Information Theory for Mobile Ad-Hoc Networks (ITMANET):

The FLoWS Project

Thrust 1 Intro:

Upper Bounds

Ralf Koetter

Joint with M. Medard

The philosophy and challenge

- Creating a compelling and workable theory of network equivalent problems
- Provide a framework in which the results of classical multiterminal information theory can be exploited for real networks
- How can we organize a tool to automatically harvest any progress on network components (MIMO broadcast, relay channel, interference channel, ...) in real networks?

We want to ask (and answer) new questions; The goal is to create a framework in which results from classical multi-terminal information theory can be used to provide scenarios and useful tools for "network engineering"

Upper bounds:

While the efforts presented in thrust 1 aim at networks, many projects within other thrusts naturally deal with upper bounds.

Some of the synergies here are described in the summary presentation.

The goal is to give a useful theory of network information theory that is applicable to networks

Upper bounds presentations:

Michelle Effros: *"Equivalence classes in network coding"*

A principled approach to source coding problems in networks that aims at understanding the structural form of general solutions rather than a specific network.

Ralf Koetter: *"An equivalence theory of network capacity"*

Rather than trying to solve a given small network in upper bounds, the goal is to understand (and acknowledge) the main combinatorial difficulty of the network information theory

Muriel Medard: *"General capacity using network coding"*

Based on conflict graphs this talk provides insight into a tool to express the general network coding capacity of networks. This work is the natural consequence of the other presentations which prepare network information theory problems for this approach.

Upper bounds presentations:

source coding
channel coding
networks

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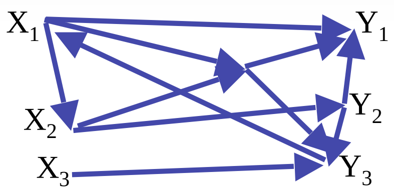
Network coding equivalence classes

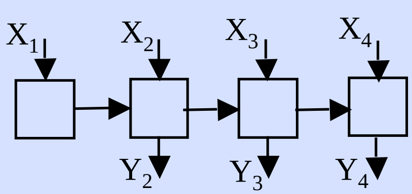
STATUS QUO



Network coding capacities are well-understood for some demands (e.g., multicast). Little theory is developed for general demands:

- ***Negative results (e.g., linearity does not suffice)***
- ***Suboptimal or constrained-optimal bounds***

NEW INSIGHTS

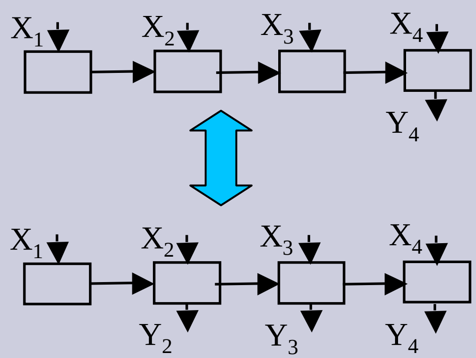


We can prove equivalence results even for networks that are unsolved. These equivalence results allow us to

- Get the biggest bang out of existing solutions
- Limit our attention to restricted classes or problems
- Understand what tools to apply to what problems

FLOWS ACHIEVEMENTS

We prove that the equivalence of the capacity region of single- and multi-demand line networks for independent sources. Using this equivalence, a solution to the simpler (single-demand) problem gives an immediate solution to the more general (multi-demand) problem.



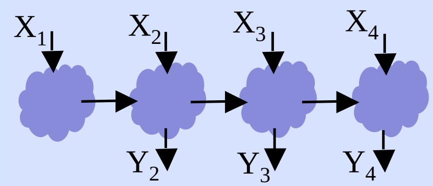
How it works:

- Decompose the multi-demand network into a sequence of single-demand subnetworks.
- Solve for one demand at a time.
- Each solved demand becomes a source.
- Concatenate the subnetwork codes to give a code for the original network.
- Prove optimality.

Assumptions and limitations:

- Sources assumed independent; result sometimes fails for dependent sources.
- Currently restricted to line networks.
- Assumes the lossless-link model.

