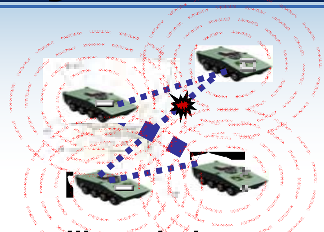
Adaptive modulation with smoothed flow utility
Boyd, Akuiyibo, O'Neill

FLoWS

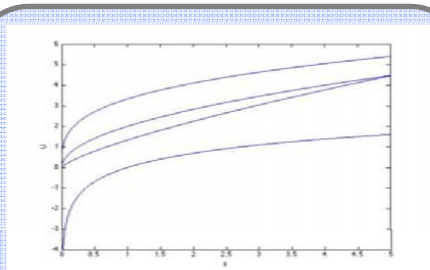
STATUS QUO



Prevailing wireless network utility maximization and resource allocation methods focus on per period optimization

These methods ignore the heterogeneous time scales over which network applications need resources

NEW INSIGHTS



- *Derive network utility from smoothed flows*

•Smoothing allows us to model the demands of an application that can tolerate variations in flow it receives over a time interval

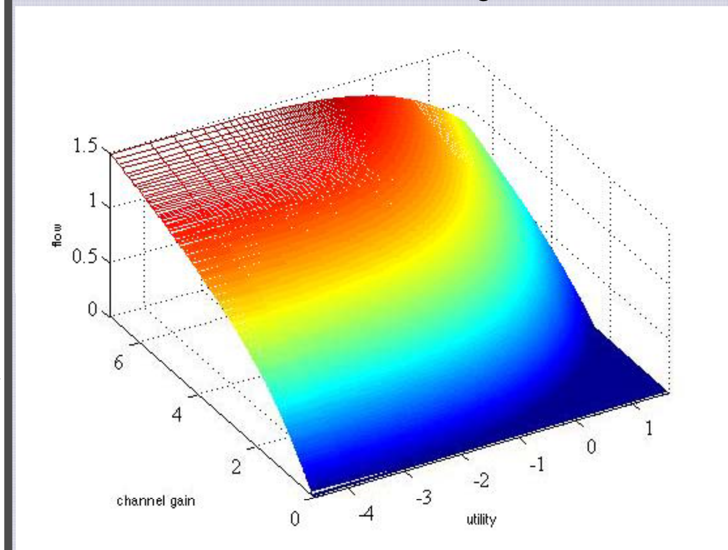
ACHIEVEMENT DESCRIPTION

MAIN RESULT:

Flow allocation to optimally trade off average smoothed flow utility and power.

HOW IT WORKS:

Optimal flow policy is a complicated function of smoothed flow and channel gain

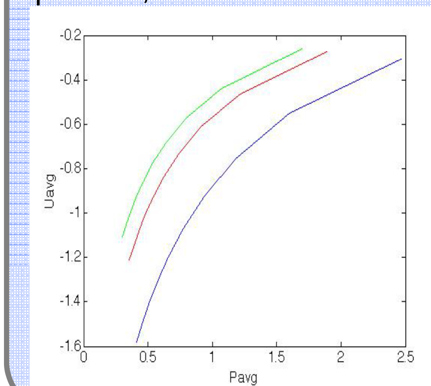


ASSUMPTIONS AND LIMITATIONS:

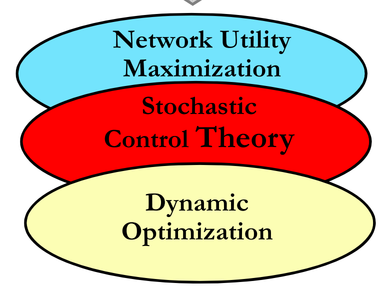
- Utilities are strictly concave, power is strictly convex; linear dynamics represent time averaging
- At each time period, assumes the transmitter learns random channel state through feedback

IMPACT

Different levels of smoothing lead to different optimal policies; different trade offs



NEXT-PHASE GOALS



Approximate dynamic programming (ADP) for MANETs

• *computationally tractable*

Optimally trade off average utility and power using smoothed flow utilities

Adaptive Modulation with Smoothed Flow Utility

Ekine Akuiyibo **Stephen Boyd** Dan O'Neill

Electrical Engineering Department, Stanford University

ITMANET PI meeting 09/14–15/09

The problem

- choose transmit power(s) and flow rate(s) to optimally trade off average utility and power
- utilities are functions of *smoothed* flow rates
- with each flow we associate a
  - smoothing time scale
  - concave increasing utility function
- our model:
  - channel gains are random
  - no interference

Smoothed flow utility

- wireless link supports n flows in period t
- $f_t \in \mathbf{R}^n$ is flow rate vector
- $s_t \in \mathbf{R}^n$ is smoothed flow rate vector: $s_{t+1} = \Theta s_t + (I - \Theta) f_t$
  - $\Theta = \mathbf{diag}(\theta), \quad \theta_i \in [0, 1)$
  - $\tau_i = 1/\log(1/\theta_i)$ is smoothing time for flow i
- $U : \mathbf{R}^n \to \mathbf{R}$: separable concave utility function
- average smoothed utility is

$$\bar{U} = \lim_{T \to \infty} \mathbf{E} \frac{1}{T} \sum_{\tau=0}^{T-1} U(s_\tau)$$

