Wireless Network Coding: Some Lessons Learned in ITMANET

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Collaborators

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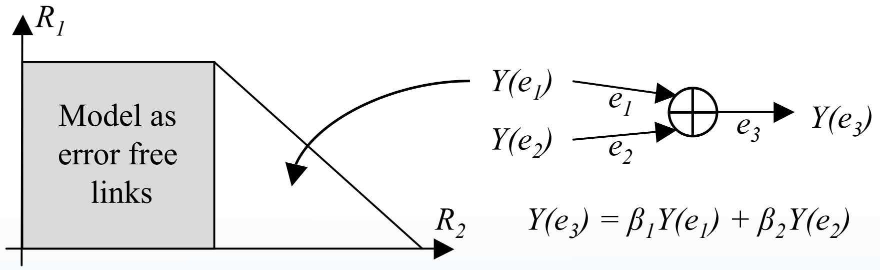
Wireless Networks

- Interference: using it as a code in the high SNR case
  - Code in deterministic model
  - Code in analog amplify and forward
  - Practical implication: coding with zig-zag decoding
- Broadcast: building subgraphs in low SNR networks
  - Optimality of decode-and-forward
  - Practical implication: low-SNR optimization
- Dealing with uncertainty: network combining though coding

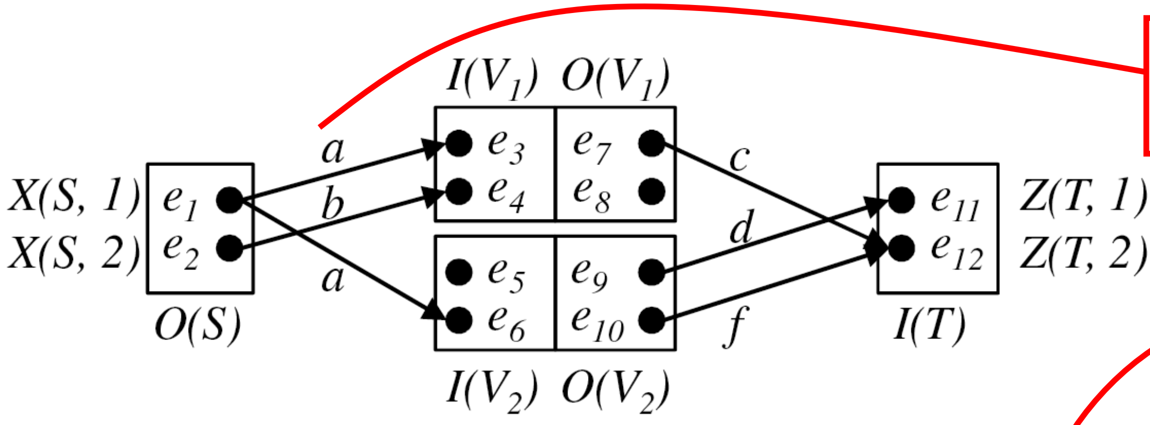
Dealing with Interference

- [Avestimehr et al. ‘07]“Deterministic model” (ADT model)
  - Interference
  - Does not take into account channel noise
  - In essence, high SNR regime
  - Requires optimization over a large set of matrices
  - Code construction algorithms [Amaudruz et al. ‘09][Erez et al. ’10]
  - Matroidal [Goemans ‘09]
- High SNR: interference is the main issue
  - Noise → 0
  - Large gain
  - Large transmit power
  - Interference as a code



Network Model

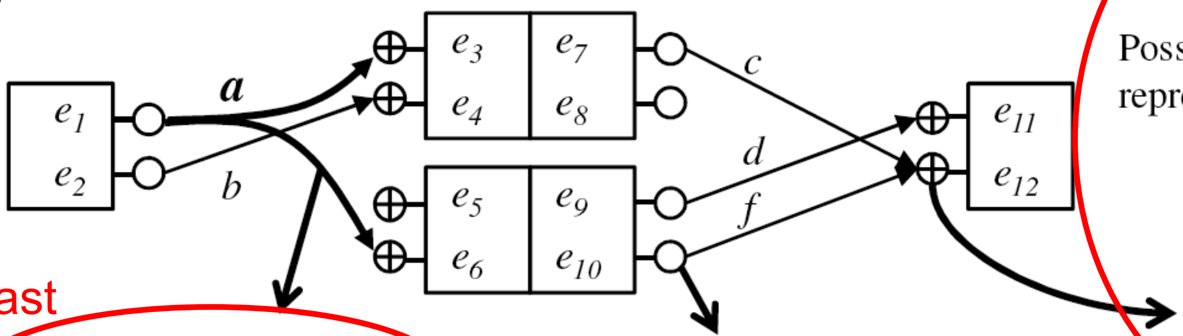
- Original ADT model:
  - Broadcast: multiple edges (bit pipes) from the same node
  - Interference: additive MAC over binary field



Higher SNR: S-V_1

Higher SNR: S-V_2

- Algebraic model:



broadcast

Use hyperedges to model broadcast constraint

0/1 decision: To transmit or not

Interference is a code

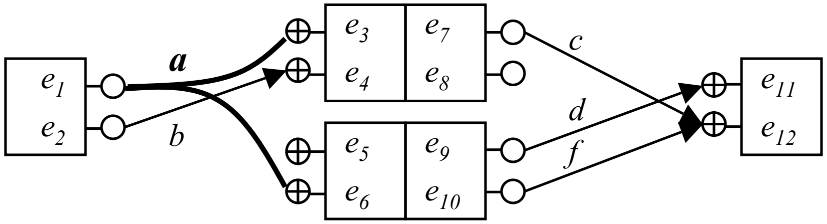
Possible "codes" at e_{12}, which represents the MAC constraint

c	f	$\oplus$
0	0	0
0	1	c
1	0	f
1	1	$c+f$

Using Network Coding with Interference Code

- Connection to Algebraic Network Coding [Koetter and Médard. ‘03]:
  - Use of higher field size
  - Model broadcast constraint with **hyper-edges**
  - Capture ADT network problem with a single ***system matrix*** M
    - Prove that min-cut of ADT networks = rank(M)
    - Prove Min-cut Max-flow for unicast/multicast holds
    - Extend optimality of linear operations to **non-multicast** sessions
    - Incorporate **failures and erasures**
    - Incorporate **cycles**
  - Show that **random linear network coding** achieves capacity
  - Do not prove/disprove ADT network model’s ability to approximate the wireless networks; but show that ADT network problems can be captured by the algebraic network coding framework

