Progress on General Capacity Using Network Coding

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Joint with Jay Kumar Sudararajan, MinJi Kim, Devavrat Shah, Ralf Koetter

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Status quo

- When separation holds, what is the benefit of having network coding?
- Major difficulty I: in non-multicast settings, codes are an open problem
- Major difficulty II: time-varying nature of traffic and of network operation, e.g. changing codes
- Major difficulty III: even without coding, performance is ill understood
- State of the art I: pick a system (say COPE) and run experimental trials to demonstrate improvement
- State of the art II: pick a multicast example and work it out by hand

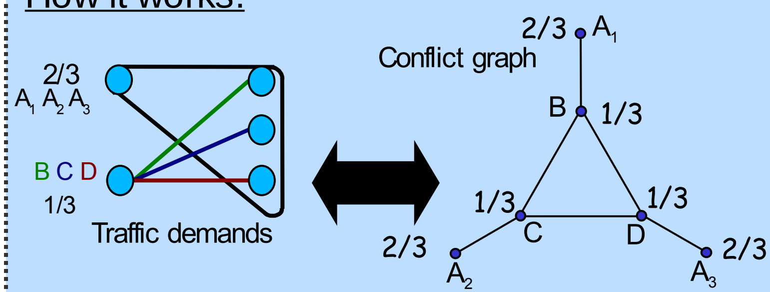
New insight/intellectual tool

- Fixing family of possible codes, give systematic representation of achievable region using conflict graph representation
- Obviates the need for finding clever schedules by hand
- Difficulty of problem now becomes one of characteristics of conflict graph (for instance, perfection)– it is a combinatorial, graph-theoretic question
- Finding schedules now comes from conflict graph

Achievement: for networks of depth one (like switch)

Provide a systematic way of characterizing achievable region and benefit of coding

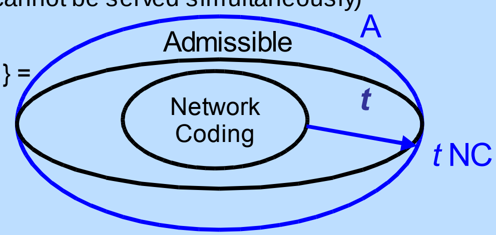
How it works:



An edge between two vertices if the two configuration cause conflict

(i.e. They cannot be served simultaneously)

$\min\{t : A \subseteq t\,NC\} =$
Speedup needed for 100% throughput



Can bound speed-up using imperfection ratio of conflict graph

Assumptions and limitations:

- Approach is general, but speedup characterized for depth one
- Works in separable settings
- Algorithms are centralized

End-of-phase goal

To obtain general results for networks with multiple layers

To incorporate MAC constraints into the conflict graphs, allowing mixture of MAC and scheduling

To provide a set of systematic approaches to determine schedules

Create online schemes, in the flavor of i-slip, to trade-off complexity and effectiveness of schedules, with possible decentralization

Community challenge

How can we approximate difficult capacity region problems?

How can we create schedules from such approximations?

What is the loss that comes from a distributed scheduling?

Ramification: bring problem to its combinatorial essence, which determines difficulty

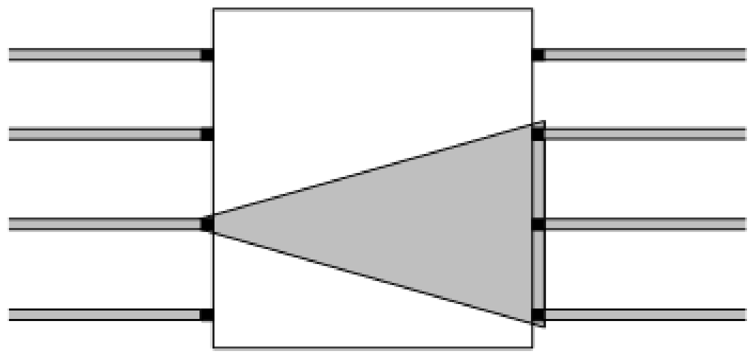
Status Quo

- When separation holds, what is the benefit of having network coding?
- In non-multicast settings, codes are an open problem
- Time-varying nature of traffic and of network operation, *e.g.* changing codes
- Even without coding, performance is ill understood
- State of the art:
  - Pick a system (say COPE) and run experimental trials to demonstrate improvement
  - Pick a multicast example and work it out by hand

