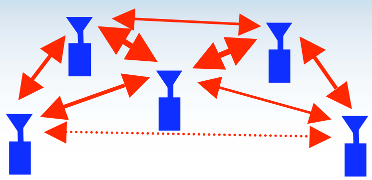
Capacity region equivalence classes
Ralf Koetter, TUM

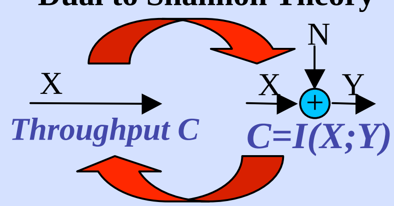
STATUS QUO



Finding capacity of wireless networks is a hard problem

- ***Good achievable rate regions unknown since we don't know how to do "network" relaying or how to deal with interference***
- ***Only have very loose cutset upper bounds that can't be achieved.***

NEW INSIGHTS



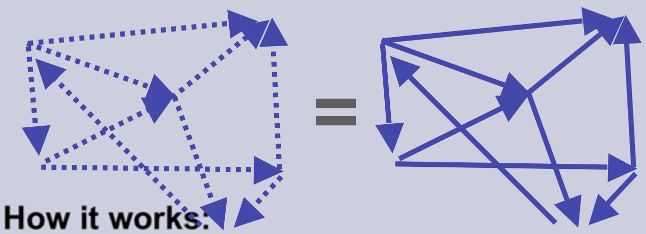
Dual to Shannon Theory:

By emulating noisy channels as noiseless channels with same link capacity, can apply existing tools for noiseless channels (e.g. network coding) to obtain new results for networks with noisy links. This provides *a new method* for finding network capacity

FLOWS ACHIEVEMENT(S)

We prove that the capacity regions of networks with noisy links and networks with noiseless links with a hard rate constraint on each link equal to the noisy link channel capacity are the same.

We can solve for the capacity of a network with noiseless links via network coding



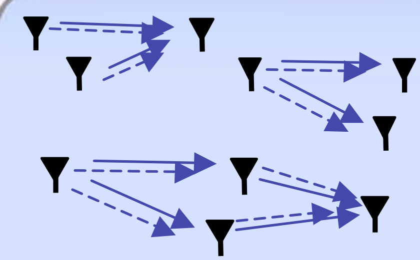
How it works:

- $R_{noiseless} \subseteq R_{noisy}$ easy since the maximum rate on the noiseless channels equals the capacity of the noisy links: can transmit at same rates on both.
- $R_{noisy} \subseteq R_{noiseless}$ hard since must show the capacity region is not increased by transmitting over links at rates above the noisy link capacity. We prove this using theory of "types" to show equivalent capacity

Assumptions and limitations:

- Link-oriented, not broadcast (no interference)
- Assumes links are memoryless and discrete
- Assumes we can solve combinatorial network coding problem (high complexity for large networks)
- Metrics other than capacity may not be the same for both networks (e.g. error exponents).

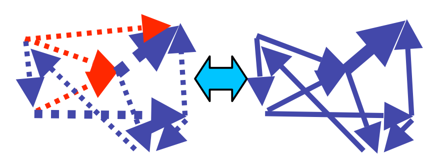
END-OF-PHASE GOAL



Extend analysis to multiple access channels and possibly broadcast channels and multihop networks

Determine capacity orderings for networks where equivalence cannot be established