Channel model and average power

- capacity in period t is (up to a constant) $\log(1 + g_t p_t)$
  - $p_t \geq 0$ is transmit power
  - g_t is channel gain (up to constant)
- power required to support flow f_t: $p_t = \phi(\mathbf{1}^T f_t, g_t) = (e^{\mathbf{1}^T f_t} - 1)/g_t$
- average power is $\bar{P} = \lim_{T \to \infty} \mathbf{E}\, \dfrac{1}{T} \sum_{\tau=0}^{T-1} p_\tau$
- g_t IID exponential (for example)
- f_t (and therefore p_t) can depend on g_t, but not $g_{t+1}, g_{t+2}, \ldots$

Optimal policy

- (state feedback) policy: $f_t = \varphi(s_t, g_t)$
- goal: choose policy φ to maximize $\bar{U} - \lambda \bar{P}$
- $\lambda > 0$ is used to trade off average utility and power
- a convex stochastic control problem
- optimal value is $J^\star$

‘Solution’ via dynamic programming

- optimal policy is

$$\varphi^\star(z,g) = \operatorname*{argmax}_{w\geq 0}\{V^\star(\Theta z + (I-\Theta)w) - \lambda\phi(\mathbf{1}^T w, g)\}$$

- $V^\star$ is value function, (any) solution of Bellman equation

$$J^\star + V^\star(z) = \mathbf{E}\left\{U(z) + \max_{w\geq 0}\left\{V(\Theta z + (I-\Theta)w) - \lambda\phi(\mathbf{1}^T w, g)\right\}\right\}$$

- can numerically compute V (and $\varphi^\star$) for n very small (say, 1 or 2)

No trasmit region

- $V^\star$ is concave, increasing
- from convex analysis, $\phi^\star(z, g) = 0$ if and only if

$$g\nabla V^\star(\Theta z) \leq \left(\frac{\lambda}{1-\theta_1}, \ldots, \frac{\lambda}{1-\theta_n}\right)$$

  (assuming here $V^\star$ is differentiable)

- interpretation: don't transmit if
  - channel is bad (g small)
  - or, smoothed flows are large (z large $\Rightarrow \nabla V^\star(\Theta z)$ small)

Suboptimal policies

- greedy policy:

$$\varphi^{\text{greedy}}(z, g) = \underset{w \geq 0}{\text{argmax}} \{U(\Theta z + (I - \Theta)w) - \lambda\phi(\mathbf{1}^T w, g)\}$$

- approximate dynamic programming (ADP) policy:

$$\varphi^{\text{adp}}(z, g) = \underset{w \geq 0}{\text{argmax}} \{\hat{V}(\Theta z + (I - \Theta)w) - \lambda\phi(\mathbf{1}^T w, g)\}$$

where $\hat{V}$ is an approximate or surrogate value function, *e.g.*,

$$\hat{V}(z) = V_1^\star(z_1) + \cdots + V_n^\star(z_n)$$

where $V_i^\star$ is optimal for associated single flow problem

Single-flow examples

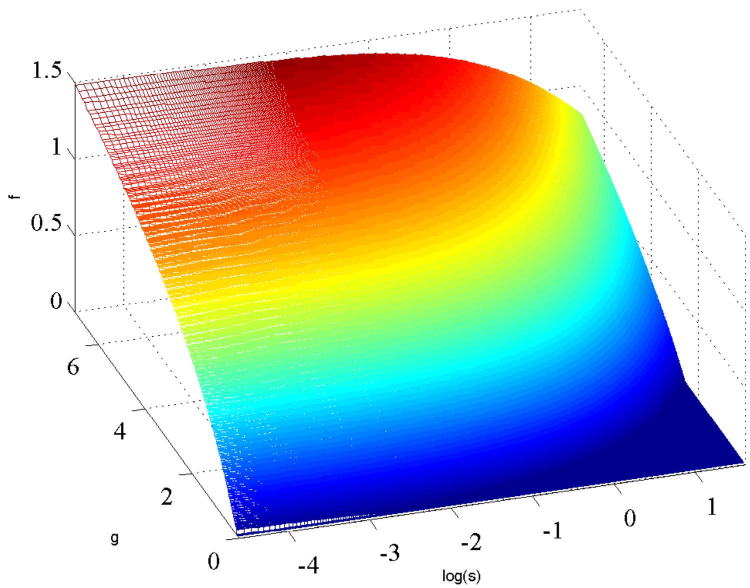
- compare two examples with
  - light smoothing ($\tau = 1$; $\theta = 0.37$)
  - heavy smoothing ($\tau = 20$; $\theta = 0.95$)

