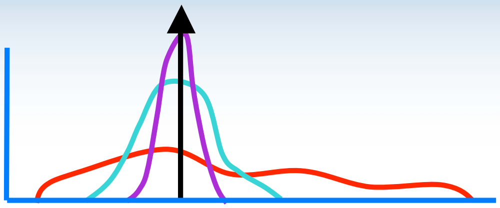
# Capacity, Distortion and Source-Channel Coding (Liang, Effros, Goldsmith)

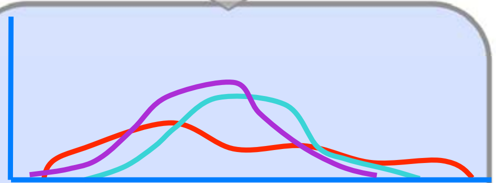
## STATUS QUO



*Classical capacity/distortion metrics focus on stationary, ergodic channels/sources. More realistic models to address non-ergodic time-varying nature of MANET*

*Stringent delay constraint prohibits coding over long block lengths. Lack of channel state information at transmitter limits use of adaptive schemes*

## NEW INSIGHTS



Definition of performance metrics should evolve with system models. Be fundamental or at least useful, e.g. to coincide with ubiquitous design practice, or communication strategies outperforming traditional schemes.

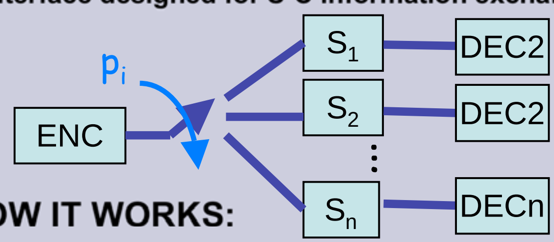
Source-channel (S-C) code design should consider the end-to-end metrics and choose appropriate S-C interface for information exchange.

## FLOWS ACHIEVEMENT(S)

**We propose generalized performance metrics (outage and expected capacity/distortion) to cope with information transmission over non-ergodic channels.**

**Flexible notions of capacity can be combined with various source coding strategies (e.g. multiple resolution code) to improve reconstruction fidelity.**

**End-to-end distortion metrics dictate optimality of separate source-channel (S-C) coding. Appropriate interface designed for S-C information exchange.**



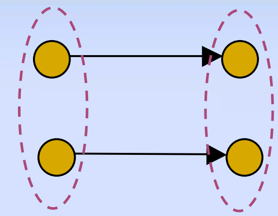

### HOW IT WORKS:

- Relax the constraint all transmitted info be decoded. Meaningful metrics for applications where (1) partial received information useful, e.g. MR source code, (2) certain outage acceptable or with ARQ mechanism
- Single encoder without CSIT. Choice of decoders based on CSIR. Map to BC with virtual Rx indexed by CSI.
- Source-channel interface more than a single-value rate, instead capacity vs. outage curve or BC capacity region

### ASSUMPTIONS AND LIMITATIONS:

- Decoder choice depends crucially on PERFECT CSIR. Performance usually degrades under CSI mismatch.
- No CSI feedback. Potential improvement w/ partial CSIT.
- Source is stationary ergodic. A dual problem is to incorporate time-varying source statistics.

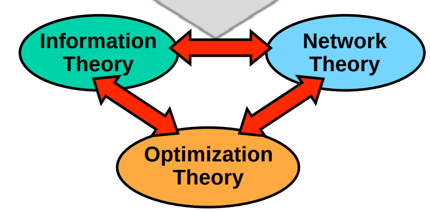
## END-OF-PHASE GOAL



Extend analysis to channel codes that transmit equal-priority information and match the multi-description source code.

Apply source-channel codes to MIMO systems, including nodes with physically mounted multiple antennas, and virtual MIMO systems formed by node cooperation in MANET.