COMMUNITY CHALLENGE

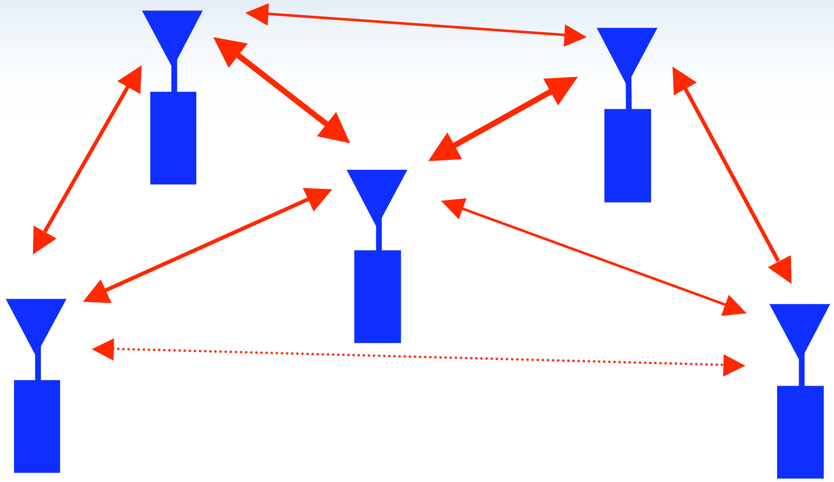


Graduate level: Identify additional equivalences and hierarchies

Prize level: understand limits of capacity ordering as a practical intellectual tool

Equivalence classes provide a new paradigm for characterizing capacity limits

Wireless networks: the status quo



Finding capacity of wireless networks is a hard problem

-*Good achievable rate regions unknown since we don't know how to do "network" relaying or how to deal with interference*

-*Only have very loose cutset upper bounds that can't be achieved.*

Previous results:

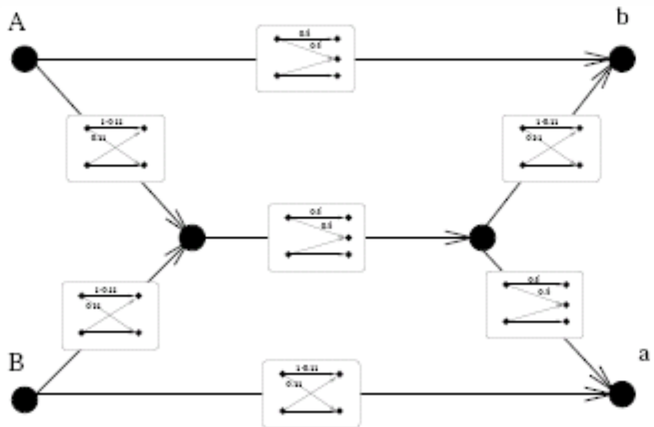
L. Song and R. W. Yeung and N. Cai, "A separation theorem for single source network coding," IEEE Transactions on Information Theory, vol. 52, no. 5, pp. 1861-1871, May 2006

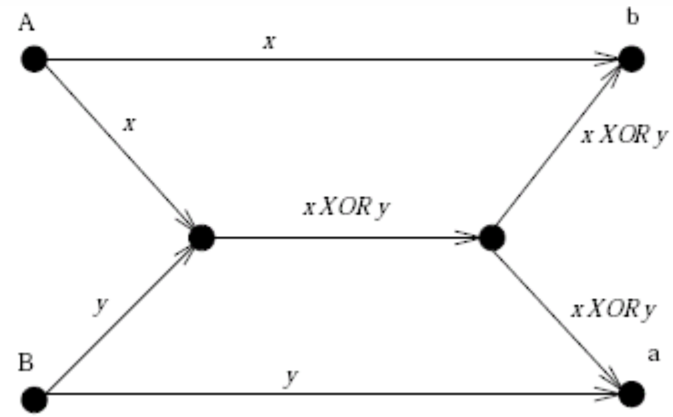
Shashibhushan Borade, "Network Information Flow: Limits and Achievability," Proc. IEEE International Symposium on Information Theory, July 2002.

Both papers address the multicast, in which case the tightness of the min-cut max-flow bounds can be exploited

