Positive Recurrent Medium Access Algorithm

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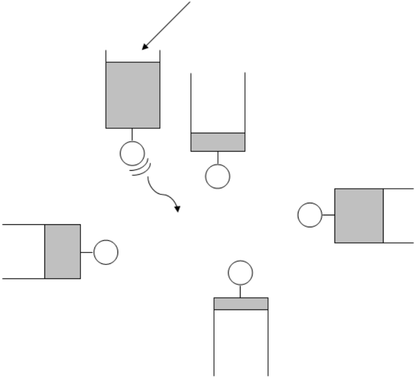
Medium Access Control (MAC) in Wireless Network



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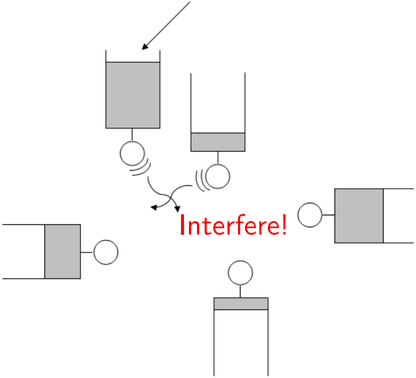

Constraints

- Interfering nodes can not transmit simultaneously.

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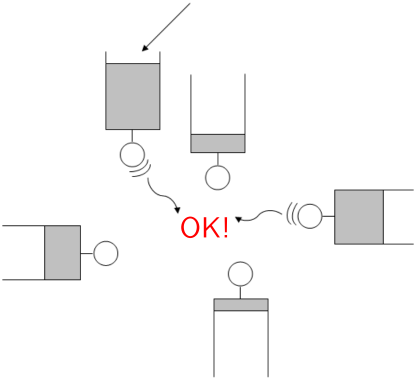

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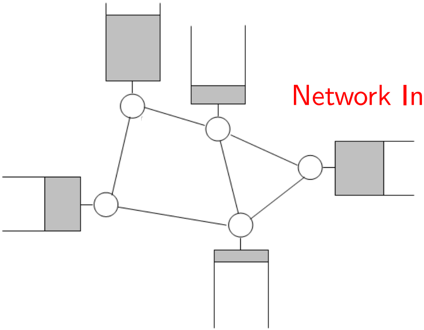

Network Interference Graph

Constraints

- Interfering nodes can not transmit simultaneously.
- Nodes have only "local" information
    - Is any interfering neighbor transmitting ?

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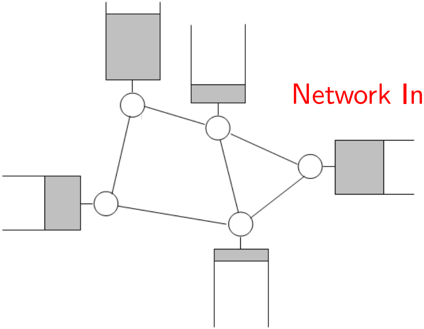

Network Interference Graph

Question

- Which nodes should transmit simultaneously using local information.
- So that performance is not compromised.

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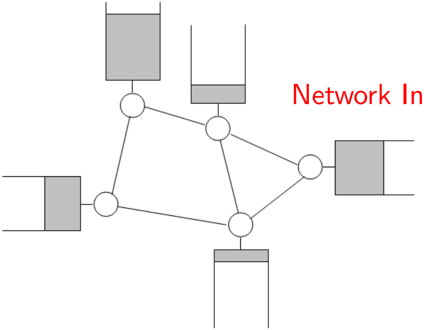
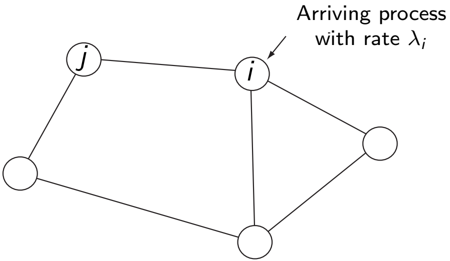


Network Interference Graph

Goal

- Design an ‘distributed’, ‘efficient’ scheduling algorithm of ‘high performance’
    - Decides transmission of non-interfering nodes

Model



- Network interference graph $G = (V, E)$ with n queues as nodes
  - $E = \{(i, j) : i \text{ and } j \text{ cannot transmit simultaneously}\}$.
  - A packet arrives at queue i with probability λ_i at time $t \in \mathbb{Z}_+$.

