LYAPONOV EXPONENTS ANALYSIS OF THE POSTERIOR MATCHING SCHEME

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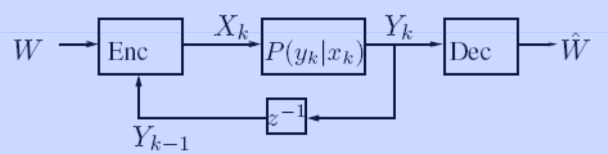
STATUS QUO

- The use of feedback is of the utmost importance in designing scalable, robust, reliable communication schemes.
- Posterior Matching Scheme generalizes previous feedback communication schemes for any memoryless channel. [1][2]
- A deep understanding of feedback is still lacking. Are there simple, provably good iterative feedback encoders/decoders?

THE POSTERIOR MATCHING SCHEME

- A real number W_0 in [0,1] represents an infinite sequence of bits to be communicated across a channel $P(y_k|x_k)$ with noiseless feedback.



- The information still missing at the receiver is extracted from the a-posteriori density function.

$$W_n = F_{W|Y_1^{n-1}}(W_0 \mid Y_1^{n-1})$$

- The signal is then reshaped to capacity-achieving (optimal) distributions.

$$X_n = F_Q^{-1}(W_n)$$

RECURSIVE FORM OF THE PM Scheme

$$X_1 = F_Q^{-1}(W_0)$$

$$X_{n+1} = S(X_n, Y_n)$$

- Converse to coding theorem: next input should be independent of everything decoder has seen so far
- With an optimal decoder, such "posterior matching" schemes achieve capacity
- Does this motivate a simple iterative decoder, that achieves capacity?
- How do we analyze this easily, exploiting the dynamics?

BINARY SYMETRIC CHANNEL

Consider the binary symmetric channel with error probability p:

- Figure 1 to the right shows the trajectories of the decoder when started with different values of W_{n+1}.
- The recursive transmission functions used for encoding and decoding are shown in Figure 1 below.

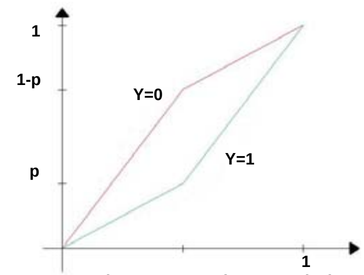


Figure 1. Recursive transmission functions

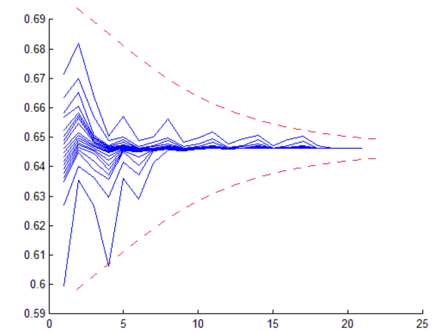


Figure 2. Trajectories of decoder with different initial condition

ASSUMPTIONS AND LIMITATIONS:

- Noiseless feedback
- Memoryless Channels
- Side benefit: provides a conceptually simple understanding of how PM scheme uses feedback to achieves capacity

MAIN RESULT

For the PM scheme, under appropriate technical assumptions, the following dynamical system decoder achieves capacity:

- Guess the state at time n to be u~Unif(0,1)

$$\hat{W}[n \mid n] = u$$

- Then recursively solve previous states

$$\hat{W}[i \mid n] = S_{Y_i}^{-1}(\hat{W}[i+1 \mid n])$$

- Finally, we arrive at our estimate of W_0

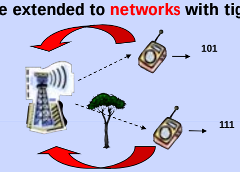
$$\hat{W}_0 = \hat{W}[0 \mid n]$$

As long as our estimate is within $2^{-(nC+1)}$ of our actual message, we can decode nC bits.

Since the Y's are independent and identically distributed, the decoder dynamical system is a Markov chain and has a Lyapunov exponent of -C for any initial condition u on (0,1).

IMPACT

- Provides explicit capacity-achieving recursvie encoders and decoders
- Can be extended to networks with tight converses



FUTURE GOALS

Use Stochastic Control methodology for a principled, canonical approach to address:

- noisy feedback (POMDP)
- Unknown channel (Q-learning)
- Delayed feedback

REFERENCES

[1] O. Shayevitz and M. Feder, "Communication with feedback via posterior matching," in *IEEE International Symposium on Information Theory,* Nice, France, June 2007.

[2] ——, "The posterior matching feedback scheme: Capacity achieving and error analysis," in *IEEE International Symposium on Information Theory, Toronto, Canada, July 2008.*

[3] H. Ebeid and T. P. Coleman, "Lyapunov Exponents and Reverse Iterated Function System Decoders for Channels with Feedback", in preparation for submission, IEEE International Conference on Communications, September 2009.

[4] T. P. Coleman "A Stochastic Control Approach to 'Posterior Matching'-style Feedback Communication Systems', submitted to *IEEE International Symposium on Information Theory, Seoul, Korea, July 2009.*

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