



# **Incomplete information, dynamics, and wireless games**

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# Incomplete information, dynamics, and wireless games



STATUS QUO

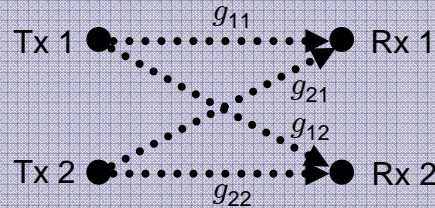
Previous work studied ad hoc wireless resource competition among multiple nodes using game theoretic techniques, but typically in a stationary setting, where each node knows all other's channel conditions (see Huang et al., Etkin et al.)

We aim to understand the importance of a lack of information about channel conditions over time.

## FLAWS ACHIEVEMENT(S)

**MAIN RESULT:** *The presence of incomplete channel information among nodes, as well as dynamic interaction among nodes, can dramatically alter the game theoretic conclusions drawn in standard complete information settings.*

*Example: A primary user may deter entry by secondary users at some cost to himself, even if it is not immediately in his best interest to do so.*



**HOW IT WORKS:** We use the theory of *Bayesian games* to find symmetric equilibria of a Bayesian Gaussian interference game.

We use the theory of *reputation effects in dynamic games of incomplete information* model to study the behavior of a primary user interacting with multiple secondary users.

### ASSUMPTIONS AND LIMITATIONS:

We assume one primary and several secondaries arriving over time; we assume the channel remains stationary over several periods of interaction between primary and secondary.

Key assumption (and limitation): there is no "protocol" for transmission, so all other transmission treated as pure noise (hence the Gaussian interference model).

END-OF-PHASE GOAL

We need to extend the model to handle not only a finite horizon model, but also an infinite horizon model with changing channel conditions.

Journal paper is being prepared for submission to JSAC.

Longer term: we need to focus more on implications for *algorithm design for ad hoc wireless nodes in a reactive environment*. Our insights set a foundation for this.

COMMUNITY CHALLENGE

*Status quo is useless for designing node strategies.*

*Employ methods from learning and dynamic equilibrium in large games to build better algorithms for competition and cooperation.*

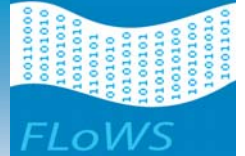
NEW INSIGHTS

We bring in the importance of incomplete channel information via the use of both static and dynamic *Bayesian games*, and in particular exploit results on reputation effects in economics to study primary/secondary competition.

(S. Adlakha, R. Johari, A. Goldsmith)

**Real environments are reactive and non-stationary; this dramatically changes incentives and game theoretic predictions**

# Motivation and overview

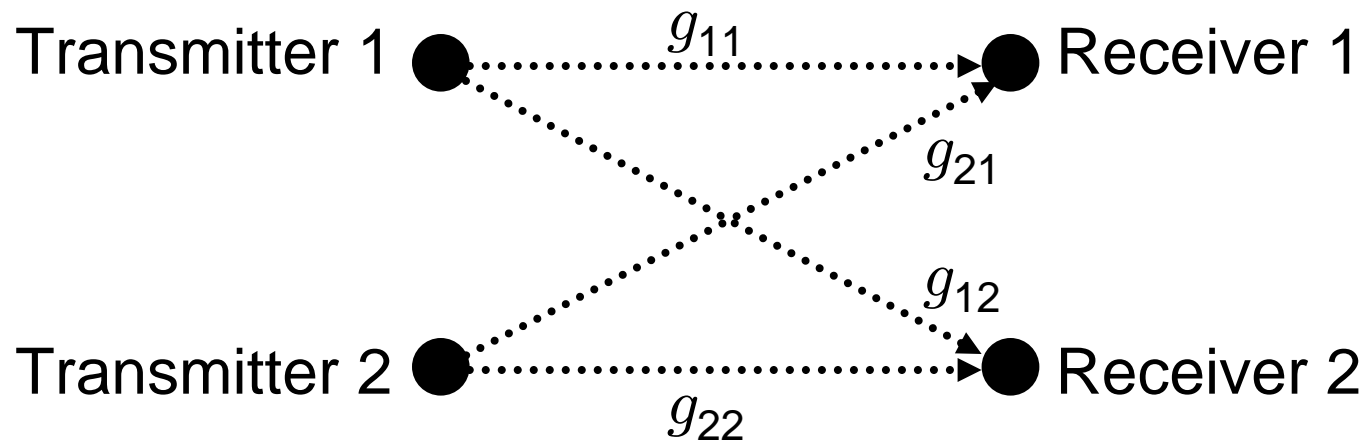


- In game theory, “incomplete information” refers to uncertainty players have about the payoff structure (and thus, behavior) of their opponents
- In wireless systems: incomplete information = nodes are *uncertain* about each other’s channel conditions
- Most prior work assumes *complete information* among nodes, and often only works with *static* models
- Our work studies the implications of imperfect information in wireless game settings
- Part I: Static Bayesian Gaussian interference game
- Part II: Reputation effects in a dynamic interference game  
(*Joint with S. Adlakha and A. Goldsmith*)

