# Adaptive Modulation with Smoothed Flow Utility

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# The problem

- choose transmit power(s) and flow rate(s) to optimally trade off average utility and power
- utilities are functions of *time-smoothed* flow rates
- with each flow we associate a
  - smoothing time scale
  - concave increasing utility function
- our model:
  - channel gains are random
  - no interference

# Smoothed flow utility

- wireless link supports $n$ flows in period $t$
- $f_t \in \mathbf{R}^n$ is flow rate vector
- $s_t \in \mathbf{R}^n$ is smoothed flow rate vector: $s_{t+1} = \Theta s_t + (I - \Theta) f_t$
  - $\Theta = \mathbf{diag}(\theta), \quad \theta_j \in [0,1)$
  - $T_j = 1/\log(1/\theta_j)$ is *smoothing time* for flow $j$
- $U : \mathbf{R}^n \to \mathbf{R}$: separable concave utility function
- smoothed flow utility is

$$\bar{U} = \lim_{N\to\infty} \mathbf{E} \frac{1}{N} \sum_{\tau=0}^{N-1} U(s_\tau)$$

# Channel model and average power

- capacity in period $t$ is (up to a constant) $\log(1 + g_t p_t)$
  - $p_t \geq 0$ is transmit power
  - $g_t$ is channel gain (up to constant)

- power required to support flow $f_t$: $p_t = \phi(\mathbf{1}^T f_t, g_t) = (e^{\mathbf{1}^T f_t} - 1)/g_t$

- average power is $\bar{P} = \lim_{N \to \infty} \mathbf{E} \dfrac{1}{N} \displaystyle\sum_{\tau=0}^{N-1} p_\tau$

- $g_t$ IID exponential (for example)

- $f_t$ (and therefore $p_t$) can depend on $g_t$, but not $g_{t+1}, g_{t+2}, \ldots$

# Optimal policy

- (state feedback) policy: $f_t = \varphi(s_t, g_t)$
- goal: choose policy $\varphi$ to maximize $\bar{U} - \lambda \bar{P}$
- $\lambda > 0$ is used to trade off average utility and power
- a convex stochastic control problem
- optimal value is $J^\star$

# General ‘solution’ via dynamic programming

- optimal policy is

$$\varphi^\star(z,g) = \operatorname*{argmax}_{w\geq 0}\{V^\star(\Theta z + (I-\Theta)w) - \lambda\phi(\mathbf{1}^T w, g)\}$$

- $V^\star$ is value function, (any) solution of Bellman equation

$$J^\star + V^\star(z) = \mathbf{E}\left\{U(z) + \max_{w\geq 0}\left\{V(\Theta z + (I-\Theta)w) - \lambda\phi(\mathbf{1}^T w, g)\right\}\right\}$$

# Value iteration

1. update unnormalized estimate of $V^\star$:

$$\tilde{V}^{(k+1)}(z) := \mathbf{E}\left\{ U(z) + \max_{w \geq 0} \left\{ V^{(k)}(\Theta z + (I - \Theta)w) - \lambda\phi(\mathbf{1}^T w, g) \right\} \right\}$$

2. normalize (and get new estimate of $J^\star$):

$$J^{(k+1)}(z) := \tilde{V}^{(k+1)}(0); \qquad V^{(k+1)}(z) := \tilde{V}^{(k+1)}(z) - J^{(k+1)}$$

- $V^{(k)} \to V^\star$, $J^{(k)} \to J^\star$
- iteration preserves concavity, monotonicity, so $V^\star$ is concave, increasing
- can carry out numerically for $n$ very small (say, 1 or 2)

# No transmit region

- from convex analysis, $\varphi^\star(z, g) = 0$ if and only if

$$g \nabla V^\star(\Theta z) \leq \left( \frac{\lambda}{1-\theta_1}, \ldots, \frac{\lambda}{1-\theta_n} \right)$$

  (assuming here $V^\star$ is differentiable)

- interpretation: don't transmit if
  - channel is bad ($g$ small)
  - or, smoothed flows are large ($z$ large $\Rightarrow \nabla V^\star(\Theta z)$ small)