## COMMUNITY CHALLENGE

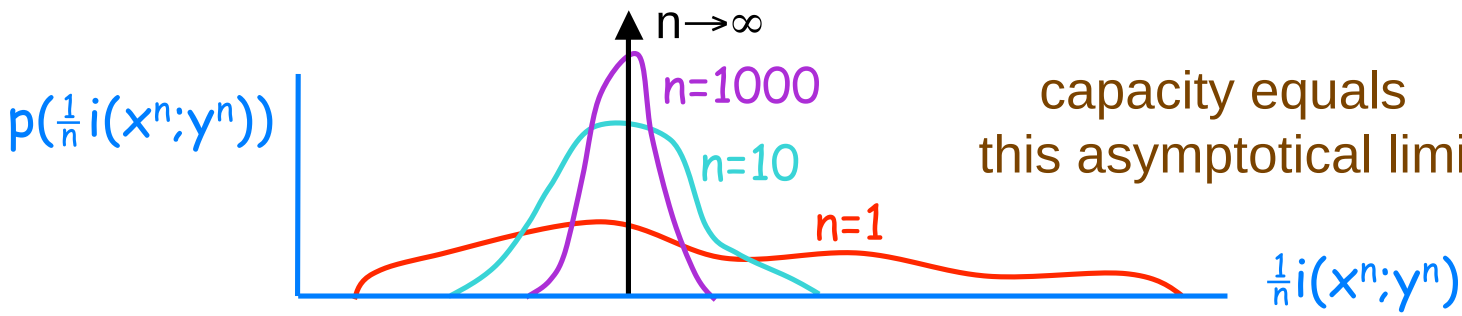



***Design criterions go beyond information theory and call for perspectives with root in network and cross-layer theory. Optimization provides a rich interface through analytical and numerical tools in addition to classical statistical tools.***

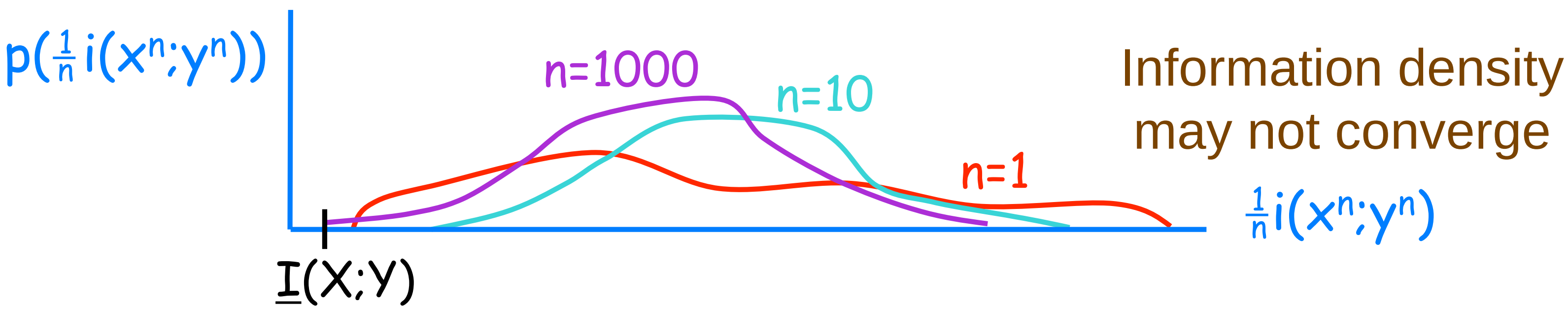
**Generalized capacity and distortion metrics are more suitable for generalized system models.**

# Information-Stable and General Channels

- Information density of a channel (random variable) $\frac{1}{n} i_{X^n Y^n}(x^n; y^n) = \frac{1}{n} \log \frac{P_{Y^n|X^n}(y^n|x^n)}{P_{Y^n}(y^n)}$
- For information-stable channels, converge in distribution to constant