END-OF-PHASE GOAL



Extend result to dependent sources

- Bound loss when decomposition fails
- Consider network generalizations (e.g., trees)
- Consider generalization from nodes to component networks

COMMUNITY CHALLENGE

Categorize the family of network coding problems

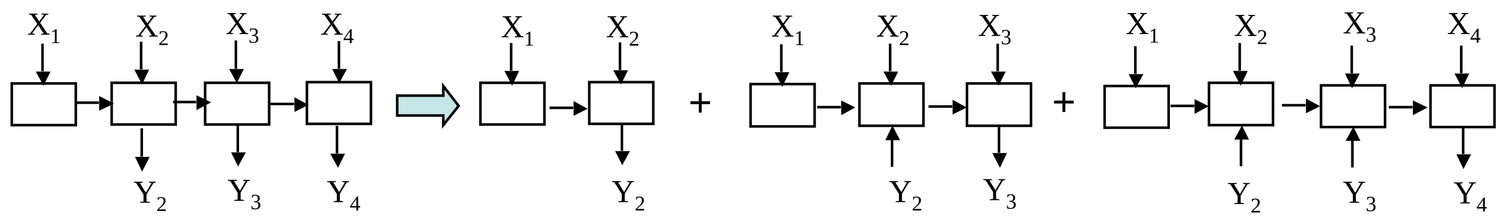
- Network equivalence.
- Network implication.
- Network hardness.
- Network completeness.

Network coding equivalence may yield progress even for unsolved network demands

Key Ideas and Results

Key ideas

- Decomposition approach to network coding
  - Make use of information already available at a node



  - Reduce line networks to line networks with one demand - **easier to tackle using Shannon Theoretic tools**
  - Simplify capacity calculations

Key Results

- Works for a large collection of line networks
  - Independent sources
  - Multicast
  - Dependent sources that have a special structure
  - A large class of three-node networks
- Sub-optimal for some networks
  - Counterexamples

Future Goals

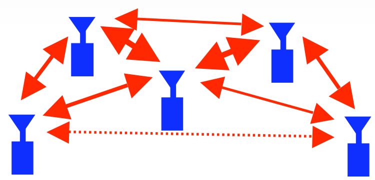
- Decomposition for general network topologies *e.g. trees*
  - Extension of the same strategy works for networks with independent sources.
  - Does not work in general, e.g. multicast
  - Can replace nodes by networks?

- Make full use of the codeword present at a node, rather than just the sources/demands:
  - **some other form of decomposition?**

- **Broaden** the class of source/demand structures for which the decomposition is optimal
- Bound the loss when decomposition is sub-optiml
- *Equivalence classes of Network Coding problems*

An equivalence theory of network capacity

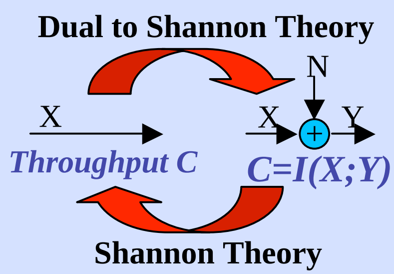
STATUS QUO



Finding capacity of wireless networks is a hard problem

- ***Good achievable rate regions unknown since we don't know how to do "network" relaying or how to deal with interference***
- ***Only have very loose cutset upper bounds that can't be achieved.***

NEW INSIGHTS



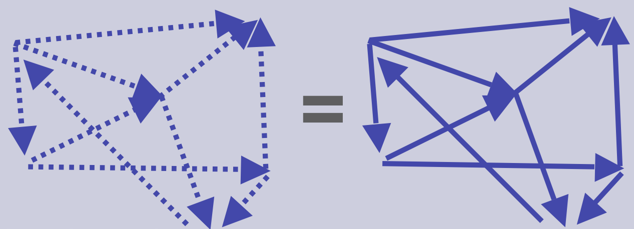
Dual to Shannon Theory:

By emulating noisy channels as noiseless channels with same link capacity, can apply existing tools for noiseless channels (e.g. network coding) to obtain new results for networks with noisy links. This provides *a new method* for finding network capacity

FLOWS ACHIEVEMENT(S)

We prove that the capacity regions of networks with noisy links and networks with noiseless links with a hard rate constraint on each link equal to the noisy link channel capacity are the same.

We can solve for the capacity of a network with noiseless links via network coding



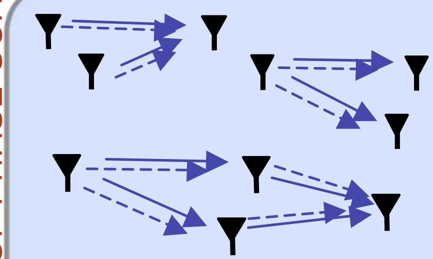
How it works:

- $R_{noiseless} \subseteq R_{noisy}$ easy since the maximum rate on the noiseless channels equals the capacity of the noisy links: can transmit at same rates on both.
- $R_{noisy} \subseteq R_{noiseless}$ hard since must show the capacity region is not increased by transmitting over links at rates above the noisy link capacity. We prove this using theory of "types" to show equivalent capacity

Assumptions and limitations:

- Link-oriented, not broadcast (no interference)
- Assumes links are memoryless and discrete
- Assumes we can solve combinatorial network coding problem (high complexity for large networks)
- Metrics other than capacity may not be the same for both networks (e.g. error exponents).