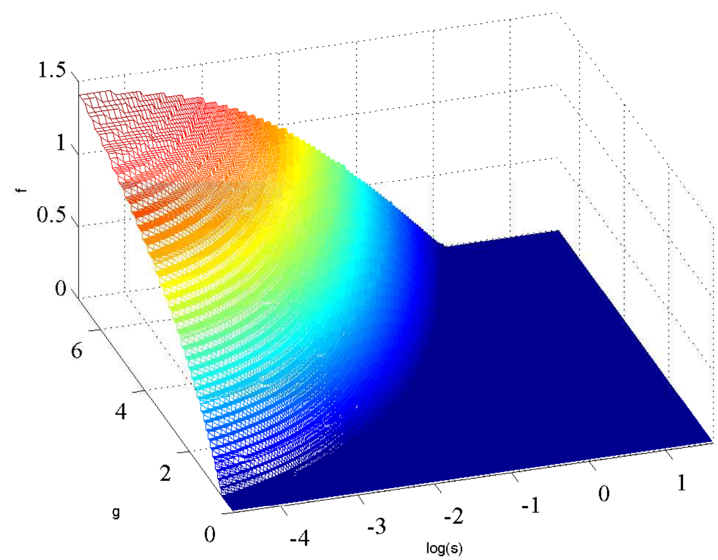
- log utility function $U(s) = \log s$; g_t exponential with $\mathbf{E}\, g_t = 1$

- λs chosen to yield $\bar{U} = -1.8$
  - $\lambda = 2.3$ for light smoothing
  - $\lambda = 10$ for heavy smoothing

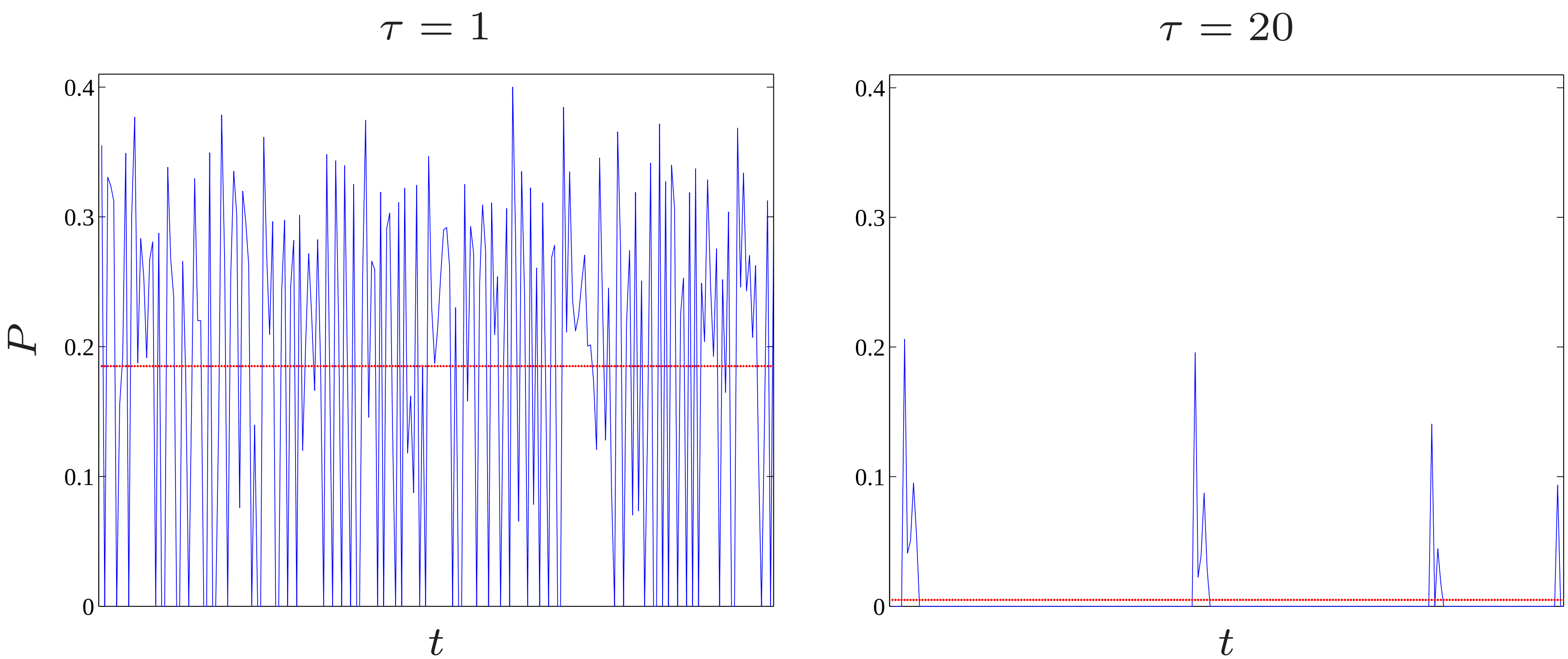
- (optimal) average power is
  - $\bar{P} = 0.1845$ for light smoothing
  - $\bar{P} = 0.0049$ for heavy smoothing

Optimal policies





Power trajectories



Observations

- smoothing has great affect on
  - optimal policy
  - average power needed
- rough interpretation of optimal policy:
  - with smoothing, wait for good channel, unless desperate
  - with more smoothing, can afford to wait longer
  - and so, save power