System Matrix $M = A(I - F)^{-1}B^T$



- Linear operations
  - Coding at the nodes V: $\beta(e_j, e_{j'})$
    - F represents physical structure of the ADT network
    - F^k: non-zero entry = path of length k between nodes exists
    - $(I-F)^{-1} = I + F + F^2 + F^3 + \ldots$: connectivity of the network (impulse response of the network)

$$F = \begin{pmatrix}
0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \beta_{(e_3,e_7)} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \beta_{(e_4,e_7)} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_{(e_6,e_9)} & \beta_{(e_6,e_{10})} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

Broadcast constraint (hyperedge)

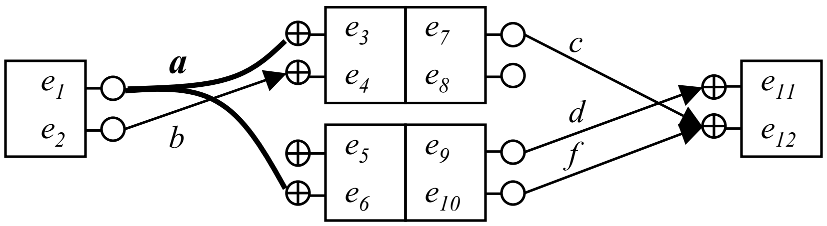
MAC constraint (addition)

Internal operations (network code)

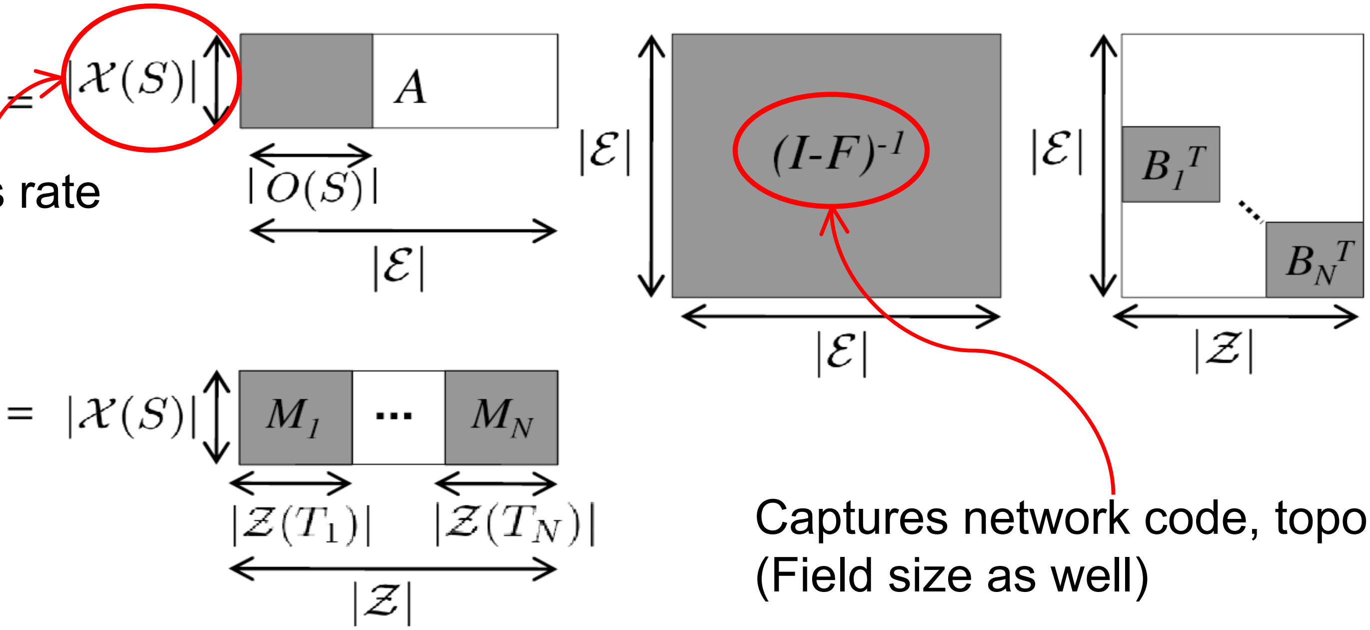
System Matrix $M = A(I - F)^{-1}B^T$

- Input-output relationship of the network

$$\boxed{Z = X(S)\quad}$$
$$M$$



System matrix $M = A\,(I\text{-}F)^{-1}\,B^T$



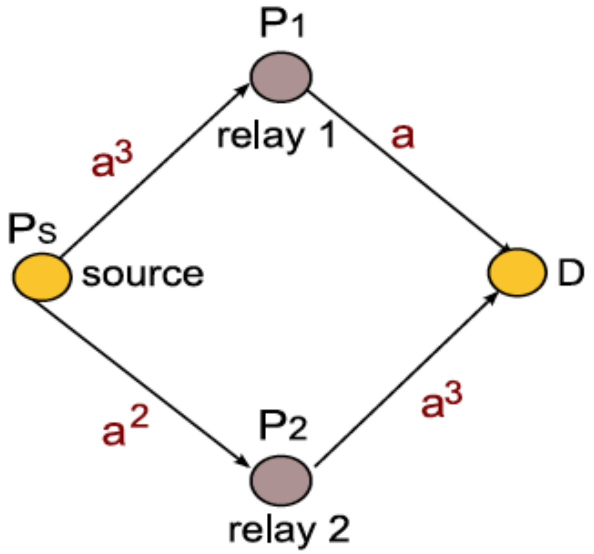
Captures rate

Captures network code, topology
(Field size as well)