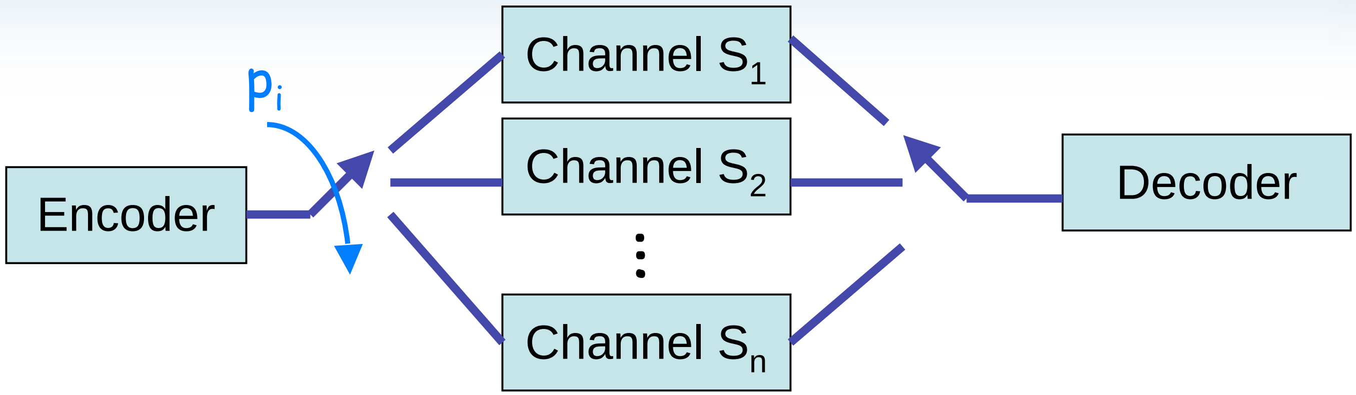

- General channels not necessarily stationary, ergodic or info-stable




- Capacity of general channels (Verdu/Han '94)

$$C = \sup_{X} \underline{\boldsymbol{I}}(\boldsymbol{X};\boldsymbol{Y}) = \sup_{X} \sup \left\{ \alpha : \lim_{n\to\infty} \Pr\left[ \frac{1}{n} i_{X^n Y^n}(x^n; y^n) \le \alpha \right] = 0 \right\}$$

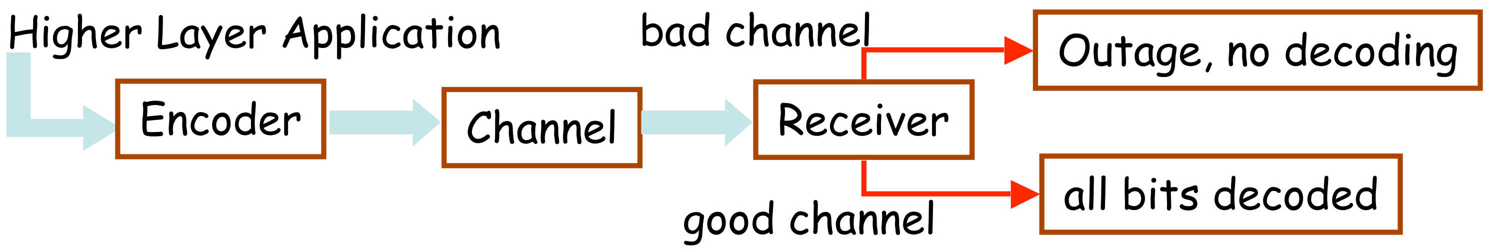
# A Composite Channel



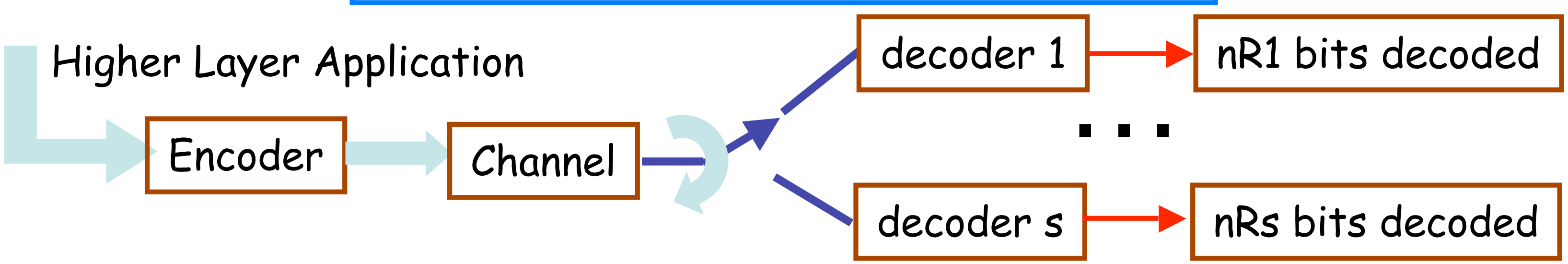

- Channel chosen randomly at the beginning of transmission, then fixed for the entire duration of the codeword, $p(S_i)=p_i$
  - Component channel $S_i$ stationary and ergodic
  - Overall channel is a mixture of stationary and ergodic components
- Assume decoder knows channel state, but the encoder only knows the state distribution
  - CDIT and CSIR
- Common assumptions in slowly-fading channels

