On Jamming in the Wideband Regime

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***Abstract*— We consider the problem of jamming in non-coherent wideband fading channels. While the problem is well understood for coherent channels, the results for the coherent case do not generalize in the non-coherent regime. We show that energy-limited jammers do not affect capacity in the wideband regime. We also propose a training based transmission scheme that is able to achieve the wideband limit in the presence of a jammer.**

I. INTRODUCTION

Traditional game-theoretic approaches for coherent additive white Gaussian noise (AWGN) channels with an additive per codeword energy-limited jammer show that a saddle point is reached with a WGN input signal distributions are optimal, while the jammer itself uses additive AWGN to jam the channel. Such approaches readily reveal, in the wideband limit, the fact that an additive jammer with limited per codeword energy cannot affect capacity, since the jammer's contribution to the noise vanishes - the energy per Hz of the additive noise goes to 0, while that of the channel's AWGN remains fixed. However, for non-coherent wideband fading channels, WGN distributions for inputs not only fail to achieve capacity, but, in the limit of infinite bandwidth, provide a null rate [1], [2]. Instead, the optimum input distributions are peaky - using low duty cycle, high power symbols [3], [4], [5].

Questions that naturally arise from the above remarks are the following: in the wideband limit, what effect does a jammer have on capacity? Should a jammer still seek to mimic AWGN, as in the coherent case? If not, what strategy should the transmitter adopt to counter the jammer?

We use recent results on capacity-achieving schemes in [7], [8] for low signal-to-noise (SNR) fading channels, which we overview in section III, to show in section IV that a jammer with no information regarding the transmission scheme has a vanishing effect on capacity in the wideband limit. The channel model we use is described in section II.

Reference [9] shows that if the transmitter uses an impulsive training-based transmission scheme, as is often done in practice, the jammer may affect capacity appreciably if it is able to jam the training symbols. In section V of this paper, we propose a training based transmission scheme that is able to achieve the wideband limit even in the presence of a jammer. We present our conclusions in section VI.

II. MODEL

We model the wideband channel as a set of N parallel narrowband channels. In general, the narrowband channels will be correlated. We restrict our analysis in this paper to channels having independent and identical statistics. We also assume that the coherence bandwidth is much larger than the bandwidth of the narrowband channel. Hence, each narrowband channel is modeled as being flat faded. This model is the same as the one used in [7]. Reference [7] shows that low SNR channels are robust to reasonable modeling assumptions. Hence, the results for a more precise channel model may well not differ significantly from that of the simple model we consider in this paper.

Using the sampling theorem, the m^{th} narrowband channel at symbol time k can be represented as:

$$\mathbf{y}[k,m] = \mathbf{h}[k,m] \cdot \mathbf{x}[k,m] + \mathbf{w}[k,m] + \mathbf{z}[k,m],$$

where $\mathbf{h}[k,m]$, $\mathbf{x}[k,m]$, $\mathbf{w}[k,m]$, $\mathbf{y}[k,m]$ and $\mathbf{z}[k,m] \in \mathcal{C}$ are the channel gain (state), input, noise, output and jammer signals, respectively, for the m^{th} narrowband channel at symbol time k. These variables take values in the complex plane. The pair (k,m) may be considered as an index for the time-frequency slot, or degree of freedom, to communicate. The fading is Rayleigh, i.e., the channel gain, $\mathbf{h}[k,m]$, is a zero-mean complex Gaussian random variable with independent real and imaginary components. We set the variance of the channel gain to 1, i.e., $\mathbf{h}[k,m] \backsim \mathcal{CN}(0,1)$. The channel gain is unknown at the transmitter and the receiver. However, its statistics are known to both. Hence, we consider a non-coherent channel. The noise signal, $\mathbf{w}[k,m]$, is a zero-mean complex Gaussian random variable with variance 1, $\mathbf{w}[k,m] \backsim \mathcal{CN}(0,1)$. Since the narrowband channels are assumed to be independent, we will omit the narrowband channel index, m, to simplify notation. The capacity of the wideband channel with average input power constraint, P, is thus N times the capacity of each narrowband channel with power constraint P/N. We focus on the narrowband channel alone.