Network Coding and ADT

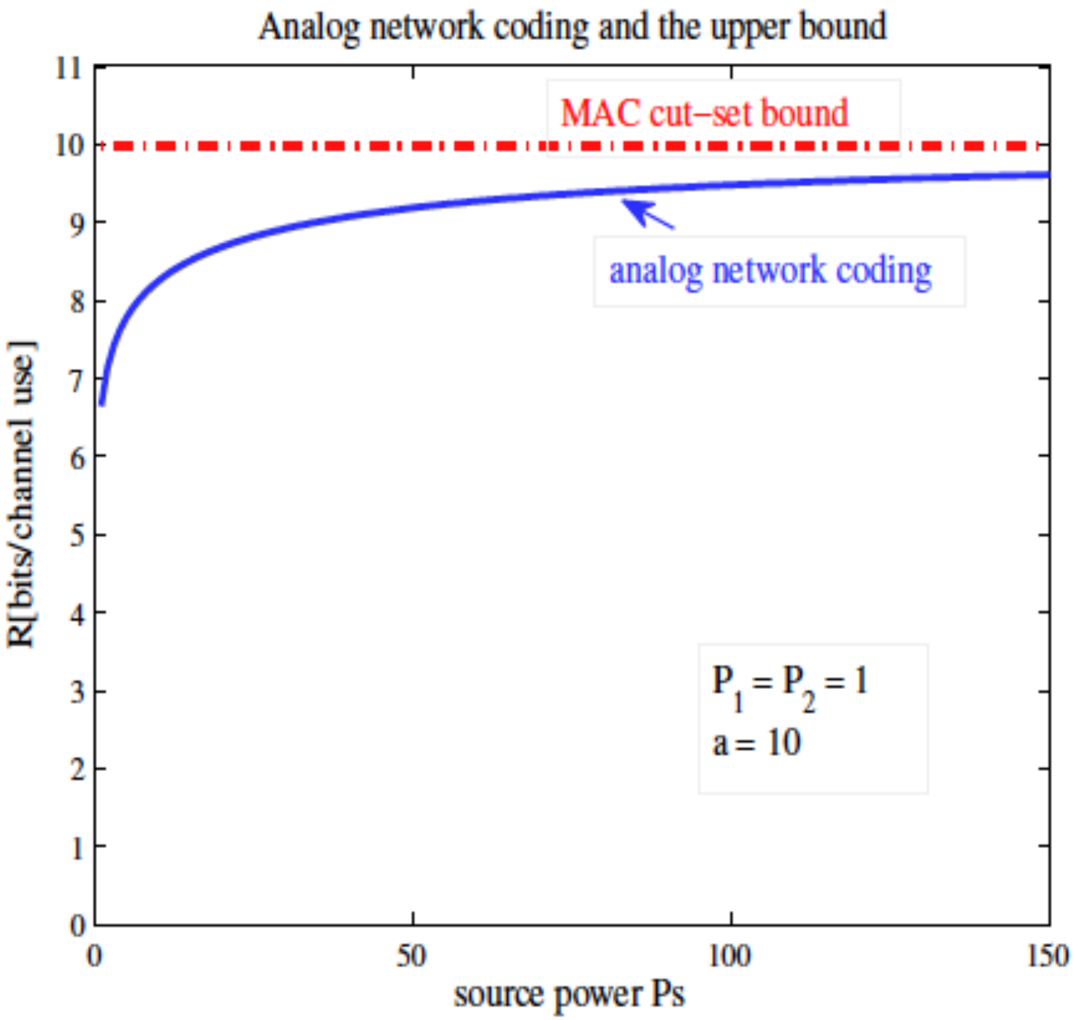
- ADT network can be expressed with Algebraic Network Coding Formulation [Koetter and Médard ‘03].
  - Use of higher field size
  - Model broadcast constraint with **hyper-edge**
  - Capture ADT network problem with a single ***system matrix*** M
- For a unicast/multicast connection from source S to destination T, the following are equivalent:
  1. A unicast/multicast connection of rate R is feasible.
  2. $mincut(S,T_i) \geq R$ for all destinations T_i.
  3. There exists an assignment of variables such that M is invertible.
- Show that **random linear network coding** achieves capacity
- Extend optimality of linear operations to **non-multicast** sessions
  - Disjoint multicast, Two-level multicast, multiple source multicast, generalized min-cut max-flow theorem
- Incorporate **delay** and **failures** (allows cycles within the network)
- **BUT IS IT THE RIGHT MODEL**?

Different Types of SNR



- Diamond network [Schein]
- As *a* increases: the gap between analog network coding and cut set increases
- In networks, increasing the gain and the transmit power are not equivalent, unlike in point-to-point links

Let SNR Increase with Input Power



Analog Network Coding is Optimal at High SNR

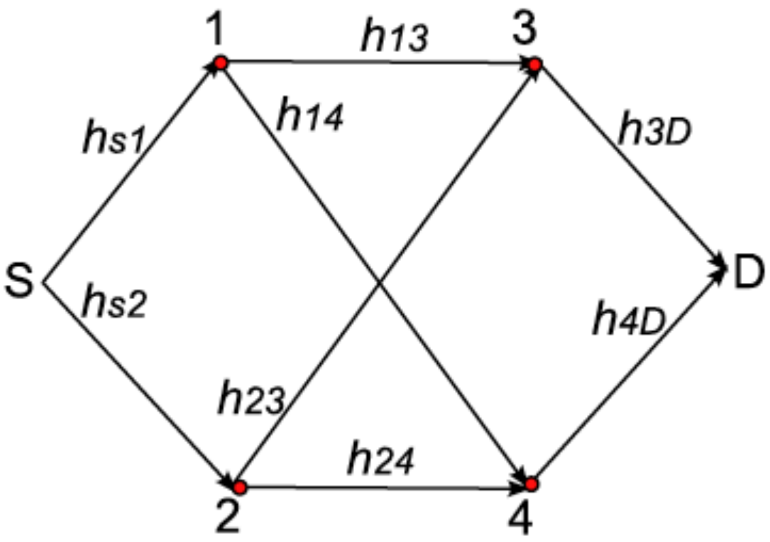
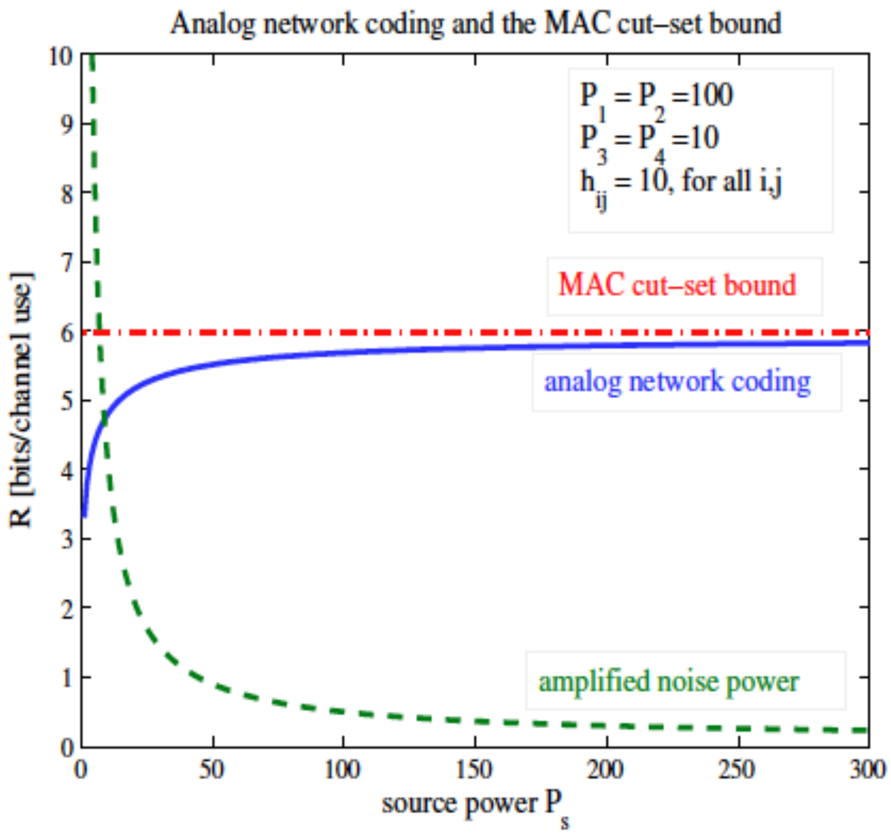
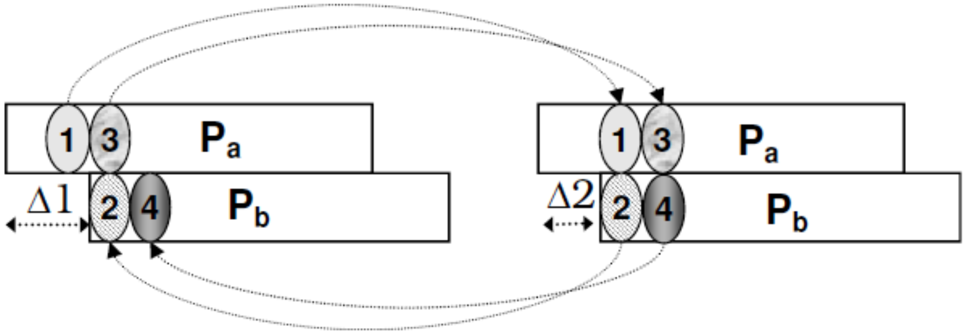