# Rethinking Capacity: Question the Questions

- Should $P_e \to 0$ always? Should all transmitted data be decoded?
- Changing these requirements leads to new capacity definitions
  - Decode some of the time: Outage
  - Decode partial data: Expected capacity
- Outage capacity $C_q^o = (1-q)\sup_X \sup\{\alpha : \lim_{n\to\infty} \Pr[i(X^n;Y^n \mid S) \le n\alpha] = q\} = (1-q)C_q$

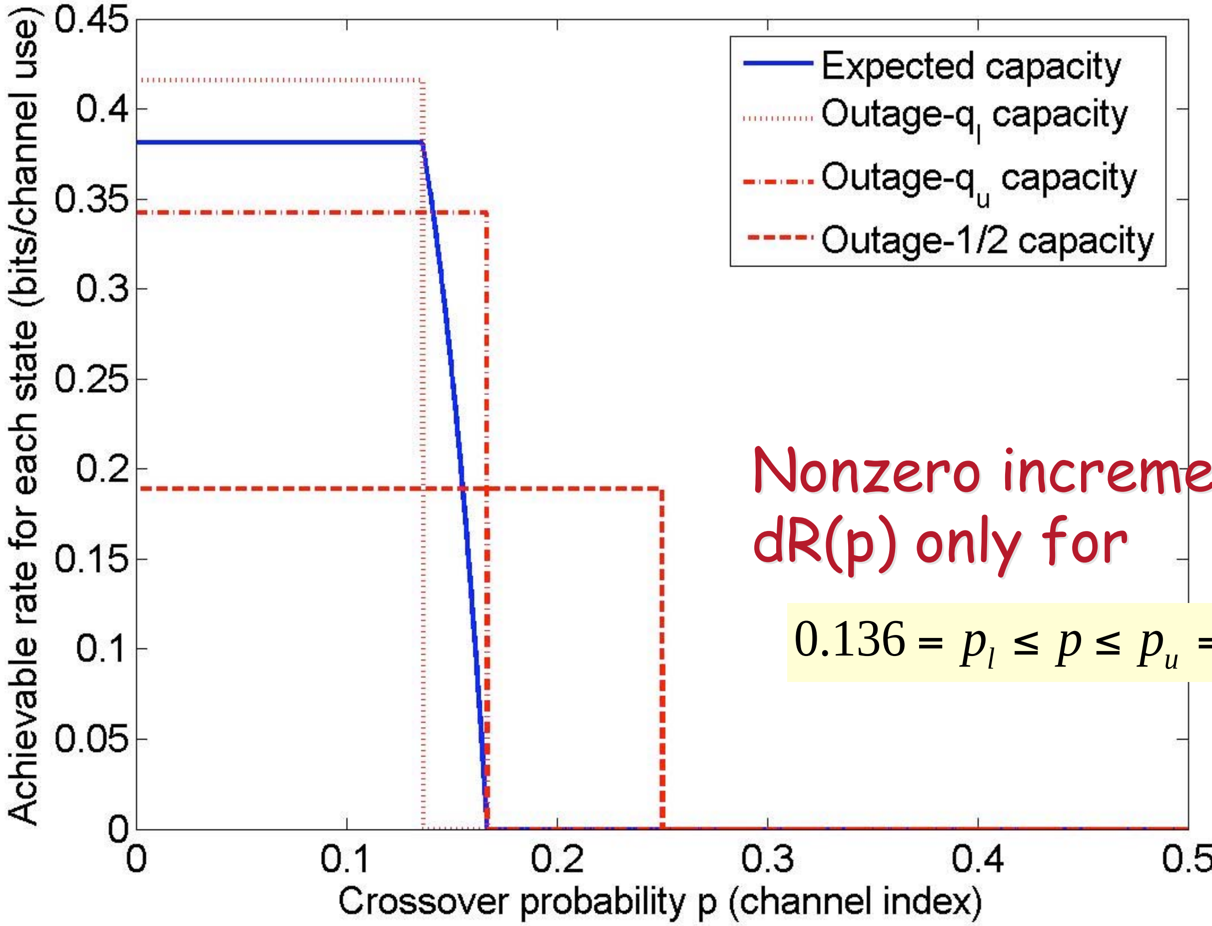



- Expected capacity $C^e = \sup_{(R_p)\in\mathcal{C}_{BC}} \sum_{p\in\mathcal{P}} R_p \sum_{s\in p} P_S(s) = \sup_{(R_p)\in\mathcal{C}_{BC}} \sum_{s\in\mathcal{S}} P_S(s) \sum_{s\in p} R_p$