New insights / Intellectual Tool

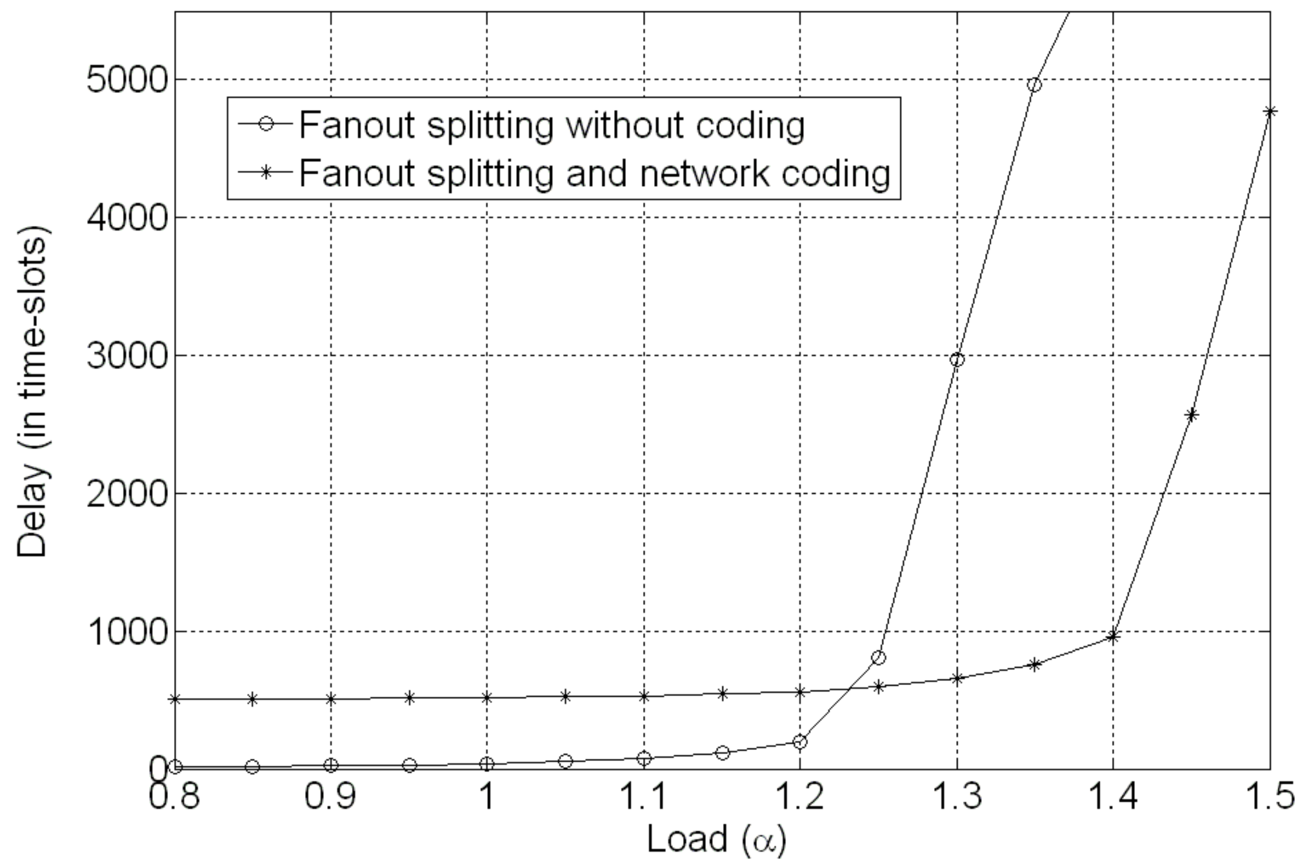
- Transform scheduling problem into a combinatorial graph-theoretic question
- Fix family of possible codes; give systematic representation of achievable region using conflict graphs
- Obviates the need for finding clever schedules by hand
- Difficulty of problem now depends on the characteristics of conflict graph (for instance, perfection)
- Finding schedules now comes from conflict graph