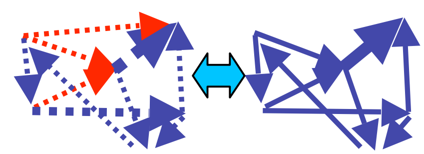
END-OF-PHASE GOAL



Extend analysis to multiple access channels and possibly broadcast channels and multihop networks

Determine capacity orderings for networks where equivalence cannot be established

COMMUNITY CHALLENGE



Graduate level: Identify additional equivalences and hierarchies

Prize level: understand limits of capacity ordering as a practical intellectual tool

Equivalence classes provide a new paradigm for characterizing capacity limits

General capacity using network coding

FLoWS

Status quo

- When separation holds, what is the benefit of having network coding?
- Major difficulty I: in non-multicast settings, codes are an open problem
- Major difficulty II: time-varying nature of traffic and of network operation, e.g. changing codes
- Major difficulty III: even without coding, performance is ill understood
- State of the art I: pick a system (say COPE) and run experimental trials to demonstrate improvement
- State of the art II: pick a multicast example and work it out by hand

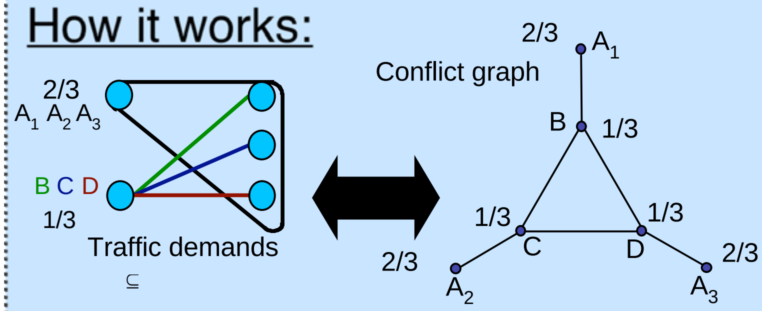
New insight/intellectual tool

- Fixing family of possible codes, give systematic representation of achievable region using conflict graph representation
- Obviates the need for finding clever schedules by hand
- Difficulty of problem now becomes one of characteristics of conflict graph (for instance, perfection)– it is a combinatorial, graph-theoretic question
- Finding schedules now comes from conflict graph

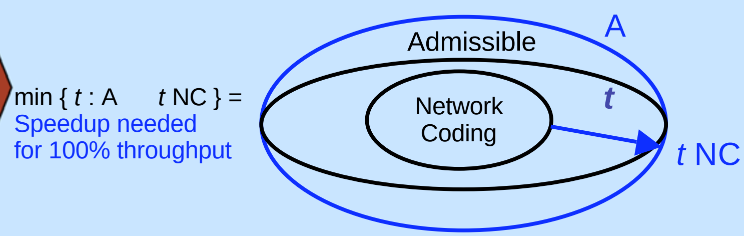
Achievement: for networks of depth one (like switch)

Provide a systematic way of characterizing achievable region and benefit of coding

How it works:



An edge between two vertices if the two configuration cause conflict (i.e. They cannot be served simultaneously)



Assumptions and Limitations

Can bound speed-up using imperfection ratio of conflict graph

- Approach is general, but speedup characterized for depth one
- Works in separable settings
- Algorithms are centralized

End-of-phase goal

To obtain general results for networks with multiple layers

To incorporate MAC constraints into the conflict graphs, allowing mixture of MAC and scheduling

To provide a set of systematic approaches to determine schedules

Create online schemes, in the flavor of i-slip, to trade-off complexity and effectiveness of schedules, with possible decentralization

Community challenge

How can we approximate difficult capacity region problems?

How can we create schedules from such approximations?

What is the loss that comes from a distributed scheduling?