# Single-flow examples

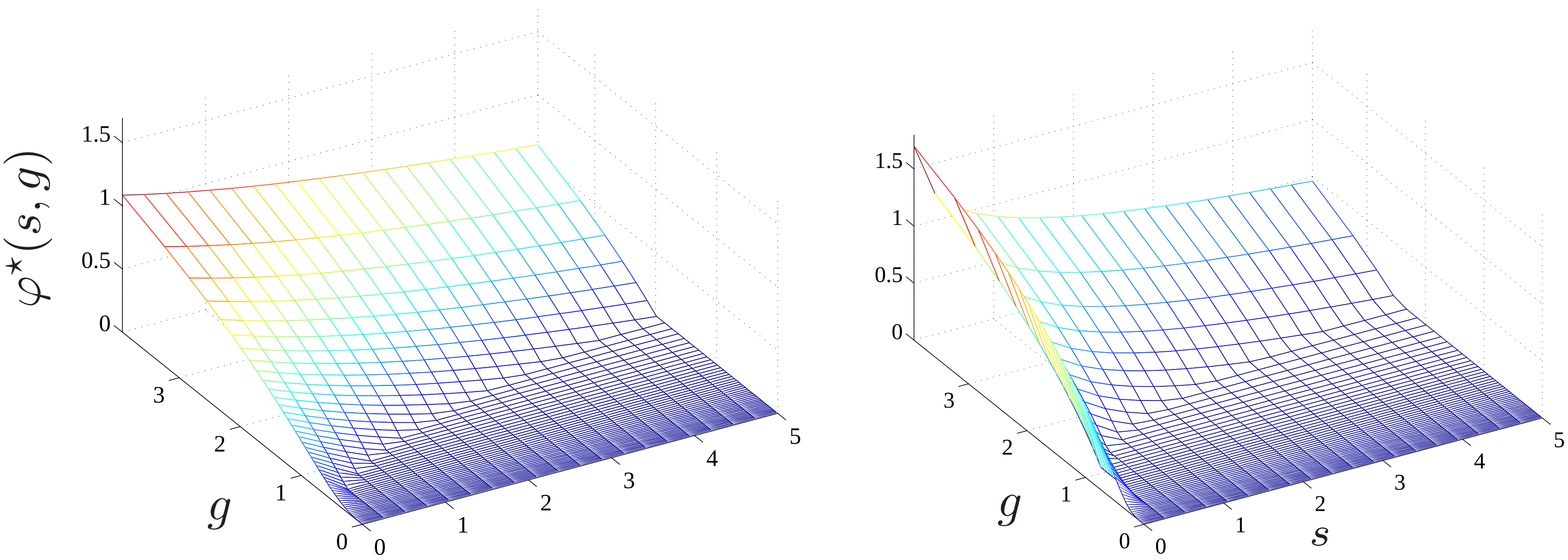
- two examples:
  - light smoothing ($T = 1$; $\theta = 0.37$)
  - heavy smoothing ($T = 50$; $\theta = 0.98$)

- $U(s) = s^{1/2}$; $g_t \sim \mathcal{E}(1)$

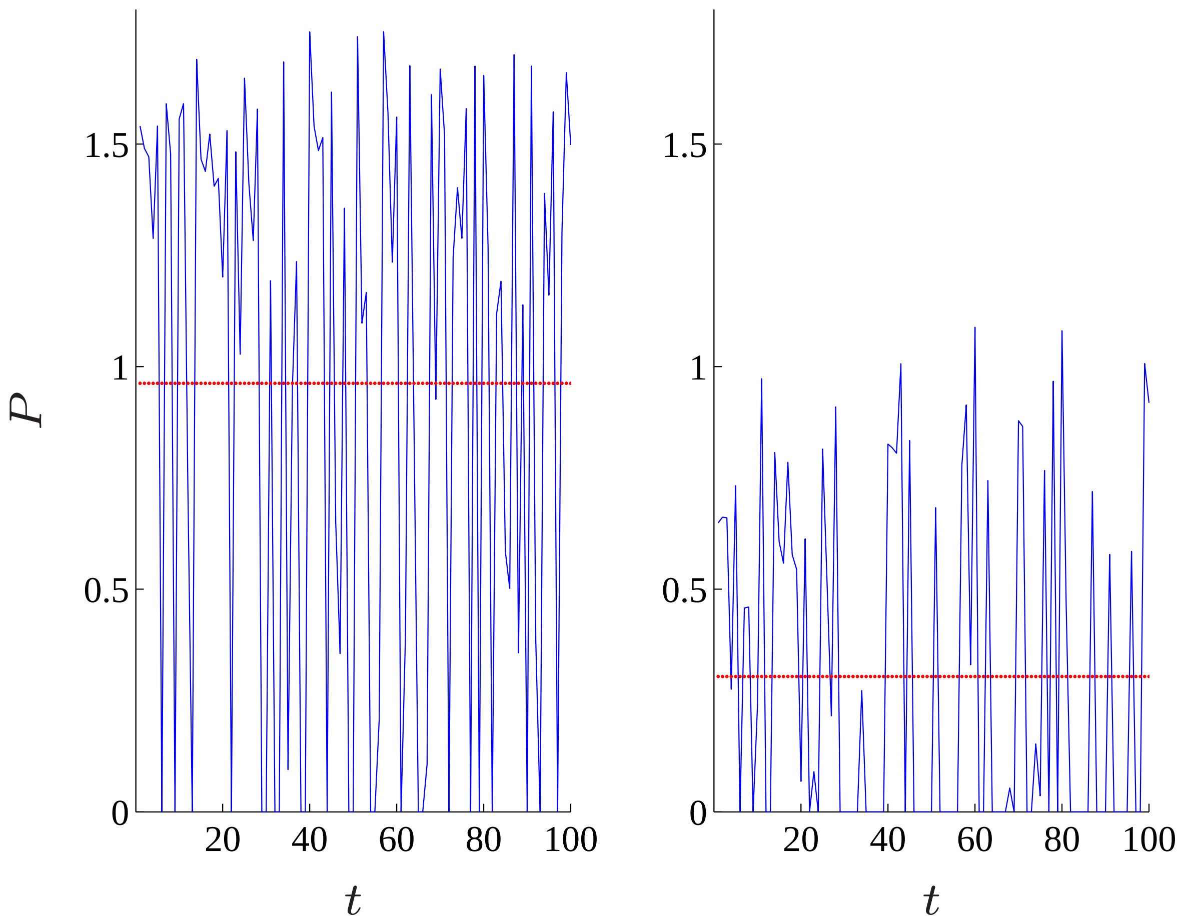
- $\lambda$s chosen to yield $\bar{U} = 0.8$