We further assume a block fading channel model, i.e., the channel gain is random but fixed for the duration of the coherence time of the channel, and is i.i.d across blocks. Hence, we may omit the time index, k, and express the narrowband channel within a coherence block of length l symbols as:

$$\vec{\mathbf{y}} = \mathbf{h}\vec{\mathbf{x}} + \vec{\mathbf{w}} + \vec{\mathbf{z}}.$$

The input, $\vec{\mathbf{x}}$, satisfies the average power constraint:

$$\frac{1}{l} E\left[\|\vec{\mathbf{x}}\|^2\right] = \mathsf{SNR}.$$

The jammer also has an average power constraint and spreads its power uniformly across the N narrowband channels. The jammer's signal is independent across blocks. Hence, the average power constraint on the jammer for each narrowband channel is

$$\frac{1}{l}E\left[\|\vec{\mathbf{z}}\|^2\right] = J\mathsf{SNR}.$$

where, J is a real and positive constant. As N tends to ∞, SNR tends to 0, and the narrowband channel is in the low SNR regime.

III. Capacity in the absence of a jammer

In the absence of a jammer, [7], [8] show that a non-coherent channel with coherence length

$$\frac{1}{4}\mathsf{SNR}^{-2\nu} < l \leq \frac{1}{4}\mathsf{SNR}^{-2(\nu+\epsilon)},$$

where, $\nu > 0$ and $\epsilon \in (0, \min\{1,\nu\})$, has capacity

$$C(\mathsf{SNR}) = \mathsf{SNR} - \mathsf{SNR}^{1+\min\{1,\nu\}} + O(\mathsf{SNR}^{1+\min\{1,\nu\}+\epsilon}).$$

The channel capacity and coherence length are related through ν. The expression, $\min\{1,\nu\}$, is the coherence level and is an indicator of the amount of coherence in the channel. A detailed treatment of coherence level can be found in [7].

A. Signaling scheme

References [7], [8] introduce a near-capacity achieving input signaling scheme - the *Peaky Gaussian* signaling scheme. We present this scheme here for completeness. For a channel with coherence length

$$l = \frac{1}{4}\mathsf{SNR}^{-2\nu}, \quad \nu > 0,$$

transmit in only $\delta(\mathsf{SNR}) = \mathsf{SNR}^{1-\min\{1,\nu\}}$ fraction of the blocks. The receiver has perfect knowledge of the blocks used for transmission. Since the power is concentrated in only a fraction of the blocks, the signal to noise ratio for the blocks used for transmission increases to SNR_b, where

$$\mathsf{SNR}_b \triangleq \frac{\mathsf{SNR}}{\delta(\mathsf{SNR})} = \mathsf{SNR}^{\min\{1,\nu\}}.$$

In the blocks chosen for transmission, let $\vec{\mathbf{x}} \sim \mathcal{CN}(0, \mathsf{SNR}_b I_l)$. Note that as the coherence length increases, the fraction of blocks used for transmission increases from SNR to 1, i.e., the signaling changes from a peaky to a continuous one.
In this paper, we restrict our attention to the *Peaky Gaussian* signaling scheme since it achieves the linear as well as the sublinear capacity term, $\mathsf{SNR}^{1+\min\{1,\nu\}}$.

IV. Capacity in the presence of a jammer

In this section, we examine the effect of a jammer on capacity. For a coherence block, let

$$\vec{\mathbf{s}} = \vec{\mathbf{w}} + \vec{\mathbf{z}},$$

denote the sum of the background noise and jammer's signal. This sum signal is independent across blocks. Capacity is then achieved by an optimal input distribution $\mathbf{Q}$ over complex vectors $\vec{\mathbf{x}}$. The pdf of $\vec{\mathbf{s}}$ is denoted by $\mathbf{V}$ and is the convolution of the Gaussian pdf for the background noise and the jammer's pdf. (This defines a feasible set $\mathcal{V}(J)$ for the pdf $\mathbf{V}$.) Capacity is given as the solution to the mutual-information game

$$C(\mathsf{SNR}, J) = \max_{\mathbf{Q}} \min_{\mathbf{V}} I(\mathbf{Q}, \mathbf{V}),$$

where $I(\mathbf{Q}, \mathbf{V})$ denotes the mutual information between channel input $\vec{\mathbf{x}}$ and output $\vec{\mathbf{y}}$ when distributions $\mathbf{Q}$ and $\mathbf{V}$ are in effect. The minimization and maximization are over convex sets. We establish the following theorem (proof is omitted for brevity):