# Example 1: Composite Binary Symmetric Channel

- BSC: random crossover probability p ~ distribution f(p), $0 \le p \le (1/2)$




Nonzero incremental rate dR(p) only for

$$0.136 = p_l \le p \le p_u = 1/6$$

# Generalized End-to-End Distortion Metrics

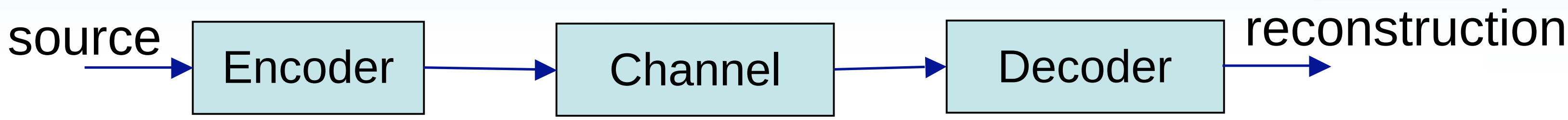
- Stationary ergodic source transmitted over stationary ergodic channel
  - A single distortion level maintained for all transmissions
  - Lossless reconstruction (D=0), lossy (D>0)

- Outage distortion: Dq achieved with max outage probability q
  - Encoder $f_n : V^n \to X^n$, decoder $\phi_n : (Y^n, S) \to \hat{V}^n$
  - Require $\Pr\left\{(V^n, \hat{V}^n) : d(V^n, \hat{V}^n) \le D_q\right\} \ge 1 - q$
- Expected distortion $\mathrm{E}_{(V^n, \hat{V}^n)}\left\{d(V^n, \hat{V}^n)\right\} = \sum_S P(S) D_S \le D^e$

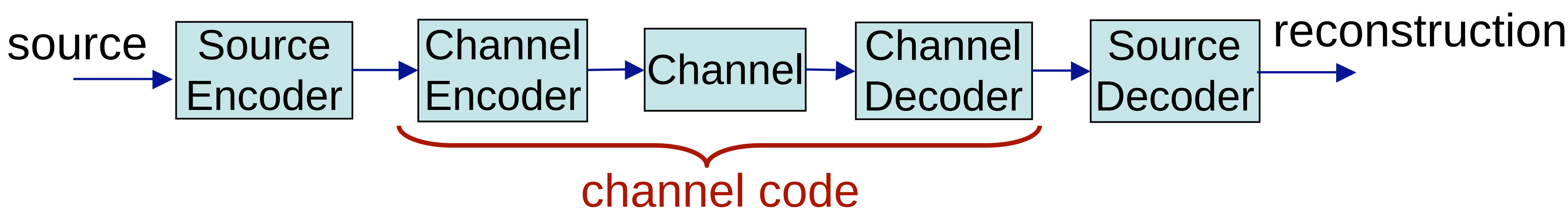
- For transmission over stationary ergodic channels, we can design source-channel codes such that $d(V^n, \hat{V}^n)$ approaches the same limit.
- For non-ergodic channels, $d(V^n, \hat{V}^n)$ may approach different limits depending on channel realization S

# Joint and Separate Source-Channel Coding

- General system: three blocks




- Separate source-channel coding: five blocks




  - separate source-channel encoder $V^n \to \{1, 2, \cdots, M\} \to X^n$
  - A built-in channel code including three blocks
  - A built-in source code
    - Output of source encoder may be different from input of source decoder, as a result of the noisy channel
    - slightly different from stand-alone source code

- In contrast, joint source-channel coding is a loose label that encompasses all coding techniques w/o the intermediate index set

# Optimality of Separate Source-Channel Coding

- Well-known result: Separate source-channel coding optimal if
  - point-to-point link
  - stationary ergodic source and channels
  - single distortion level (D=0, lossless) maintained for all transmissions