Ramification: bring problem to its combinatorial essence, which determines difficulty

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Upper Bounds:
Thrust area summary and conceptual ideas

Thrust 1: Upper bounds

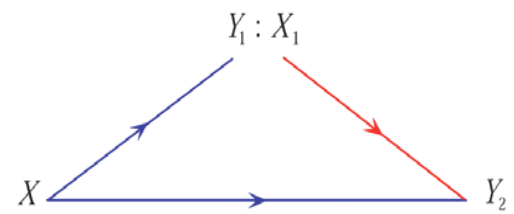
- Achievability results are usually demonstrated by concrete schemes.
- The goal of upper bounds is to give an objective approach to assessing performance.
- Upper bounds should be concrete, computable, helpful and practically meaningful!
- The main mathematical tool for upper bounds are ***Fano's lemma***, stating that any quantity that we want to estimate with a small error must be determined by our evidence. As a consequence of Fano's lemma we get the ***min-cut max flow theorem***

Thrust 1: Upper bounds - the classical focus

- R= H(W)<I(W;Z)+H(W|Z)<.... < I(X;Y) +n epsilon
- This has been reformulated many times leading to results as sophisticated as:

$$\bar{\mathcal{R}} = \bigcup_{p(t,u,x_1,x,y_1,y_2)} \left\{ \begin{array}{l} (R_0, R_1, R_2) : \\ R_0 \geq 0, R_1 \geq 0, R_2 \geq 0, \\ R_0 \leq \min\left\{ I(T;Y_1|X_1), I(T,X_1;Y_2) \right\}, \\ R_0 + R_1 \leq I(X;Y_1|X_1), \\ R_0 + R_2 \leq \min\left\{ I(T,U,X_1;Y_2), I(X,X_1;Y_2) \right\}, \\ R_0 + R_1 + R_2 \leq I(T;Y_1|X_1) + I(X;Y_1|T,U,X_1) + I(U;Y_2|T,X_1), \\ R_0 + R_1 + R_2 \leq I(X;Y_1|T,U,X_1) + I(T,U,X_1;Y_2), \\ R_0 + R_1 + R_2 \leq I(X;Y_1,Y_2|X_1) \end{array} \right\}$$

for a “simple” three node network



from Y.Liang G. Kramer, “Rate Regions for Relay Broadcast Channels”

What answer do we want?

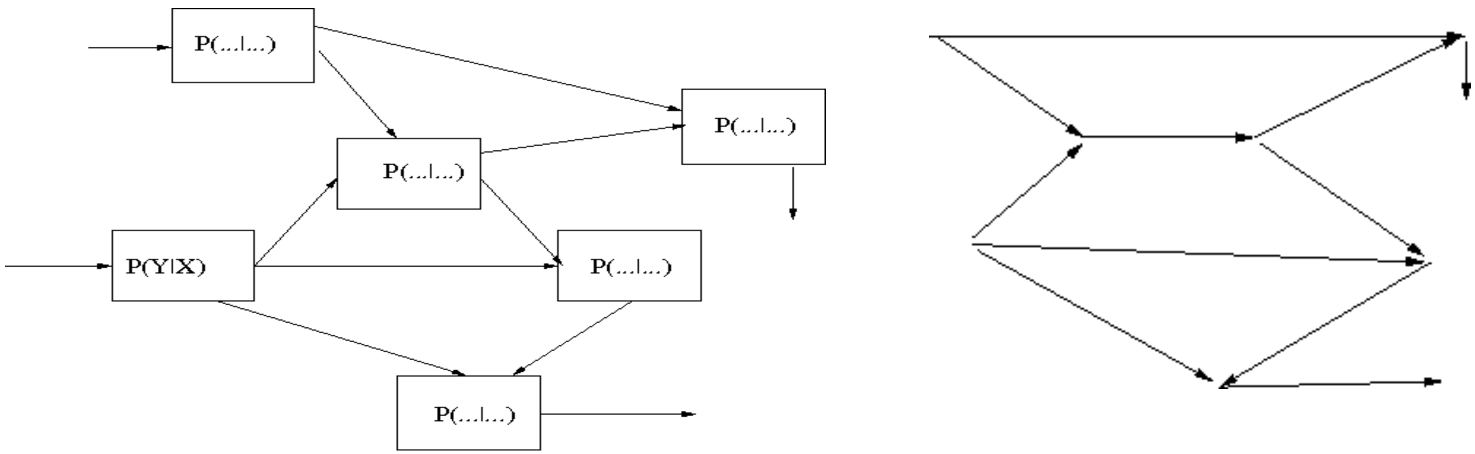
The “user's” question

- A typical question that a “user” of information theory may ask is: “Is the rate tuple (0.5,0.5,1.3) supportable in principle?”
- How can I think about the capacity of a wireless network?
- What should I do?
- What about non-ergodic properties (e.g. delay, variability)?