Achievement



- Focus on networks of depth one (*eg.* multicast switch)
- Provide a systematic way of characterizing the achievable rate region and the benefit of coding – using a simple and intuitive graph theoretic formulation
- Scheduling algorithms using rate decomposition approach

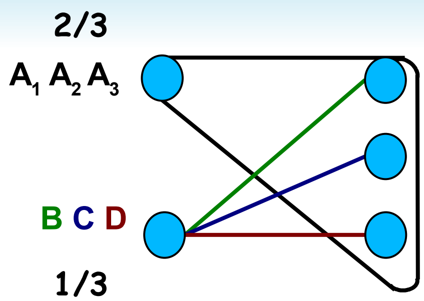
Simulation Result



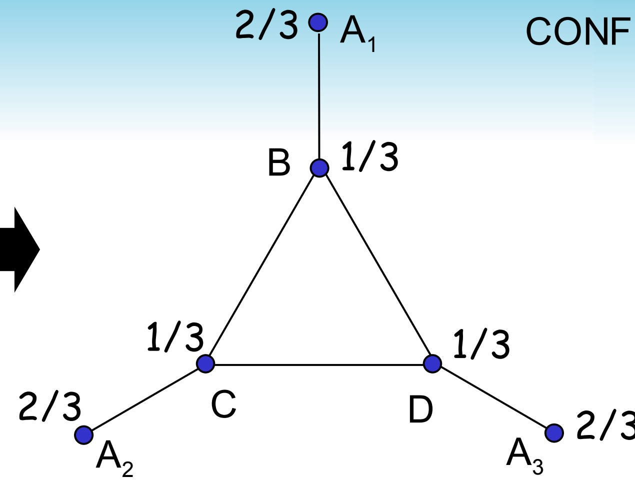
- 4 x 3 switch
- Traffic: mix of unicast and special pattern
- Used randomized variant of max weight scheduling

How it works

TRAFFIC DEMANDS

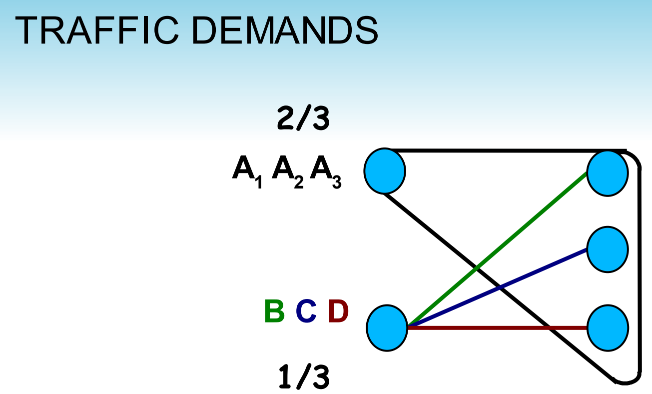


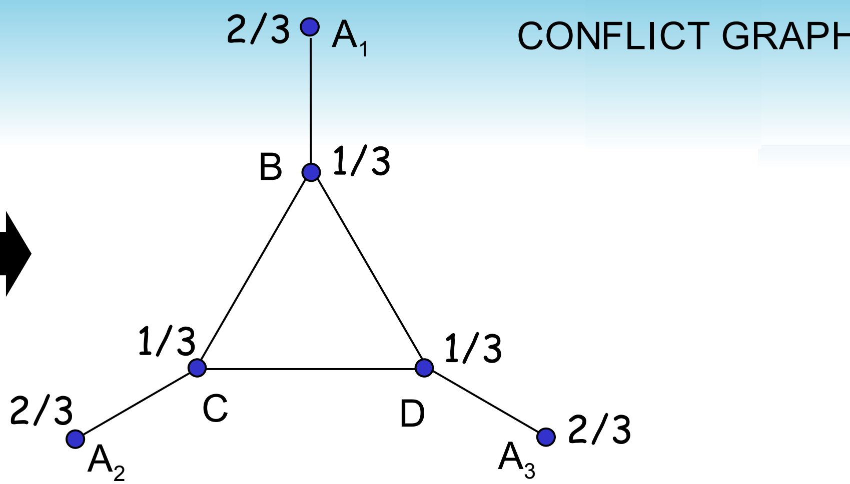
CONFLICT GRAPH



- Conflict graph:
  - A vertex for every network state (*i.e.* switch configuration)
  - An edge between two vertices if the two states cause conflict (*i.e.* two flows that cannot be served simultaneously)
- Map valid service configurations of the system of queues to stable sets of conflict graph