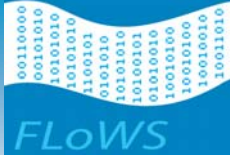


# Part I: Bayesian Gaussian interference game

- Two devices,  $N$  non-overlapping channels
- Both devices have same power constraint  $P$
- Flat fading (i.e., same gains in each channel)
- Gains randomly drawn



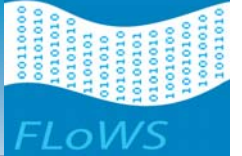
# Part I: Bayesian Gaussian interference game



- Assume transmit/receive pair 1 observes the incident gains  $g_{11}$ ,  $g_{21}$ , but not  $g_{22}$  or  $g_{12}$  (similarly for Tx/Rx pair 2); assume flat fading
- This is a Bayesian game:  
Once random gains are realized, each TR pair knows its own gains but not the gains of the other.
- This is a supermodular Bayesian game; in particular, local search dynamics converge (see also R. Berry's work)
- Nodes can either use a single channel, or spread power across all channels



# Part I: Bayesian Gaussian interference game



- We focus on *symmetric equilibrium*:  
As a function of channel gain, all devices have the same strategy.

Motivation:

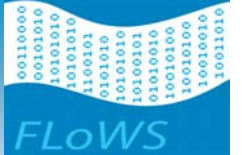
Asymmetric equilibrium requires prior coordination

- *Theorem*:  
Equal spreading is unique symmetric equilibrium

Implications: This can be quite inefficient!



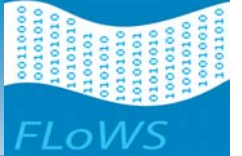
## Part II: Reputation effects in a dynamic game



- Now assume Tx/Rx 1 = primary, Tx/Rx 2 = secondary; same system model, but now assume only 2 channels
- Primary is *long-lived* and *fully rational*
- Secondary user is *myopic* (only optimizes one period payoff), but *history-aware* (remembers the past)
- Secondary user decides each period whether to “enter” (i.e., transmit), or “leave” (i.e., stay silent)
- Secondary user is assumed to have a cost for power consumption
- Primary user can “share” (give up a channel to secondary) or “spread” (spread power equally over channels)



## Part II: Reputation effects in a dynamic game

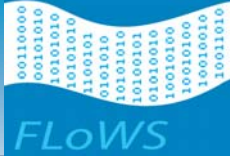


- Assume:  $g_{11} = g_{22} = 1$ , and primary knows  $g_{21}$ , secondary knows  $g_{12}$ .
- For simplicity, assume primary also knows  $g_{12}$
- Gains are constant over the time horizon of interest
- At each time period:
  - 1) Arriving secondary user decides whether to enter or leave
  - 2) Primary user decides transmission strategy: either all power in 1 channel, or power spread equally across both channels
  - 3) Secondary user chooses transmission strategy: same options as primary user





## Part II: Reputation effects in a dynamic game

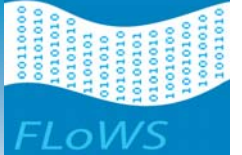


In a single period game, when the secondary chooses to enter:

- $g_{12}$  small  $\Rightarrow$  in equilibrium, both primary and secondary user spread power
- $g_{12}$  large,  $g_{21}$  small  $\Rightarrow$  in equilibrium, both primary and secondary user spread power
- $g_{12}$  large,  $g_{21}$  large  $\Rightarrow$  in equilibrium, both primary and secondary user share the channel



## Part II: Reputation effects in a dynamic game



Now consider multiperiod horizon.

Then when both  $g_{12}$ ,  $g_{21}$  are large,  
there can be a *reputation effect*.

Despite the fact that the primary would be better off sharing (in one period) if secondary enters, the primary may choose to spread (“act” threatening) *because this deters future entry by the secondary*

Key point:

This cannot happen in a complete information model!  
(For complete information case, see Etkin et al.)



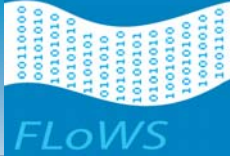
# Next steps



- Results provide insight into the role of incomplete information
- Assumptions that are problematic:
  - Supermodularity breaks down with more than two nodes
  - Primary-secondary interaction heavily stylized
  - Equilibrium is suspect:  
How did nodes coordinate on that equilibrium in the first place?



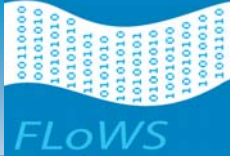
# Large scale stochastic games



- We will leverage recent results in analysis of *large scale stochastic games* to address the central question of this project:  
How should nodes behave when their environment is reactive?
- Approach:
  - Standard solution concept is *Markov perfect equilibrium*, but is hard to compute and requires too much information
  - Recent suggestion by Weintraub et al. for economic models: *oblivious equilibrium*
  - In OE, optimize *as if* the rest of the system behaved according to its stationary average (good in large scale systems)
  - We are generalizing this approach for arbitrary stochastic games (*joint with V. Abhishek, S. Adlakha, G. Weintraub*)



# Large scale stochastic games



- The OE methodology is appropriate for cognitive radio design
- Model: large system of many interacting nodes
- Use low-dimensional dynamic programming computation to solve for approximately optimal strategy
- Resulting strategy close to optimal in exact problem

Our goal: apply this approach to wireless system modeling