- Classical answers: rate regions, mutual information characterizations, error-exponents

A different set of questions

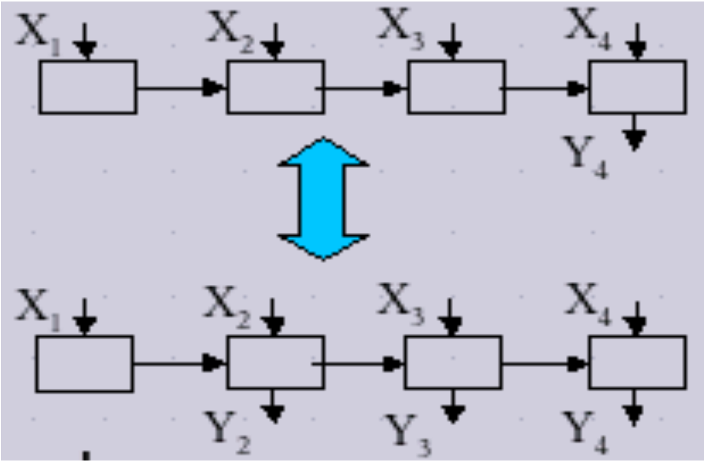
- In order to bridge the gap between networking and multiterminal information theory we want to utilize noise-free networks and equivalence classes of networks as framework for deriving results.



- Networking aspect of the solutions are emphasized rather than multiterminal information theoretic aspects. The idea is to bring out the essential nature of an information theory for networks (statistical vs combinatorial)

Precise equivalence

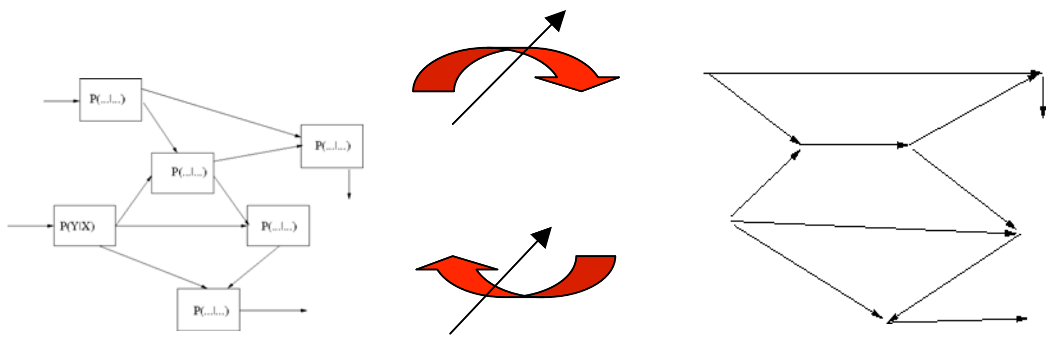
- Precise relations can be given for networks of noisy links with noise-free bit pipes
- Degraded broadcast and message degraded multiple access (?)
- Source coding line networks



- Capacity results with and without feedback
- Network coding prototypes (multiple unicasts)