The initial question

All channels have capacity 1 bit/unit time



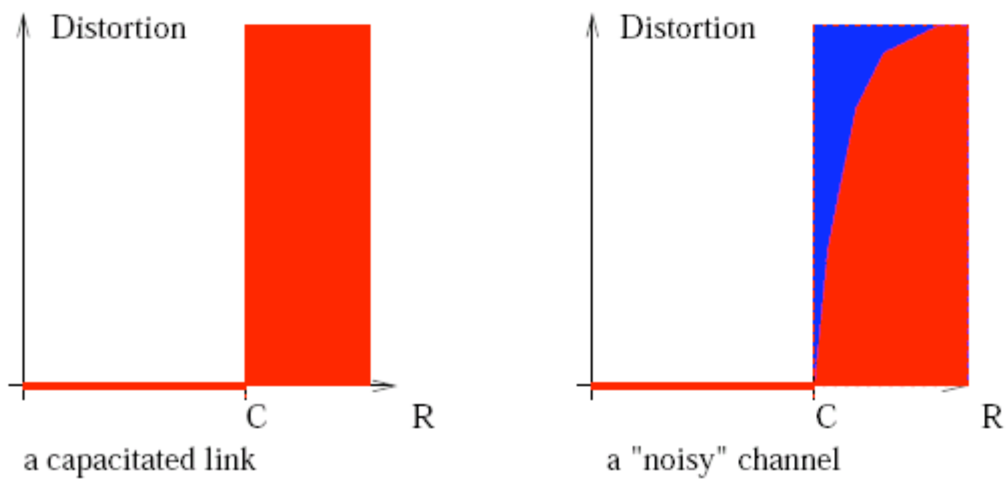


Are these two networks essentially the same?

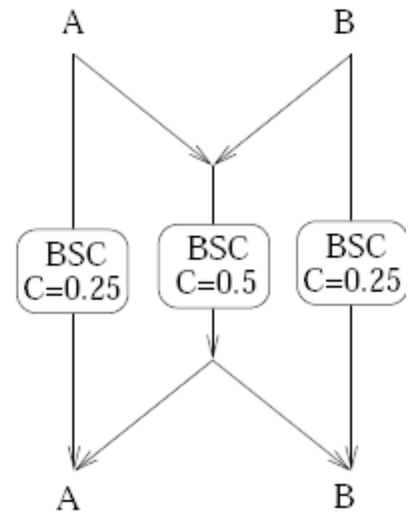
Intuitively, since the “noise” is uncorrelated to any other random variable it cannot help.....

The main technical problem:

The characteristic of a noisy link vs a capacitated bit-pipe



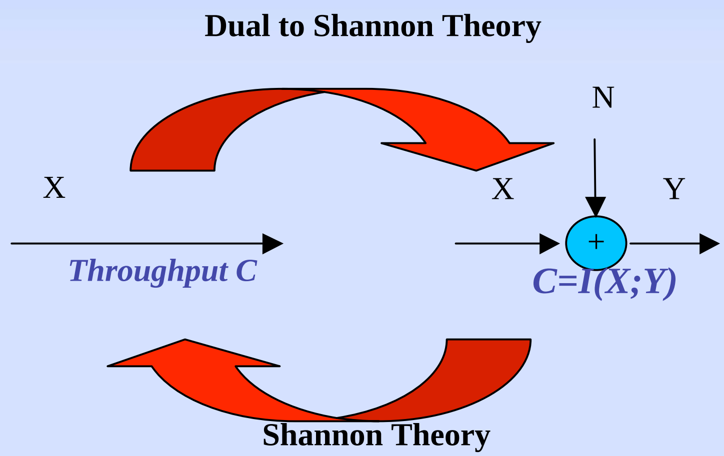
Can the blue area help?



A noisy channel allows for a larger set of strategies than a bit-pipe

Using the center link uncoded approaches capacity, too!

The main technical step:



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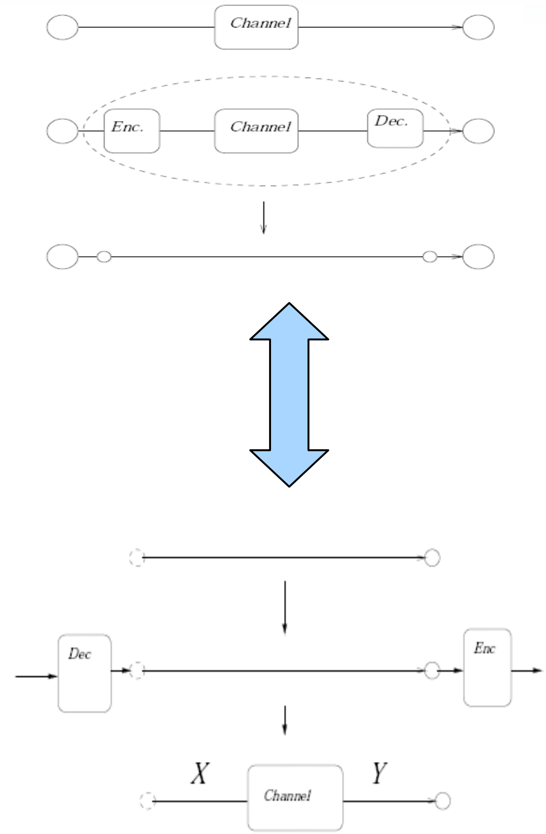
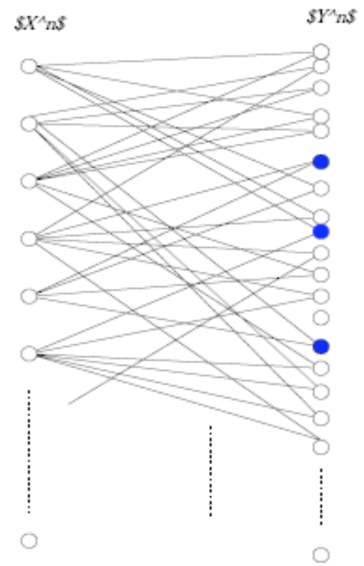