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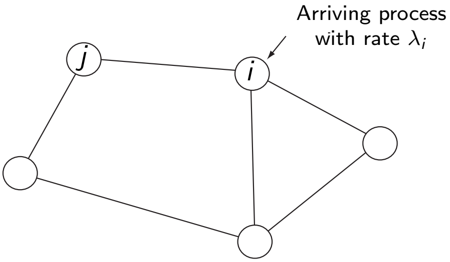

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- Scheduling algorithm: at each time instance $t \in \mathbb{Z}_+$
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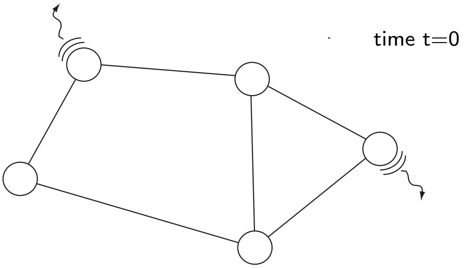

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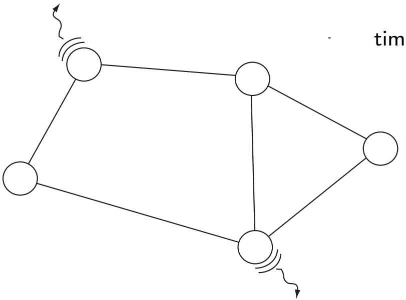

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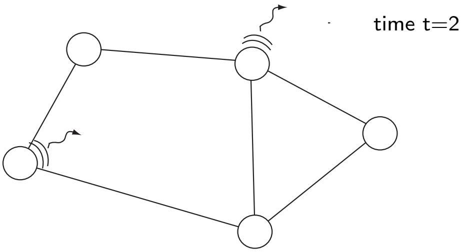

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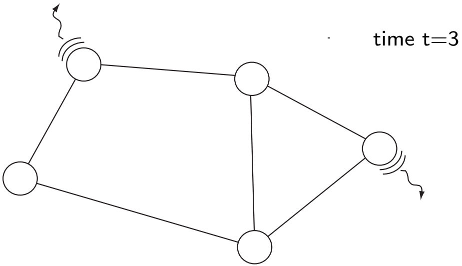

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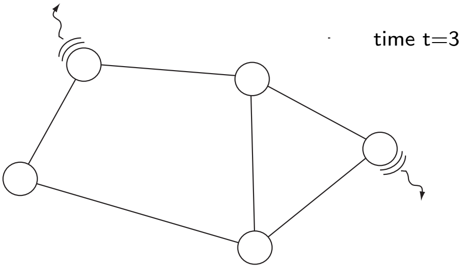

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  - A packet in each selected queue departs (or serviced) from the network.
  - Our primary interest is to design a distributed algorithm minimizing cooperations.

Queueing Evolution

Notations

- $\mathbf{Q}(t) = [Q_i(t)]$ be the queue-sizes at time t.
- $\sigma_i(t) = \begin{cases} 1 & \text{if queue } i \text{ is transmitting (successfully) at time } t \\ 0 & \text{otherwise} \end{cases}.$
  - $\boldsymbol{\sigma}(t) = [\sigma_i(t)] \in \mathcal{I}(G) :=$ Collection of all independent sets in G.

$$Q_i(t+1) = Q_i(t) + A_i(t) - \sigma_i(t) \cdot 1_{\{Q_i(t)>0\}}$$

- $\mathbf{A}(t) = [A_i(t)]$ be the number of arrival packets at queue i at time t.
  - $\mathbb{E}[A_i(t)] = \lambda_i$.

Performance Metric

No common notion in the literature

- Want queues to be kept small under large arrival rate $\boldsymbol{\lambda} = [\lambda_i]$.
- In our model, $\boldsymbol{\lambda} \in \mathrm{conv}(\mathcal{I}(G))$.
  - Otherwise, queues should grow linearly over time.

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Scheduling algorithm is

- Positive recurrent if Markov chain $(\mathbf{Q}(t), \boldsymbol{\sigma}(t))$ is pos. rec. for $\boldsymbol{\lambda} \in \text{conv}^o(\mathcal{I}(G))$.
  - Hence, queues remain finite with probability 1.
- Rate stable if $\mathbf{Q}(t)/t \to 0$ with probability 1 for $\boldsymbol{\lambda} \in \text{conv}^o(\mathcal{I}(G))$.

Prior MAC Algorithms

Two Recent Research Directions

I. Starting from [Jiang and Walrand 08]
   - Based on arrival-rate information $\boldsymbol{\lambda}$

II. Starting from [Rajagopalan, Shah and Shin 09]
   - Based on queueing information $\mathbf{Q}(t)$

MAC Algorithm: Example

Each individual queue i makes her own decision as

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- Open question: How about general G?
  - Next: some positive answers utilizing additional local information

MAC Algorithm using Carrier Sensing (CSMA)

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Attempt to transmit with probability $p_i(t)$ at time t.

$$p_i(t) = \begin{cases} 0 & \text{if some (interfering) neighbor attempted at time } t-1 \\ 1 - \frac{1}{W_i(t)} & \text{else if } i \text{ attempted (to transmit) at time } t-1 \\ \frac{1}{2} & \text{otherwise} \end{cases}.$$

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- Carrier Sensing Information
  - Knowledge whether neighbors attempted to transmit (at the previous time-slot).
- Next: Two known successful designs of $W_i(t)$
  - Using arrival-rate information $\boldsymbol{\lambda}$ [Jiang and Walrand 08]
  - Using queueing information $\mathbf{Q}(t)$ [Rajagopalan, Shah and Shin 09]

CSMA I using Arrival-rate Information

Theorem (Jiang and Walrand 08)

For given arrival rate $\boldsymbol{\lambda} \in conv^o(\mathcal{I}(G))$, *there exists* $\mathbf{W}^* = \mathbf{W}^*(\lambda, G)$ *such that*

$$\textit{CSMA using } W_i(t) = W_i^* \textit{ is rate stable.}$$

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- Question: Possible to design $\mathbf{W}(t)$ using queueing information?