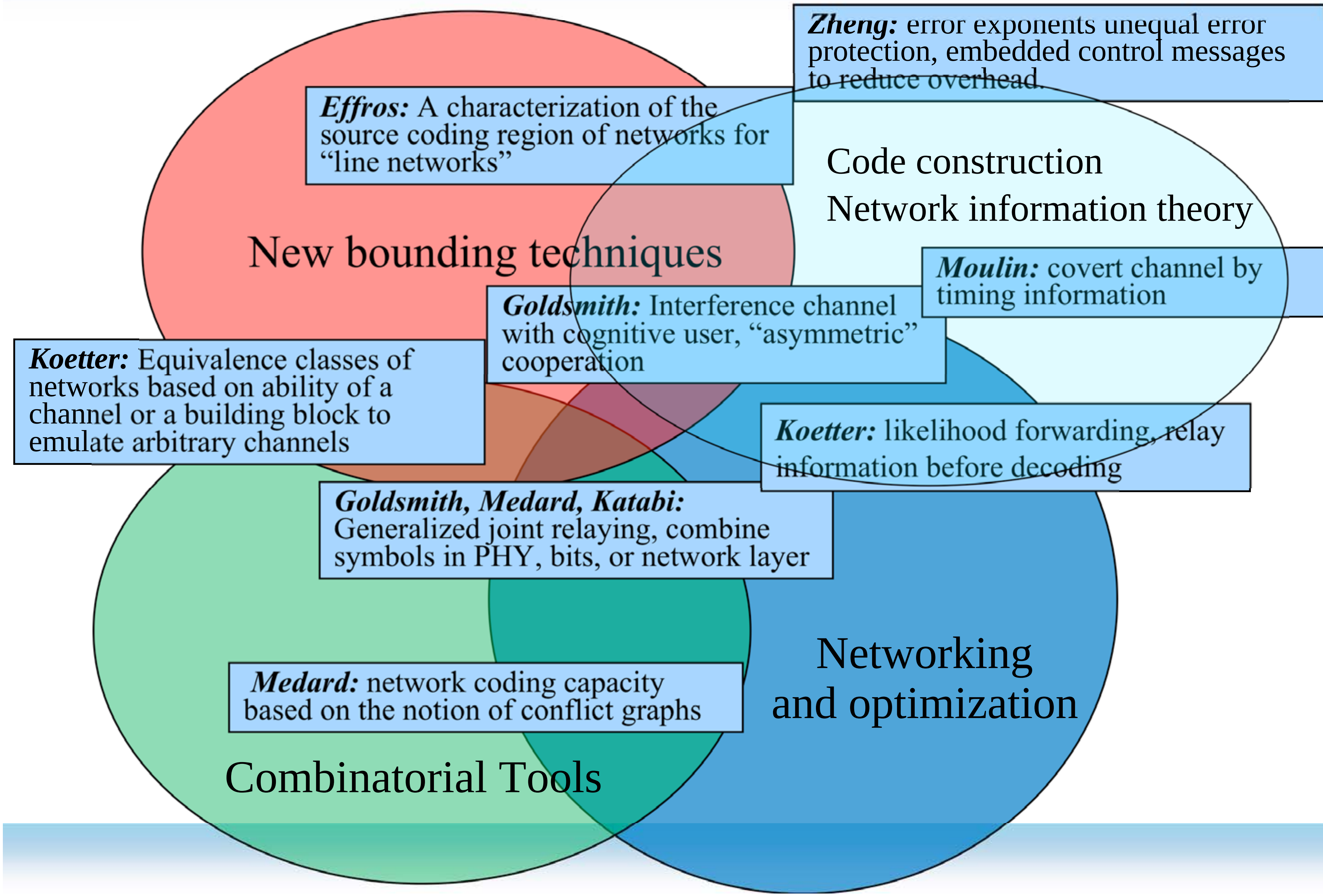
Upper and lower bounds

- Provided a multiterminal setup can be used to support a given rate demand we can use multiterminal results as a drop in for achievability results on noise-free networks
- Conversely, upper bounds can be obtained out of noise-free networks if we can upper bound the rate that we need in order to provide a statistically equivalent multi-terminal problem.



- Using bounds on the component multi-terminal networks we obtain bounds on bigger networks
- noise-free networks can be handled by combinatorial methods => conflict graphs