Figure: 3-layer network.



Practical Implications

- Interference management in wireless networks
  - –Simultaneous transmissions are are considered lost (collision) in most MAC protocols
  - –Collisions are normally avoided using centralized scheduling or Aloha-type mechanisms

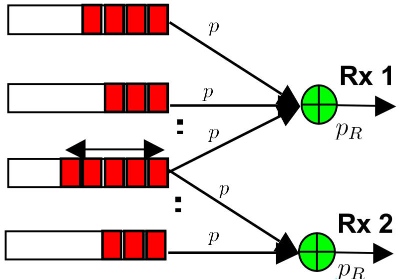


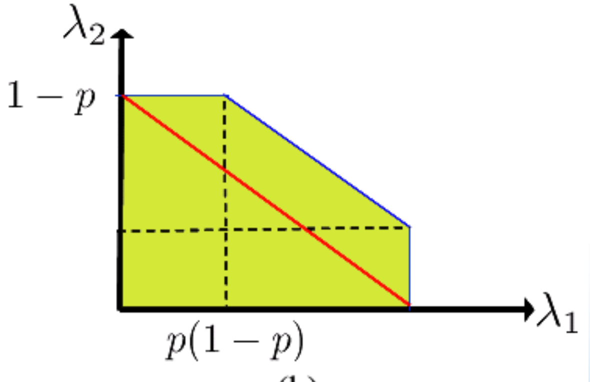
- Collision Recovery e.g. ZigZag decoding [Gollakota el 2008]
  - –Algebraic representation of the collisions
  - –Combine finite-field network coding with analog network coding (in the form of collisions)

Practical Implications

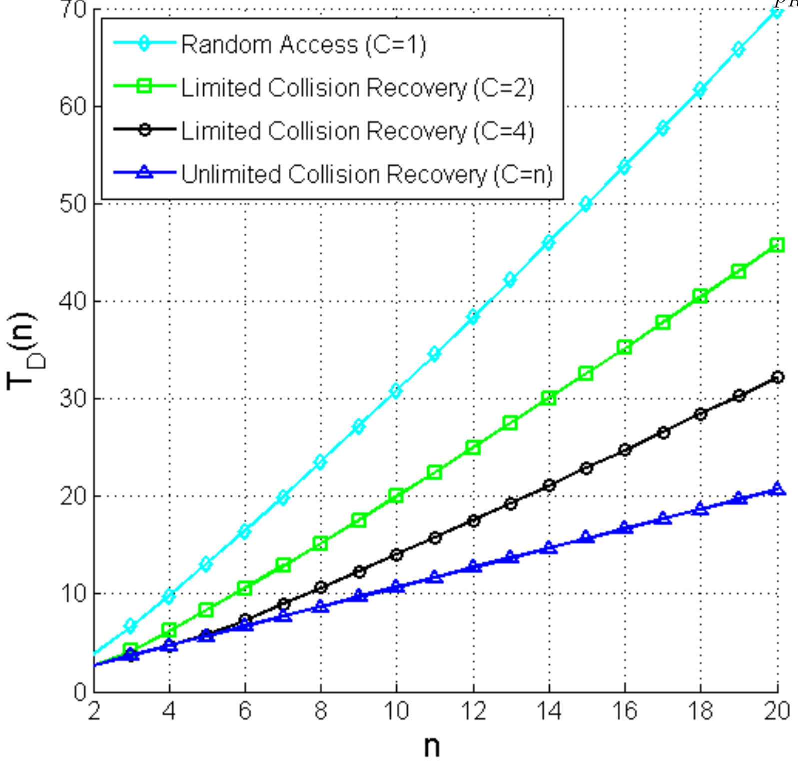
Stability Region: Achieve cut-set bound

- –Exploit the diversity gain of the links to different senders by allowing more simultaneous transmissions
- –Priority-based ack
- –Each sender broadcasts a random linear combination of packets
- –ACK seen packets
- –Throughput and completion improvement without sender coordination



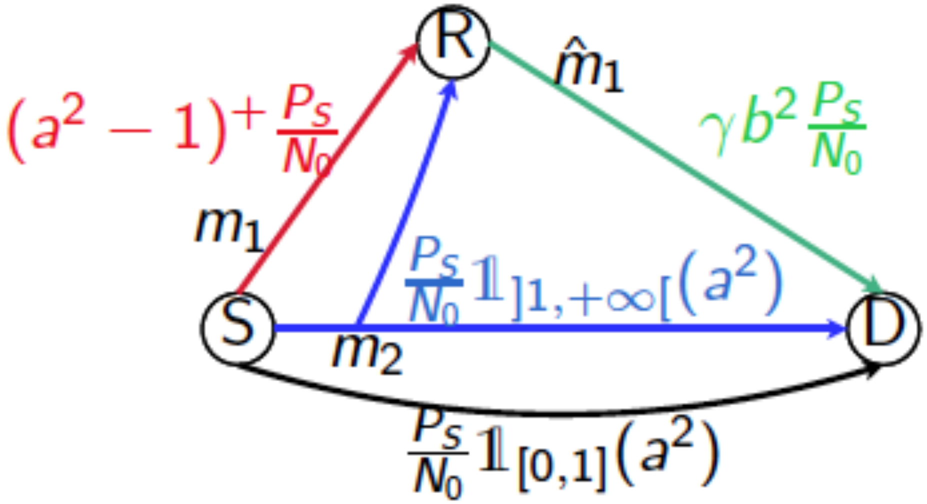


(b)