- (optimal) average power is $\bar{P} = 0.9$ for $T = 1$; $\bar{P} = 0.3$ for $T = 50$
  - smoothing allows $3\times$ reduction in power

# Optimal policies



# Sample power trajectories



# ADP policy for multiple flows

- for more than $3$ flows (say), computing $V^\star$ intractable
- *approximate dynamic programming* (ADP) policy:

$$\varphi^{\mathrm{adp}}(z,g) = \operatorname*{argmax}_{w \geq 0} \{V^{\mathrm{adp}}(\Theta z + (I - \Theta)w) - \lambda \phi(\mathbf{1}^T w, g)\}$$

  - $V^{\mathrm{adp}}$ is an *approximate* or *surrogate* value function
- ADP can work surprisingly well, even when $V^{\mathrm{adp}}$ is not a particularly good approximation of $V^\star$
- some general methods for coming up with a surrogate:
  - use exact value function for simpler problem
  - learning ($e.g.$, Q-learning) or optimization over a basis

## Separable surrogate value function

- we propose surrogate value function

$$V^{\mathrm{adp}}(z) = V_1^\star(z_1) + \cdots + V_n^\star(z_n)$$

  - $V_j^\star : \mathbf{R} \to \mathbf{R}$ is value function for $j$th flow alone
  - can evaluate $\varphi^{\mathrm{adp}}(z, g)$ very fast via waterfilling

- $V^{\mathrm{adp}}$ is separable, but policy $\varphi^{\mathrm{adp}}$ is not

- (optimizing over basis of separable surrogate value functions yields very little performance improvement)

- policy $\varphi^{\mathrm{adp}}$ seems to work well . . . but how suboptimal is it?

## An upper bound on $J^\star$

- we relax ($i.e.$, ignore) causality requirement, $i.e.$, we have complete knowledge of *future* channel gains
- for each channel gain realization, results in (large, but convex) multi-period optimization problem
- expected value of optimal cost (obtained by Monte Carlo simulation) is *upper bound* on $J^\star$
- called *prescient bound* $J^{\mathrm{pre}}$ (since it assumes future is known)

# Numerical example

- two flows on a single link, with light $(T = 1)$ and heavy $(T = 50)$ smoothing
- $U(s_1, s_2) = s_1^{1/2} + s_2^{1/2}$;  $\phi(f_t, g_t) = \lambda/g_t(e^{\mathbf{1}^T f_t} - 1)$
- $g_t \sim \mathcal{E}(1)$, $\lambda = 1$
- we run $1000$ realizations, each of length $N = 1000$
- $J^{\mathrm{adp}} = -13.9$; $J^{\mathrm{pre}} = -13.8$
- so $J^{\mathrm{adp}}$ is *at most* $0.1$-suboptimal

# Final observations

- time smoothing has great affect on
  - optimal policy
  - average power needed
- rough interpretation of optimal policy:
  - with smoothing, wait for good channel, unless desperate
  - and so, save power
  - more smoothing $\Longrightarrow$ more opportunistic, less power
- multi-flow ADP policy
  - surrogate is sum of single-flow value functions
  - performance is nearly optimal, as shown by upper bound on $J^\star$