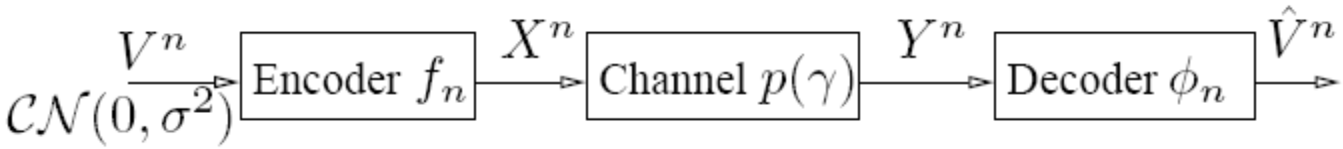
simply compare source rate distortion function R(D) and channel capacity C

- Need reexamination if any condition does not hold
  - Separation fails for multi-user systems [Cover & El Gamal]
  - Generalized source and channel models: notion of domination [Verdu et al]
  - Generalized distortion metrics?

- Source-channel coding under an outage distortion metric
  - Outage distortion criterion satisfied if $R(D_q) < C_q = C_q^o / (1-q)$
  - Also necessary condition for certain systems

# E.g. 2: Gaussian Source over Slow-Fading Gaussian Channel

- Gaussian source $CN(0,\sigma^2)$, slow-fading Gaussian channel, dist $p(\gamma)$



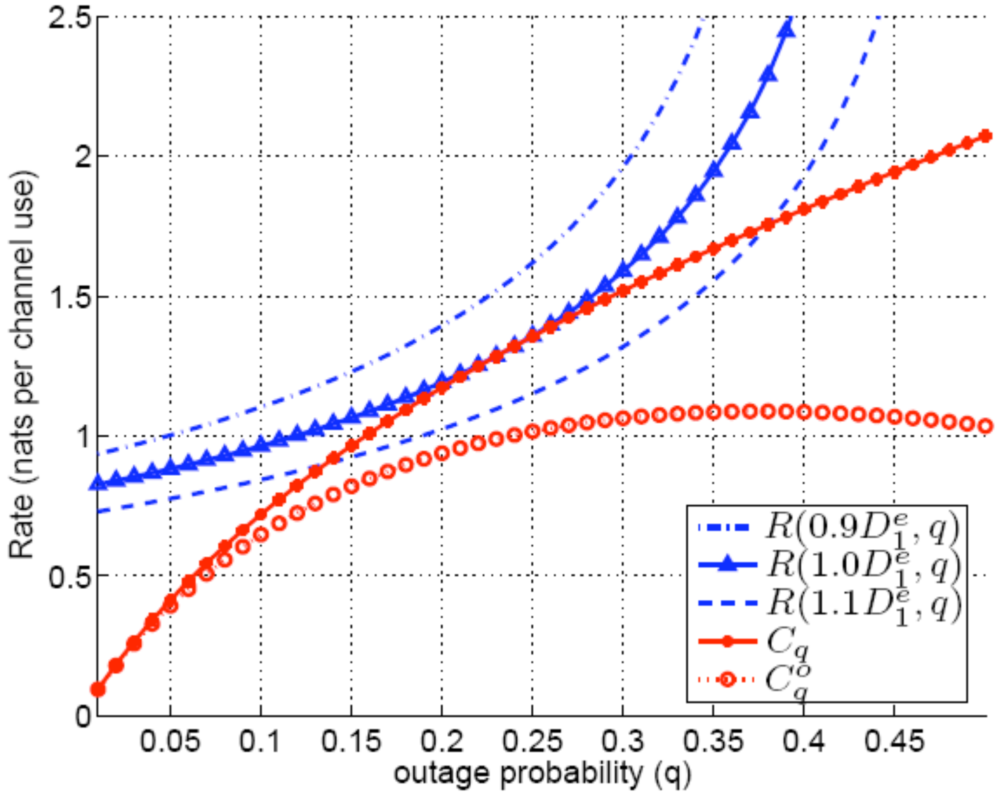

bandwidth expansion ratio b=1, power constraint P, noise variance 1

- Uncoded transmission optimal: a linear code $X = \sqrt{P/\sigma^2}V$ achieves min distortion for each channel state γ: $D_\gamma^* = \sigma^2/(1+P\gamma)$

- To satisfy outage distortion criterion, both separation scheme and optimal uncoded scheme require $D_q > \sigma^2/(1+P\gamma_q)$
  - Separation holds for this system under the outage distortion criterion