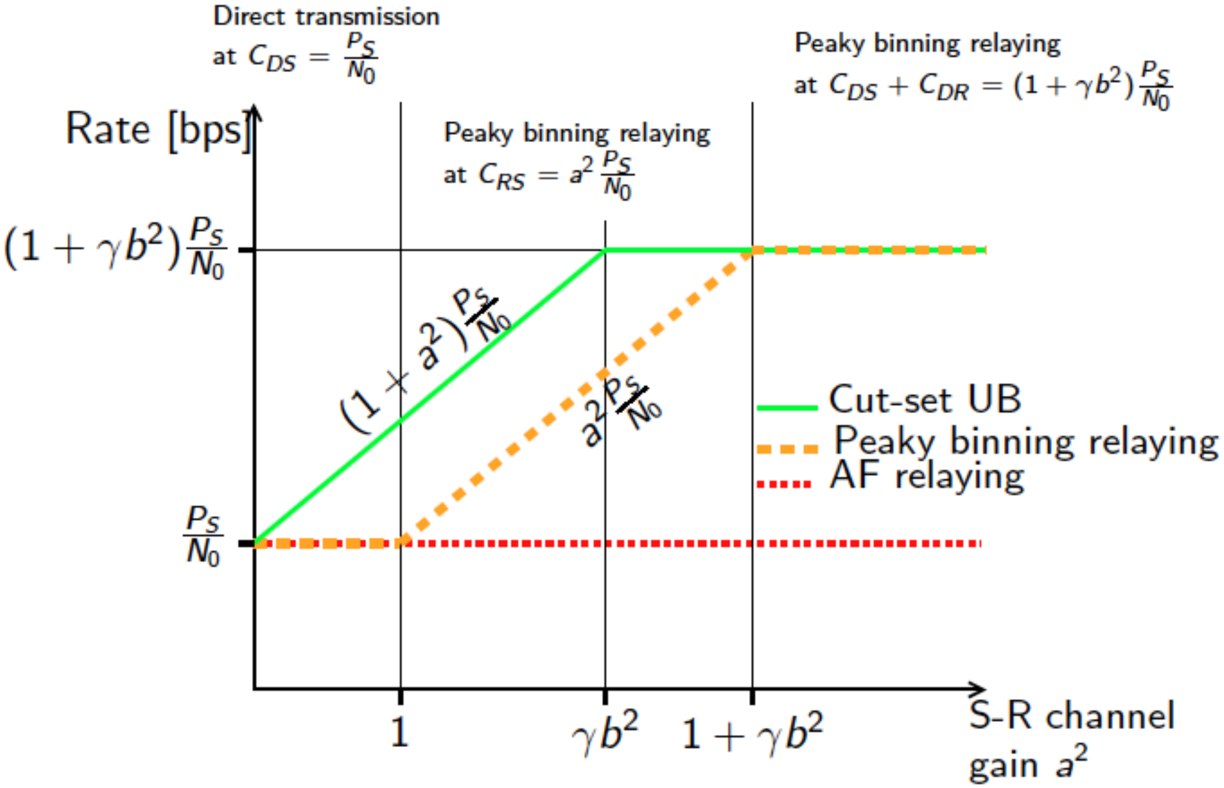
When noise is the main issue

- Consider again hyperedges
- At high SNR, interference was the main issue and analog network coding turned it into a code
- At low SNR, it is noise



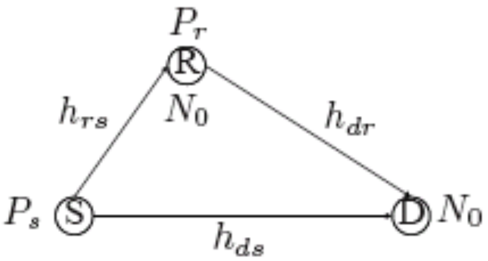
Peaky Binning Signal

- Non-coherence is not bothersome, unlike the high-SNR regime



What Min-cut?

- Open question: Can the gap to the cut-set upper-bound be closed?
- An ∞ capacity on the link R-D would be sufficient to achieve the cut like in SIMO



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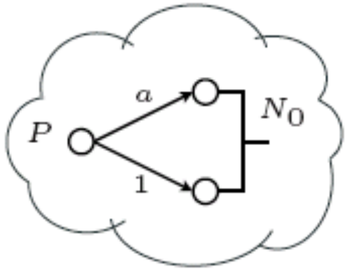


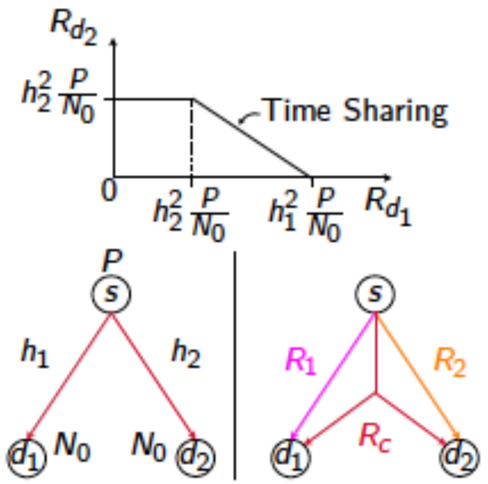
- Because of power limit at relay, it cannot make its observation fully available to destination.
- Implications for virtual MIMO scaling based arguments – simple arguments based on constant quantization do not work
- Relay channel in low SNR /wideband regime:
  - At low SNR, cut-set upper-bound = virtual MIMO with perfect channel R-D, is not achievable
  - Block Markov DF/ peaky binning hypergraph lower-bound is tight = capacity
- Converse**: cannot reach the cut-set upper-bound**