One of the key technical tools



Let a bipartite, biregular graph be given with vertex classes V_1, V_2 of degree d_1, d_2. There exists of subset U of k vertices of V_2 such that every vertex in V_1 is adjacent to U and $k \leq \frac{|V_2|}{d_1}(1 + \log(d_2))$.

(a weak form of the Johnson-Stein-Lovasz Theorem)

One of the key technical tools

$\Rightarrow$

Here $|U| \leq 2^{n(I(X,Y)+o(1))}$ $\Rightarrow$ we can emulate the “type” by transmitting not more than $I(X,Y) + o(1)$ bits.

In other words the “randmoness” due to the channel is provided by the random representations of the types...

——————-

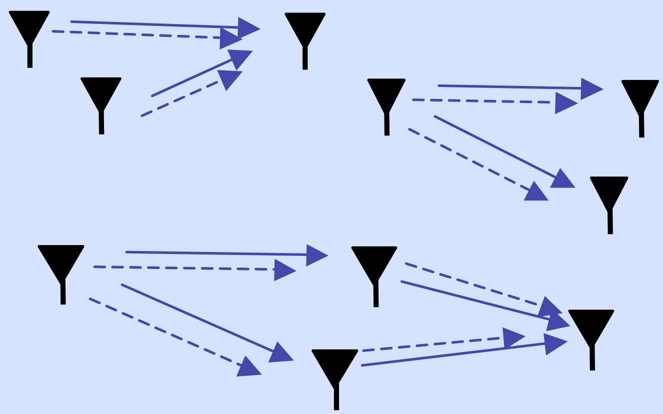
Other issues: - we have to consider all possible ways to use a channel

- error exponents are far from equal (and rather poor)

Assumptions and limitations

- Link-oriented, not broadcast (no interference)
- Assumes links are memoryless and discrete
- Assumes we can solve combinatorial network coding problem (high complexity for large networks)
- Metrics other than capacity may not be the same for both networks (e.g. error exponents).
- No framework (yet) to assess the effects of network changes, rather this let's us make statements about a given network

Extensions and end of phase goals



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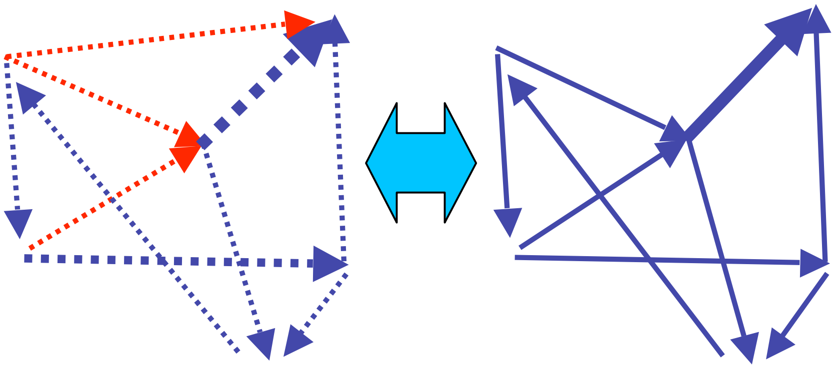
Partly done during a research visit, July 2007

Long term evolution and community service



Graduate level: Identify additional equivalences and hierarchies

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Develop a frame-work to bridge the gap between information theory and networking