CSMA II using Queueing Information

Theorem (Shah and Shin 10)

CSMA is positive recurrent if

$$W_i(t) \;=\; \max\left\{\log Q_i(t)\;,\; e^{\sqrt{\log\,\log Q_{\max}(t)}}\;\right\},$$

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- We revise the algorithm so that it does not require such message passing.
  - Motivation : Possible to learn $W_j(t)$ using carrier sensing without message passing?

Main Issue: Online Learning Problem

How to learn weights of neighbors without message passing?

- Equivalent question: how to learn access probabilities of (interfering) neighbors?
  - since recall access probability $p_i(t) = 1 - \frac{1}{W_i(t)}$.

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  - $W_j(t)$ is changing
  - # samples is affected by many random environments
  - how much error is allowed for positive recurrence?

New Algorithm with Learning

Recall the original positive recurrent algorithm

- When the medium is free, each queue i attempts to transmit with probability

$$1 - \frac{1}{W_i(t)} = 1 - \frac{1}{\max\left\{\log Q_i(t)\,,\ e^{\sqrt{\log \max_{j \in \mathcal{N}(i)} W_j(t-1)}}\right\}}.$$

New Algorithm with Learning

Revise the original positive recurrent algorithm using estimator L_j^i

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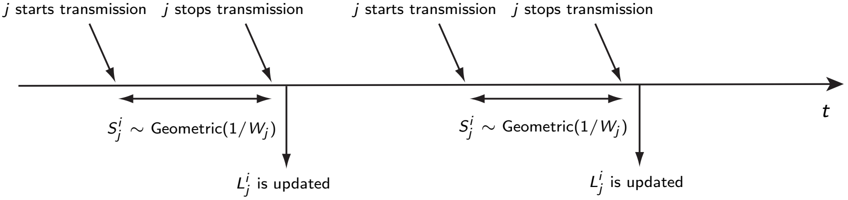
In addition, each queue i maintains $L_j^i(t)$, $S_j^i(t)$ for $j \in \mathcal{N}(i)$: for some function g

- If $\sigma_j(t-1) = 1$, $S_j^i(t) = S_j^i(t-1) + 1$.
- Else if $S_j^i(t-1) > 0$, $S_j^i(t) = 0$ and

$$L_j^i(t) = \begin{cases} L_j^i(t-1) + \Delta & \text{if } S_j^i(t-1) \geq L_j^i(t-1) \\ L_j^i(t-1) - \Delta & \text{otherwise} \end{cases}, \qquad \text{where } \Delta = \frac{1}{g(L_j^i(t-1))}.$$

$\star$ $L_j^i(t)$ and $S_j^i(t)$ are long-term and temporal estimators of $W_j(t)$ at queue i, respectively.

New Algorithm with Learning



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Theorem (Shah, **Shin** and Tetali)

The network Markov chain induced by the revised algorithm is positive recurrent if

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$$F(t) \;=\; \sum_{i} h(Q_i(t)) + \sum_{i,j} g(L_j^i(t))^2 + \sum_{i,j} g(S_j^i(t)),$$

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- Main additional techniques
  - Careful martingale arguments to control L_j^i.

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$$\boldsymbol{\lambda} \in conv^{o}(\mathcal{I}(G)) \qquad and \qquad g(x) = e^{e^{\log^{1/4} x}}.$$

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$$F(t) \;=\; \sum_i h(Q_i(t)) + \sum_{i,j} g(L_j^i(t))^2 + \sum_{i,j} g(S_j^i(t)),$$

  where $h = \int \log\log$.

- Main additional techniques
  - Careful martingale arguments to control L_j^i.
  - 'Hitting time' of Markov chains.

Summary & Wide-applicability

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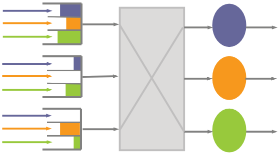
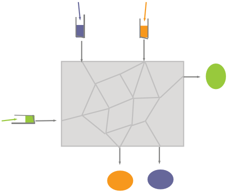
Our framework to design such algorithms has wide-applicability

- In the context of stochastic processing networks [Harrison 00].



High speed switch scheduling

(Matching constraints)



Scheduling in optical core networks

(Multi-commodity-type constraints)