Achievements Overview



The people and connections across thrusts

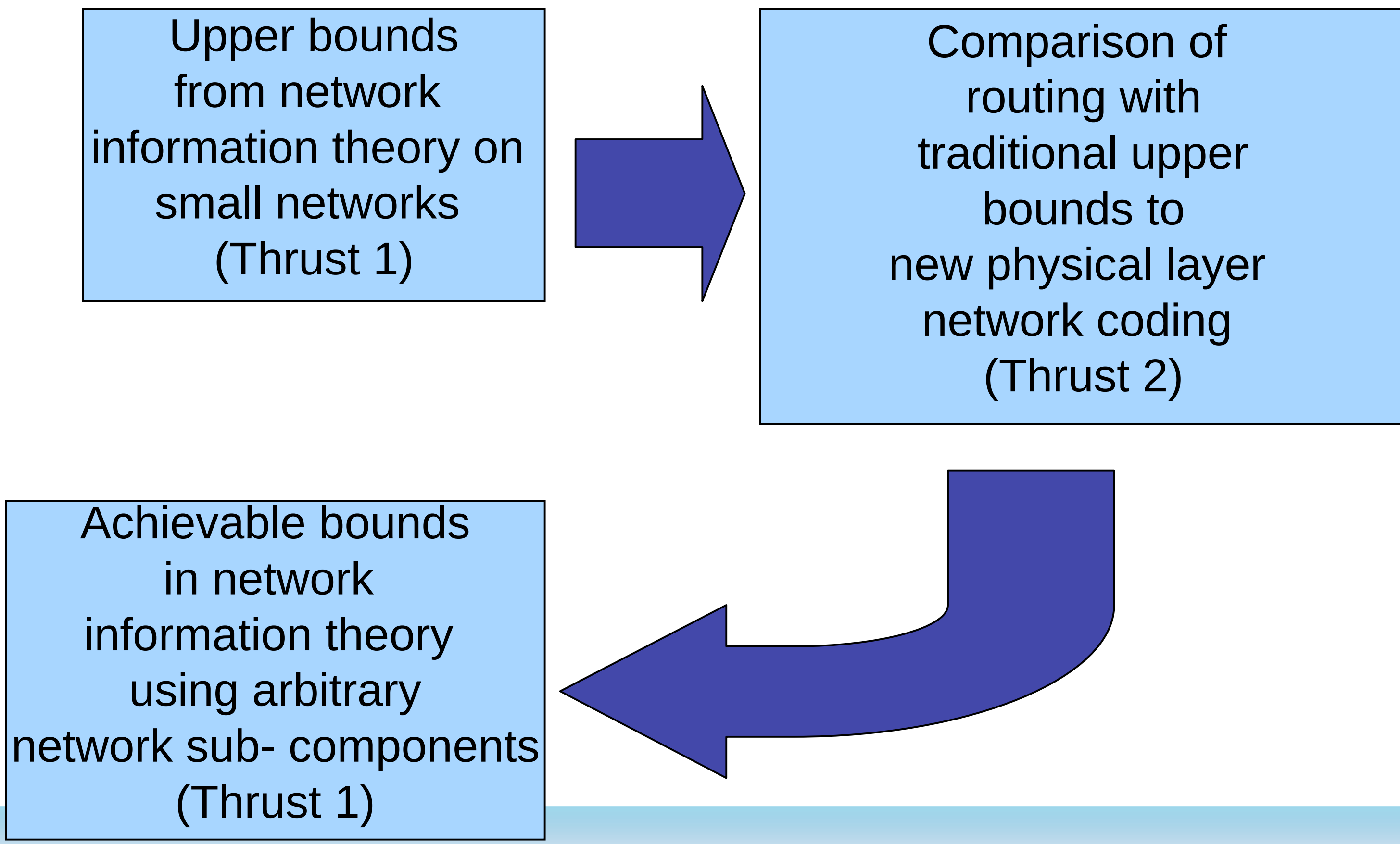
- The team for the upper thrust focus:
  Michelle Effros, Muriel Medard, Ralf Koetter
  (and everybody else)

- Strong connections across the thrusts (examples):

  - *General Relaying for Multicast in Wireless Networks*
    General relaying allows for joint encoding of multiple data streams and cooperation. It results in improved rate performance in MANETs
  - *Capacity and queue-based codes for timing channels in MANET*
    Characterize individual link capacity for timing channels in MANETs.
  - *Throughput/Delay Fundamental Performance Limits*
    Computational intractability of information theoretic capacity achieving codes for wireless networks.
  - *and many other presentations....*