# E.g. 2: Separate Scheme for Expected Distortion

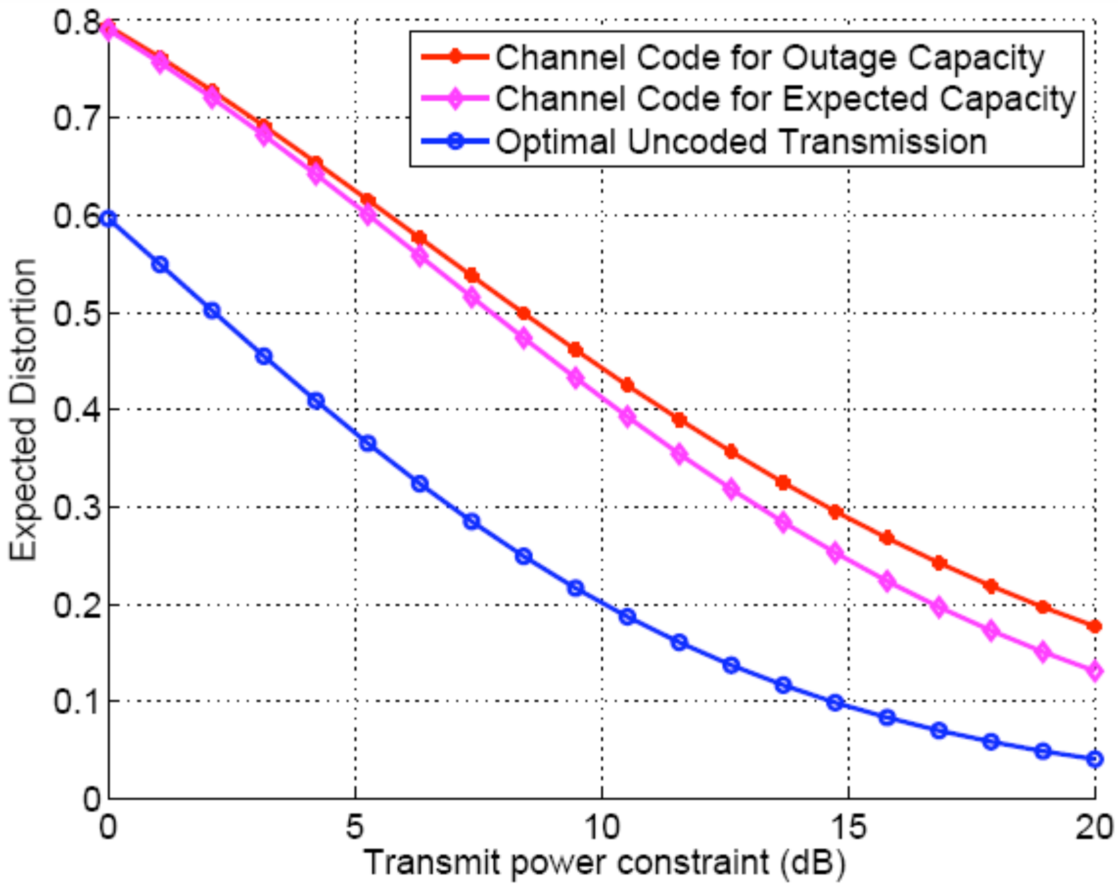
- Single-rate source code and channel code for outage capacity




channel should provide the entire (q, Cq) curve as interface instead of the single point $(q_C^*, R_{q_C^*})$ that maximizes outage capacity

- Multiple-resolution source code and broadcast channel code
  - BC capacity region boundary as interface

# E.g. 2: Comparison of Schemes for Expected Distortion




- Broadcast channel code combined with the multi-resolution source code performs slightly better than the channel code for outage capacity combined with a single rate source code,
- There is a large gap between their expected distortion and that of the optimal uncoded scheme

# Community Challenge

- Rethinking capacity
  - Shannon's definition was right for information stable channels
  - New definitions needed for more general channels
  - When outage acceptable or not all bits need be received, outage or expected capacity are valuable metrics

- Rethinking end-to-end distortion
  - Typically defined as a single guaranteed maximum distortion
  - New ideas: outage distortion and expected distortion
  - Match source codes with new capacity definitions
    - Multi-resolution source code and degraded BC channel code
    - Multiple description source code and un-prioritized expected-rate channel code

- Rethinking separation
  - Traditionally exchange of a single value between source and channel
  - What is the right source/channel interface under generalized metrics
  - How is separation and its optimality defined