Converse

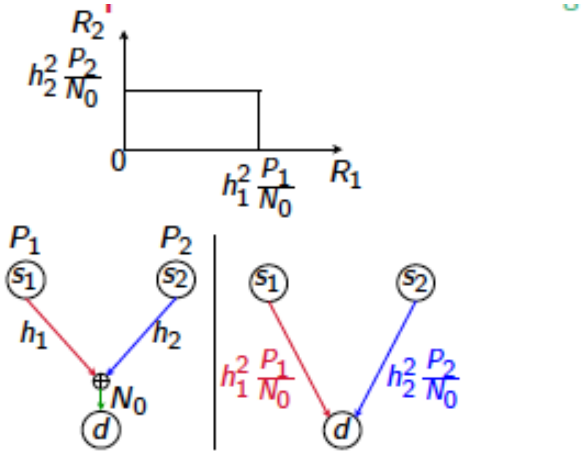
- Sketch of proof:
  - Assuming that the relay cannot decode:
  - Split total mutual information into two parts
    - contribution from relay
    - remaining contribution from source after deducting contribution from relay
  - Bound contributions **using equivalence theory** and rate distortion theory, in particular to justify
    - Gaussian input at source
    - Estimation with distortion at relay
    - Error-free R-D link with finite capacity
- Analyze the limit of these contributions in the low SNR regime and show that the total converges to the direct link capacity
- Conclusion: **the relay should decode in the low SNR and we do Network Coding in the digital domain at low SNR**

Low-SNR Approximation

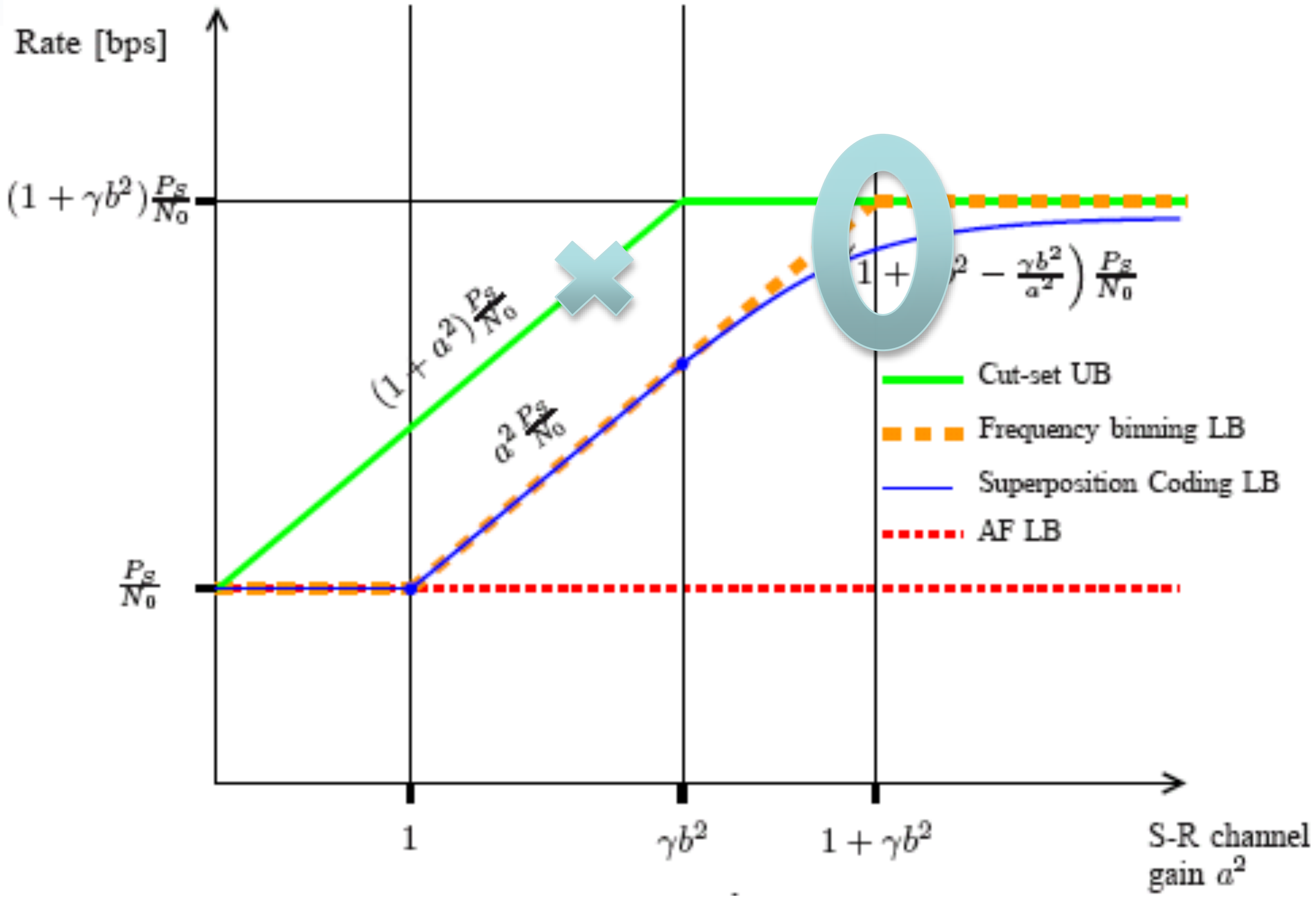
- Broadcast:
  - Superposition coding rates ~ time-sharing rates
  - Common rate received by both destinations rate received only by the most reliable destination



- Multiple access
  - No interference, FDMA
  - Both sources achieve same rate as in the absence of the other user