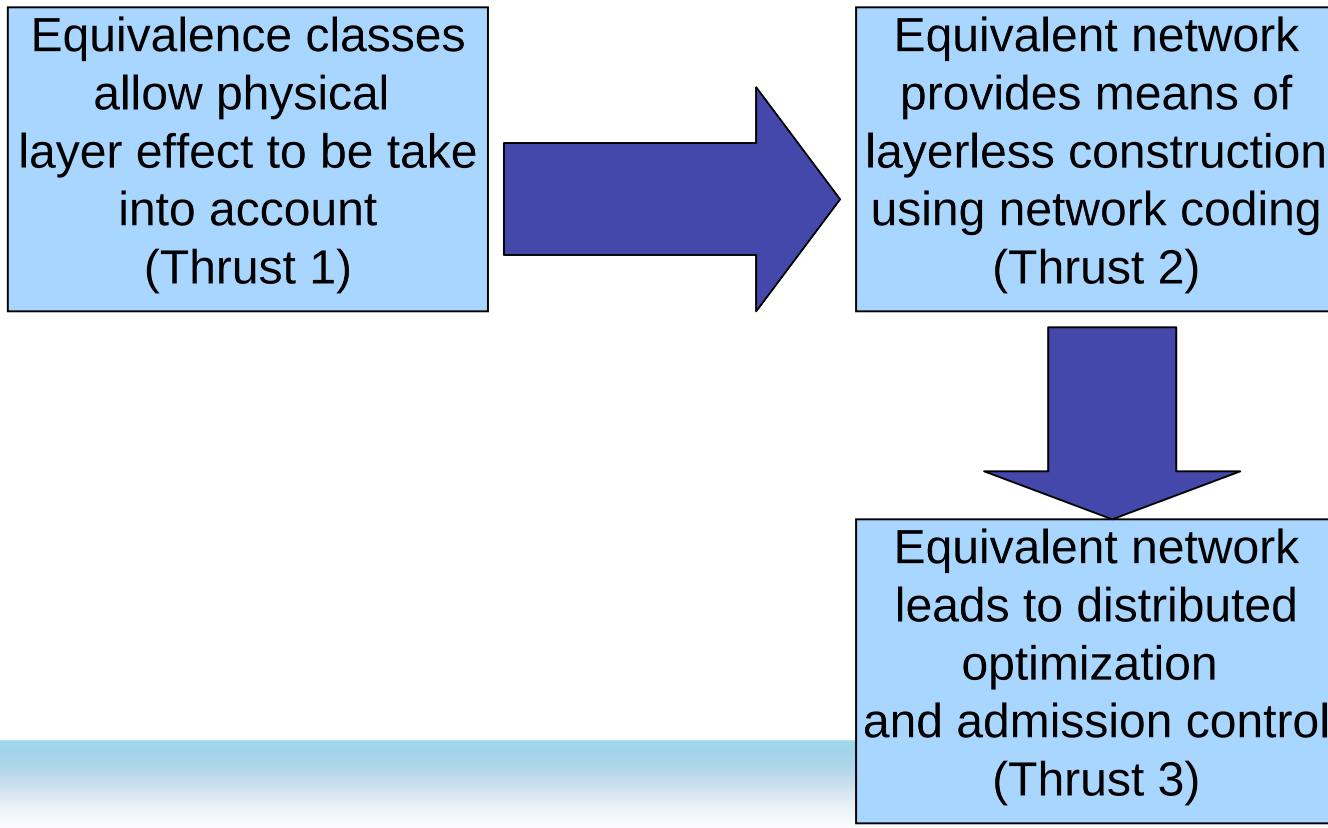
Physical layer network coding

- Feedback loop between thrusts 1 and 2 – network information theory, physical network coding and general network information theory



Equivalence classes

- The ability to incorporate physical layer in separable systems leads to equivalent networks with optimal layerless constructions
- Over such networks, network-coding based optimization and related admission control is possible under sufficient time separations between link-by-link coding and network coding



Publications

• V. Doshi, D. Shah, M. Médard, "Source Coding with Distortion through Graph Coloring", IEEE Int'l Information Theory Symposium (ISIT), Nice, France, 2007.

• S. Katti, I.Maric, A. Goldsmith, D. Katabi and M. Médard,"Joint Relaying and Network Coding in Wireless Networks", IEEE International Symposium on Information Theory /2007.

• Y. Liang, A Goldsmith, M. Effros, "Distortion Metrics of Composite Channels with Receiver Side Information".

• I. Maric, A. Goldsmith, G. Kramer and S. Shamai (Shitz), "On the Capacity of Interference Channels with a Cognitive Transmitter", 2007 Workshop on Information Theory and Applications (ITA), Jan.29 - Feb. 2, 2007.

• Chris T. K. Ng and Andrea J. Goldsmith,"The Impact of CSI and Power Allocation on Relay Channel Capacity and Cooperation Strategies," submitted to IEEE Transactions on Information Theory, 2007.

• I. Maric, A. Goldsmith, G. Kramer and S. Shamai (Shitz), "On the Capacity of Interference Channels with a Partially-Cognitive Transmitter", ISIT 2007.

• Chris T. K. Ng and Andrea J. Goldsmith, "Capacity and Cooperation in Wireless Networks," Information Theory and Applications (ITA) Workshop, February 6–10, 2006, La Jolla, CA. (Invited)

• Chris T. K. Ng, Nihar Jindal, Andrea J. Goldsmith and Urbashi Mitra, "Capacity Gain from Two-Transmitter and Two-Receiver Cooperation," accepted for publication in IEEE Transactions on Information Theory, 2007.

• S. Yang and R. Koetter, "Network Coding over a Noisy Relay : a Belief Propagation Approach", IEEE International Symposium on Information Theory 2007.

• M. Effros, A. Goldsmith and Y. Liang, "Capacity definitions of general channels with receiver side information",To appear, IEEE International Symposium on Information Theory 2007.

•M. Bakshi and M. Effros and W. Gu and R. Koetter, On network coding of independent and dependent sources in line networks, ISIT, 2007