How it works



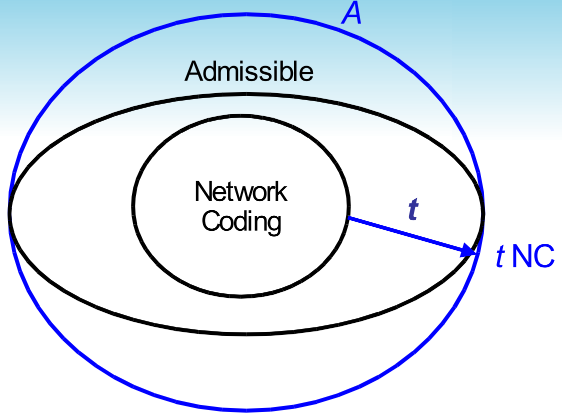


- Achievable rate region is the stable set polytope of the conflict graph
  - Closed form characterization known for several, but not all, classes of graphs
- Computing schedules based on rate decomposition – maps to weighted coloring of the graph
  - Polynomial time algorithm if the graph is perfect

How it works

Speedup needed for 100% throughput

$= \min\{t \mid \mathbf{A} \subseteq t.\mathbf{NC}\}$



- Admissible region:
  - Set of rates such that no input or output node is overloaded
  - Corresponds to the clique inequality polytope of conflict graph
- Can use switch speedup to “expand” the rate region
- Speedup needed for 100% throughput = Factor of expansion for network coding region to cover admissible region
- **Result:** Imperfection ratio of conflict graph is an upper bound on the speedup needed for 100% throughput

Assumptions and limitations

- Approach is general, but the speedup results are only for a network of depth one
- Works in settings where separation holds
- The algorithms are centralized. This could be a limitation some scenarios

End of phase goals

- To obtain general results for networks with multiple layers
- To incorporate MAC constraints into the conflict graphs, allowing mixture of MAC and scheduling
- To provide a set of systematic approaches to determine schedules
- Create online schemes, in the flavor of i-slip, to trade-off complexity and effectiveness of schedules, with possible decentralization

Community Challenge

- How can we approximate difficult capacity region problems?
- How can we create schedules from such approximations?
- What is the loss that comes from a distributed scheduling?

Conclusions and Discussion

- New systematic approach to scheduling and rate region questions in network coding
- Transforms problem into a combinatorial graph theoretic formulation
- Incorporates time sharing among different connection states and codes – scheduling using graph coloring

Summary: Progress on General Capacity Using Network Coding

FLoWS

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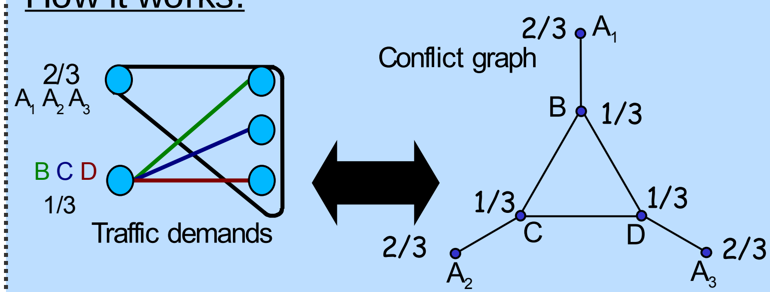

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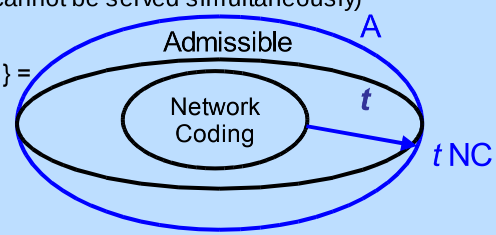

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