Theorem 1: For any $J < \infty$ and $\nu > 0$, the linear term in the asymptotic expansion of $C(\mathsf{SNR}, J)$ is identical to that of $C(\mathsf{SNR})$ and, the order of the sublinear term in the asymptotic expansion of $C(\mathsf{SNR}, J)$ is identical to that of $C(\mathsf{SNR})$. □

The theorem shows that the jammer has at best an $O(\mathsf{SNR}^{1+\min\{1,\nu\}})$ effect on capacity, in the asymptotic expansion of $C(\mathsf{SNR}, J)$. The intuitive explanation is that since the jammer's power in the narrowband channel is dominated by that of the background noise $\vec{\mathbf{w}}$, it cannot have a significant effect on capacity.

V. Training based scheme

Zheng *et al.* [7] show that in the absence of a jammer, the linear capacity term, SNR, is achievable when training is performed in the blocks chosen for transmission. In this training scheme, the location of the training symbol in the block is fixed and is the same for all blocks. This makes it easy for a jammer to detect the position of the training symbol and significantly hamper channel state estimation at the receiver by injecting all its energy in a block interval ($\mathsf{E}_{\mathsf{jammer}} = l\ J\mathsf{SNR}_b$) into the training symbol position [9].
We now consider the effect of a jammer when the transmitter does not fix the position of the training symbol but picks it uniformly and independently every block from $\{1, \ldots, l\}$. We assume that the location is perfectly known at the receiver. Since the training symbol is designed to have much higher power than the data symbols (shown later), the location of the training symbol can be easily detected by the receiver.
The jammer is assumed to know which blocks are used for transmission and spreads its energy equally over them. In order to significantly affect receiver channel state estimation, the jammer has detect the training symbol in every block and put all its energy in that position. However, in our model, the transmitter and jammer have the same bandwidth and thus operate at the same symbol rate. Hence, the jammer has to incur a minimum delay of 1 symbol and cannot jam the training symbol. Therefore, since detecting the training symbol position is not useful, it distributes its power uniformly and independently over the block.
We describe the training scheme now. In a coherence block used for transmission, the channel model is

$$\vec{\mathbf{y}} = \mathbf{h}\vec{\mathbf{x}} + \vec{\mathbf{w}} + \vec{\mathbf{z}},$$

where,

$$\begin{aligned} E[\vec{\mathbf{z}}] &= \vec{0}, \\ E[\vec{\mathbf{z}}\vec{\mathbf{z}}^\dagger] &= J\mathsf{SNR}_b\ I_l. \end{aligned}$$

I_l is the $l \times l$ identity matrix. The total energy available in the block to the transmitter is

$$\mathsf{E}_{\mathsf{total}} = l\ \mathsf{SNR}_b.$$

Let the transmitter pick the k^{th} symbol to be used as the training symbol. Training is done using $\gamma \in (0,1)$ fraction of the total block energy. The remaining fraction is used for communicating data. The energy used for training is $\gamma \mathsf{E}_{\mathsf{total}} = \gamma l \mathsf{SNR}_b$ and the training symbol is $\mathbf{x}_k = \sqrt{\gamma \mathsf{E}_{\mathsf{total}}}$. The receiver computes the minimum mean-squares error (MMSE) estimate of $\mathbf{h}$ from $\mathbf{y}_k$. Using $\hat{\mathbf{h}}$ and $\tilde{\mathbf{h}}$ to denote the estimate and estimation error of $\mathbf{h}$, respectively, we have:

$$\begin{aligned} \mathbf{h} &= \hat{\mathbf{h}} + \tilde{\mathbf{h}}, \\ \hat{\mathbf{h}} &\sim \mathcal{CN}\left(0, \frac{\frac{\gamma \mathsf{E}_{\mathsf{total}}}{1+J\mathsf{SNR}_b}}{1+\frac{\gamma \mathsf{E}_{\mathsf{total}}}{1+J\mathsf{SNR}_b}}\right), \\ \tilde{\mathbf{h}} &\sim \mathcal{CN}\left(0, \frac{1}{1+\frac{\gamma \mathsf{E}_{\mathsf{total}}}{1+J\mathsf{SNR}_b}}\right). \end{aligned}$$

Since $\mathbf{h}$ is Gaussian, $\hat{\mathbf{h}}$ and $\tilde{\mathbf{h}}$ are also independent Gaussian. The channel state estimate $\hat{\mathbf{h}}$ is used to decode the data transmitted in the remaining $l-1$ symbols. For these symbols, $(1-\gamma)\mathsf{E}_{\mathsf{total}} = (1-\gamma)l\mathsf{SNR}_b$ energy is used to send data using an i.i.d Gaussian code. The channel in this phase can be represented as

$$\mathbf{y}_i = \hat{\mathbf{h}}\mathbf{x}_i + \underbrace{\tilde{\mathbf{h}}\mathbf{x}_i + \mathbf{w}_i + \mathbf{z}_i}_{\mathbf{v}_i},$$

for $i = 1, \ldots, k-1, k+1, \ldots, l$. The channel inputs $\mathbf{x}_i$ are i.i.d complex Gaussian random variables, $\mathbf{x}_i \sim \mathcal{CN}(0, \sigma_x^2)$, where

$$\sigma_x^2 = \frac{(1-\gamma)l\mathsf{SNR}_b}{(l-1)}.$$

$\tilde{\mathbf{h}}\mathbf{x}_i$ is the noise due to the estimation error from the training phase coupled with the input signal. Combining the additive white noise and jammer's signal with the noise due to estimation error, we have

$$\mathbf{v}_i \triangleq \tilde{\mathbf{h}}\mathbf{x}_i + \mathbf{w}_i + \mathbf{z}_i.$$

Note that $\mathbf{v}_i$ is uncorrelated, but not independent of $\hat{\mathbf{h}}\mathbf{x}_i$. It is zero-mean and has a variance of

$$E[|\mathbf{v}_i|^2] = E[|\tilde{\mathbf{h}}|^2]E[|\mathbf{x}_i|^2] + E[|\mathbf{w}_i|^2] + E[|\mathbf{z}_i|^2] \triangleq \sigma_v^2.$$

Let us define $f^*(\mathsf{SNR})$ as

$$f^*(\mathsf{SNR}) \triangleq \max_{\gamma \in (0,1)} \left[\frac{E[|\hat{\mathbf{h}}|^2]\sigma_x^2}{\sigma_v^2}\right].$$

If we assume $\mathbf{v}_i$ to be Gaussian and independent of the input signal, we can lower bound the capacity of the training based scheme, $C_T(\mathsf{SNR})$, using [6],

$$\begin{aligned} &C_T(\mathsf{SNR}) \\ &\quad \geq \delta(\mathsf{SNR}) \cdot \frac{l-1}{l} \cdot \max_{\gamma \in (0,1)} E_{\hat{\mathbf{h}}}\left[\log\left(1 + \frac{|\hat{\mathbf{h}}|^2\sigma_x^2}{\sigma_v^2}\right)\right] \\ &\quad = \delta(\mathsf{SNR}) \cdot \Big[f^*(\mathsf{SNR}) + o(f^*(\mathsf{SNR}))\Big]. \end{aligned} \tag{1}$$

The following lemma is the computation of $f^*(\mathsf{SNR})$. The proof is omitted from this paper for brevity.

