarrow \big(f(x_1), f(x_2), \ldots, f(x_n)\big)$$

Data packets

Codeword

Main features:

Not capacity achieving in general

Encoding Complexity: $O(n(\log n)^2)$ (Close to linear)

Decoding Complexity: $O(n(\log n)^2)$ (Close to linear)

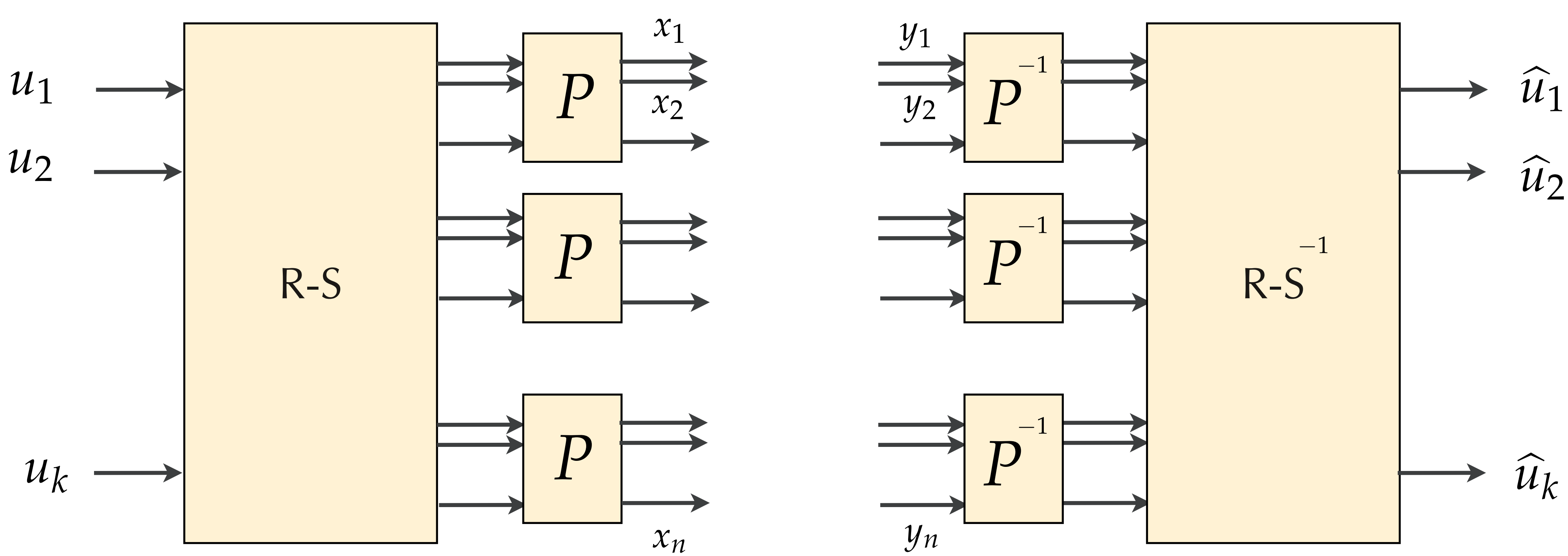
Error probability: $2^{-\alpha n}$ (short block lengths suffice to get a desired error probability)

Easily scale to large field sizes

Q: Can we get the best of both worlds?

A: Yes, almost

Concatenation



Encoding

Decoding

Concatenation

- Encode and decode in two steps

- Polarization based codes help correct channel errors at rate close to capacity

- R-S code encodes across blocks of Polar code to correct block errors when Polar codes fail

Main features:

Achieve capacity for arbitrary point-to-point channels

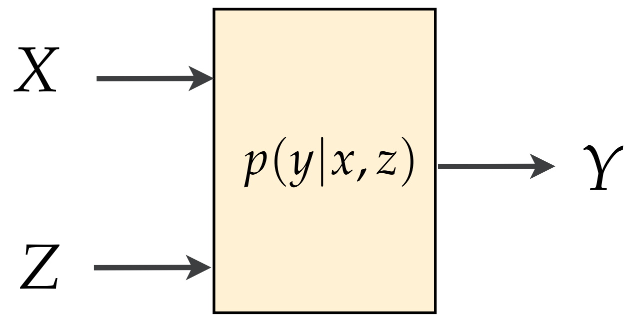
Encoding Complexity: $O(n(\log n)^2)$ (Close to linear)

Decoding Complexity: $O(n(\log n)^2)$ (Close to linear)

Error probability: $2^{-n/\log n}$ (block length required to get a desired error probability is almost of the same order as R-S)

Concatenation in multi-user channels

e.g. Multiple access channel



- Perform separate concatenation at each encoder
- R-S code adds redundancy to each message set
- Polarization based codes achieve the capacity
- By a careful choice of parameters:

  Achieve capacity

  Encoding Complexity: $O(n(\log n)^2)$

  Decoding Complexity: $O(n(\log n)^2)$

  Error probability: $2^{-n/\log n}$

Concatenation in network source coding

General idea:

- Use systematic R-S codes to compute redundancy packets at each encoder
- Encode the message symbols by an optimal code
- Transmit the redundancy packets without coding
- At each decoder, use redundancy packets to correct block errors
- Similar performance boost as in channel coding
- e.g., when combined with Polar codes for Coded Side Information problem,

  Achieve optimal rates

  Encoding Complexity: $O(n(\log n)^2)$

  Decoding Complexity: $O(n(\log n)^2)$

  Error probability: $2^{-n/\log n}$

Summary

Key ideas

- Concatenation helps reduce the error probability of coding schemes even in networked scenario
- Complexity is largely determined by outer code - R-S code
- Rate is determined by inner code - Polar Code

Results

- Efficient codes for
  - Several multi-user channels: Degraded broadcast channel, multiple-access channel
  - Network Source coding problems: e.g. Slepian-Wolf, Coded Side Information