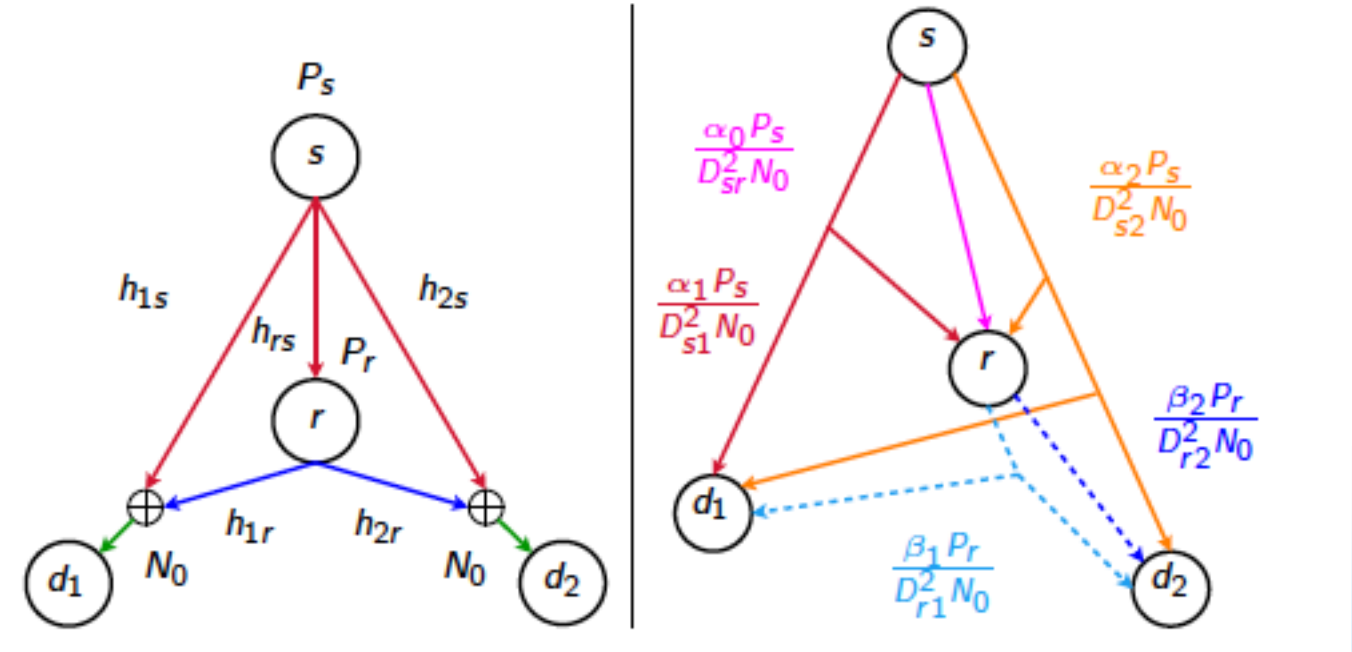


Achievable Rates



Practical Implications

- Achievable hypergraph model: Superposition coding, FDMA.
- Network coding over the subgraph
- Multicommodity flow optimization => Linear program for simple costs (network power, linear cost functions etc.).
- Separable dual => decentralized solutions.
- Hypergraph model facilitates network coding => power savings, increased throughput and reliability.



What About Other Regimes?

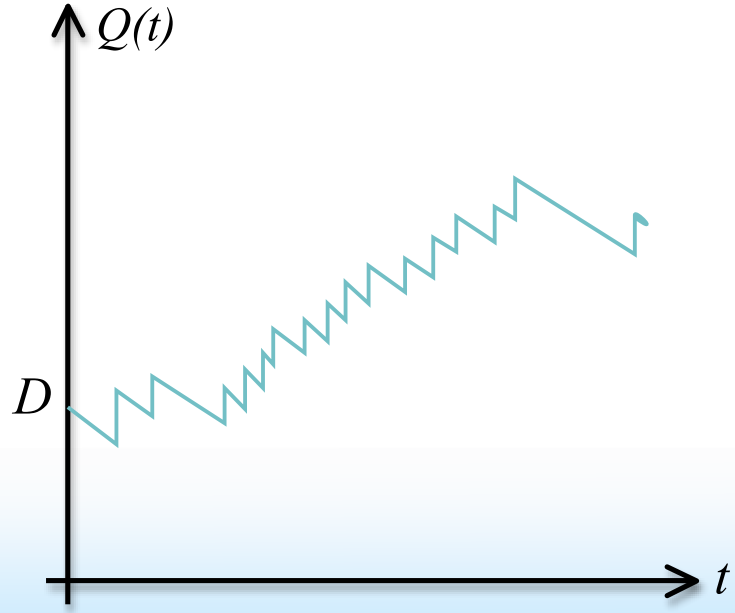
- Finding tight bounds when the model is unknown may be difficult
- We can still use coding to deal with uncertainty – **go to higher layers**
- Single server, single receiver, media streaming
- Media file consisting of T packets
- Packet arrivals: Poisson process with rate R (bandwidth)
- Media playback: Deterministic with rate R_p (resolution)
- Initially buffer D packets,

  then start the playback

$$Q(t) = D + N(t) - R_p t$$

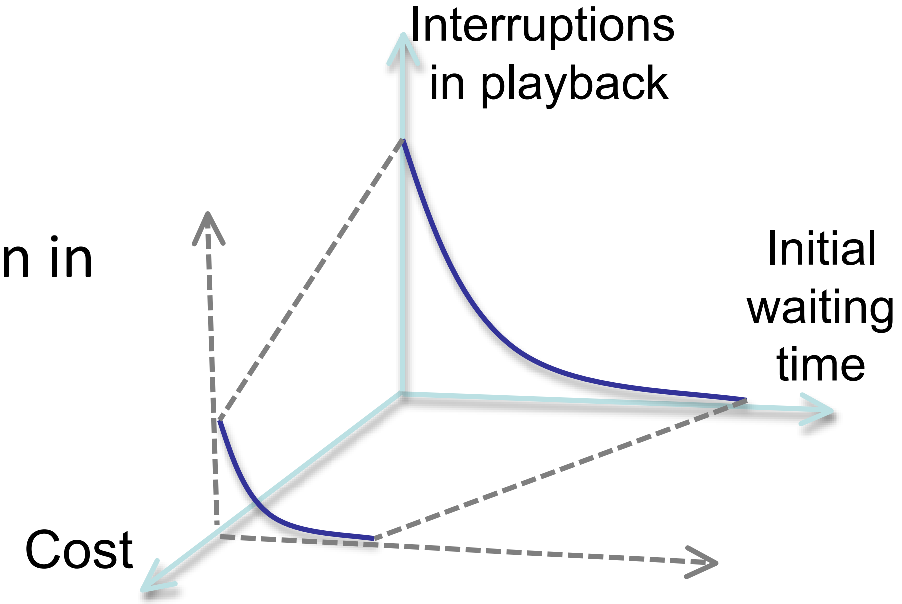
- M/D/1 queue dynamics at the receiver

$$N(t) \sim \mathrm{Poiss}(Rt)$$



What About Other Regimes?