Lemma 1:

$$\begin{aligned} &f^*(\mathsf{SNR}) = \mathsf{SNR}^{\min\{1,\nu\}} + o(\mathsf{SNR}^{\min\{1,\nu\}}), \\ &\arg\max_{\gamma \in (0,1)} E_{\hat{\mathbf{h}}}\left[\log\left(1 + \frac{|\hat{\mathbf{h}}|^2\sigma_x^2}{\sigma_v^2}\right)\right] \\ &\qquad = \mathsf{SNR}^{(\gamma - \frac{\min\{1,\gamma\}}{2})} + o(\mathsf{SNR}^{(\gamma - \frac{\min\{1,\gamma\}}{2})}).\square \end{aligned}$$

Combining this lemma with (1), we obtain the following lower bound to the capacity using the training based scheme:

$$C_T(\mathsf{SNR}) \geq \mathsf{SNR} + o(\mathsf{SNR}). \tag{2}$$

A. Discussion

We see from (2) that this training scheme achieves the linear capacity term, SNR. Hence, this scheme achieves the wideband capacity limit even in the presence of a jammer and is therefore preferable over the scheme in [9], which doesn't.

By randomly choosing the location of the training symbol, the transmitter forces the jammer to spread its energy over the entire block. This results in white noise dominating the signal of the energy limited jammer. Hence, the wideband capacity limit is unaffected. Note that the fraction of energy required for training goes to 0 as SNR tends to 0, like in the case when the jammer is absent [7].

VI. Conclusions

We have shown that a jammer does not, in principle, have the ability to reduce capacity for non-coherent fading channels in the wideband regime. This result is in effect similar to that known for coherent results, but its derivation is altogether different. The type of signalling used by the transmitter is, as for a wideband channel in the absence of a jammer, impulsive. The presence of fading does affect, to some extent, the ability of the transmitter to counter the jammer in the wideband limit. An attractive means of approaching the wideband capacity, impulsive training schemes, is affected by jamming if the location of the training signals is known by the jammer. A training scheme that randomly changes the position of the training symbol achieves the wideband capacity limit.

Our results have considered that the jammer remains independent of the transmitted signal. It is possible that a correlated jammer, even using such a simple scheme as repeat-back jamming, may have a more marked effect on capacity. While channel fading may preclude coherent cancelling of part of the input signal, a correlated jammer may be able to change, in effect, the coherence of the channel.

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REFERENCES

[1] M. Médard and R. G. Gallager, " Bandwidth Scaling for Fading Multipath Channels", *IEEE Transactions on Information Theory*, 48(4):840-852, April 2002.

[2] V. G. Subramanian and B. Hajek, "Broad-band fading channels: signal burstiness and capacity", *IEEE Trnsactions on Information Theory*, Volume: 48 Issue: 4 , April 2002 Page(s): 809 - 827.

[3] R. S. Kennedy, *Fading Dispersive Communication Channels*, Wiley Interscience, New York, NY, 1969.

[4] I. E. Telatar and D. N. C. Tse, "Capacity and mutual information of wideband multipath fading channels", *IEEE Transaqctions on Information Theory*, 46(4):1384-1400, July 2000.

[5] S. Verdú, "Spectral efficiency in the wideband regime", *IEEE Transactions on Information Theory*, 48(6):1319 -1343, June 2002.

[6] M. Médard, "The effect upon channel capacity in wireless communications of perfect and imperfect knowledge of the channel", *IEEE Transactions on Information Theory*, Vol. 46, May 2000.

[7] L. Zheng, D. N. C. Tse and M. Médard, "Channel Coherence in the low SNR Regime", submitted to *IEEE Transactions on Information Theory*. Available at http://web.mit.edu/lizhong/www/.

[8] S. Ray, M. Médard and L. Zheng, "Wideband Non-coherent MIMO Capacity", *ISIT 2005*, Adelaide, Australia, Sep. 4-9, 2005.

[9] S. Ray, P. Moulin and M. Médard, "On Optimal Signaling and Jamming Strategies in Wideband Fading Channels", *IEEE Workshop on Signal Processing Advances in Wireless Communications (SPAWC) 2006*, Cannes, France, July 2006.