- **Setup:** User initially buffers a fraction of the file, then starts the playback
- **QoE metrics:**
  1. Initial waiting time
  2. Probability of interruption in media playback



- Homogeneous access cost *:

$$P_{\text{int}} = e^{-I(R)D}, \quad \forall R > 1$$

- Heterogeneous access cost: Design resource allocation policies to minimize the access cost given QoE requirements

System Model

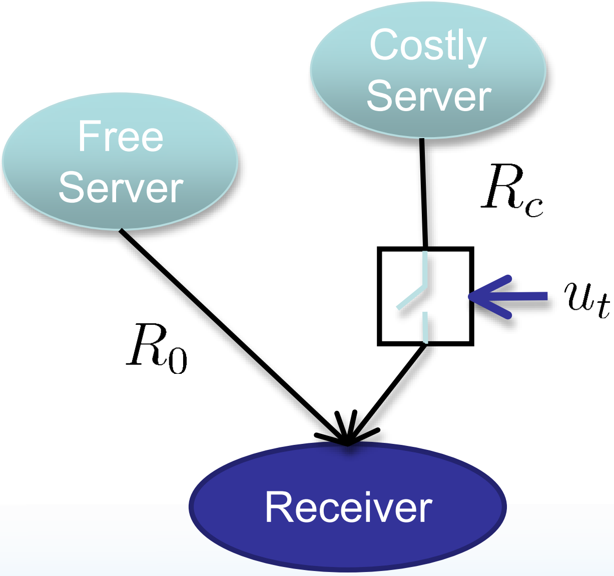
- Two classes of servers, single receiver
- Packet arrivals: Independent Poisson processes
- Media playback: Deterministic with unit rate
- Initially buffer D packets, then start the playback
- QoE requirement: $P_{\text{int}} \leq \epsilon$
- Control action:

$$u_t \in \{0, 1\}$$

$u_t = 1$ iff the costly server is used

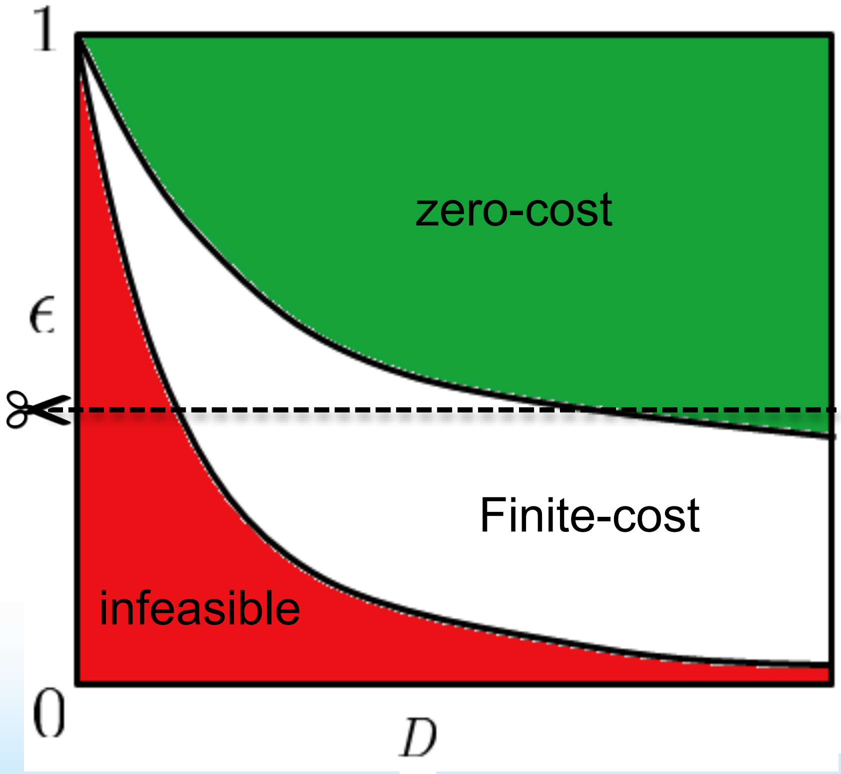
- Objective: Find control policy π to minimize the usage cost, while meeting QoE requirements

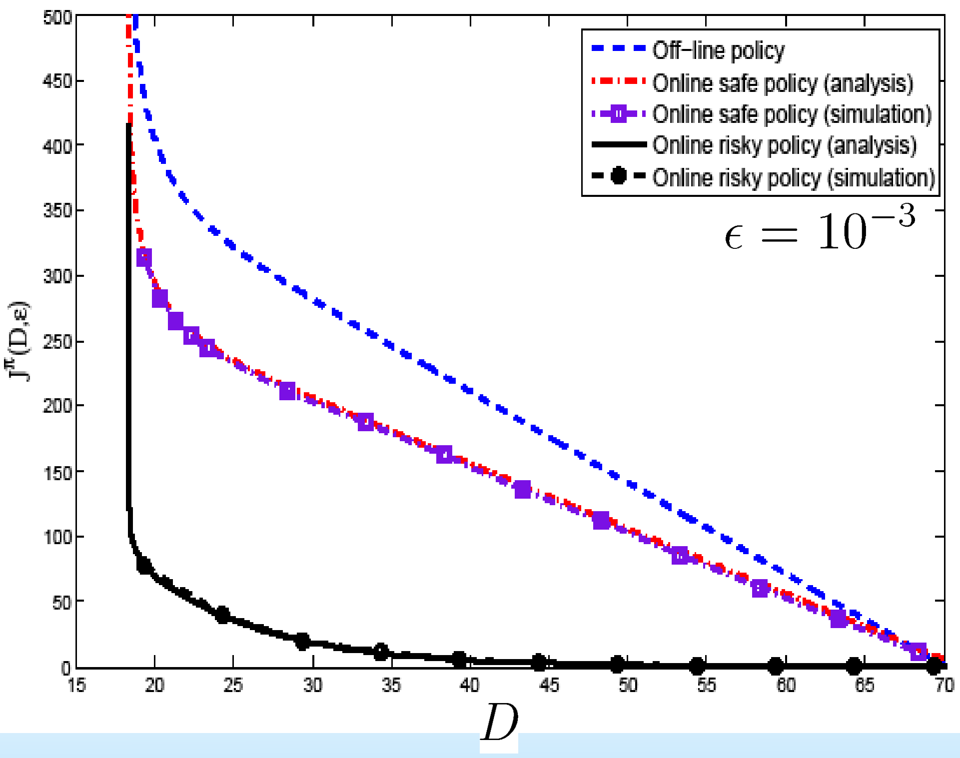
$$J^{\pi}(D, \epsilon) = \mathbf{E}\Big[\int_0^{\infty} u_t dt\Big]$$



Performance Comparison

- Three regimes for QoE metric (D, ϵ)
  1. Zero-cost $\epsilon \geq e^{-I(R_0)D}$
  2. Infeasible (infinite cost) $\epsilon \leq e^{-I(R_0+R_c)D}$
  3. Finite-cost $e^{-I(R_0+R_c)D} \leq \epsilon \leq e^{-I(R_0)D}$





Conclusions

- Interference: using it as a code in the high SNR case
  - Code in deterministic model
  - Code in analog amplify and forward
  - Practical implication: coding with zig-zag decoding
- Broadcast: building subgraphs in low SNR networks
  - Optimality of decode-and-forward
  - Practical implication: low-SNR optimization in node placement
- When physical channel models do not suffice:
  - We can still apply information theory and optimization to the higher layers effectively