Universal decoding with erasures in MANETs: Compound fading channels

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Compound Fading Multiple-Access Channel

- K transmitters, 1 receiver, block fading model
- Send $\mathbf{x}_1, \cdots, \mathbf{x}_K \in \mathbb{R}^n$, receive

$$\mathbf{y} = \sum_{k=1}^{K} h_k \, \mathbf{x}_k + \mathbf{e}$$

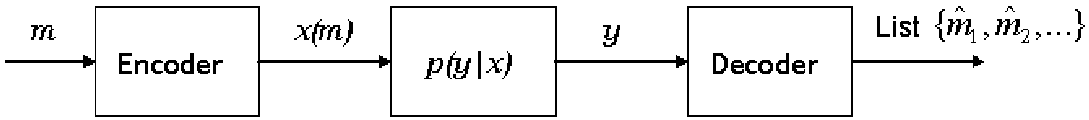
  where h_k, $1 \le k \le K$ are *unknown* real fading coefficients
  $\mathbf{e}$ is AWGN

- Cannot do ML decoding
- GLRT is universal for $K = 1$

Unreliable and Uncertain Links

- Decision space includes *erasure* option (acknowledging unreliable transmission)
- Error exponents
- Seek universal encoders and decoders
- Problem solved for discrete alphabets (Austin, TX, Dec. 07)
- Can achieve Forney's list/erasure decoding exponents for $K = 1$, at high rates

Universal List/Erasure Decoding



- Equienergetic codewords
- Type-I exponent for outputting **incorrect messages**

$$E_{\mathrm{i}}(R) \triangleq \liminf_{N\to\infty} -\frac{1}{N} \log \mathbb{E}[N_{\mathrm{i}}]$$

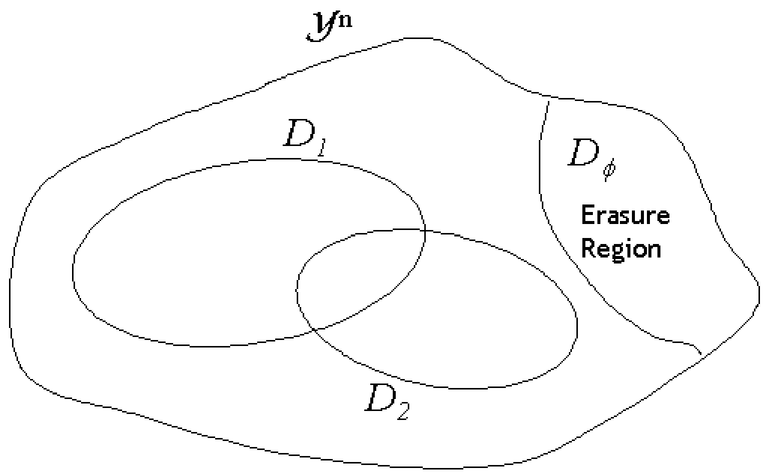
- Type-II exponent for **erasures**

$$E_{\emptyset}(R) \triangleq \liminf_{N\to\infty} -\frac{1}{N} \log Pr[\mathcal{E}_{\emptyset}]$$

- What is the (**Neyman-Pearson**-like) decoding rule that optimizes the fundamental tradeoff between these exponents?

Achievements

- Derived asymptotic NP rule for single-user channel



- This rule coincides with Forney's rule at high rates
- Derived asymptotic NP rule for multiple-access channel

Single-User Channels

- Define empirical mutual information for $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$ as

$$I(\mathbf{x};\mathbf{y}) \triangleq -\frac{1}{2}\log(1-\rho^2(\mathbf{x},\mathbf{y}))$$

where $\rho(\mathbf{x},\mathbf{y}) = \frac{\mathbf{x}\cdot\mathbf{y}}{\|\mathbf{x}\|\,\|\mathbf{y}\|}$ = normalized correlation

- Universal variable-size list **decoding rule**

$$g_F(\mathbf{y}) = \begin{cases} \hat{m} & : \text{ if } I(\mathbf{x}(\hat{m});\mathbf{y}) > R + \max_{i\neq\hat{m}} F(I(\mathbf{x}(i);\mathbf{y}) - R) \\ \emptyset & : \text{ else.} \end{cases}$$

(same as for finite alphabets) variation on MMI rule, parameterized by nondecreasing function F

- Special case: $F(t) = \Delta + \lambda\,|t|^+$, $\lambda \geq 1$ (Csiszár and Körner)
- Solve $\max_F \min_h E_{\mathrm{i}}(R,h,F)$ subj. to $\min_h E_\emptyset(R,h,F) \geq \alpha$

Error Exponents

- First assume $h_{\min} \le h \le h_{\max}$
- Sphere-packing exponent $E_{sp}(R, h)$
- *Modified random coding exponent* $E_{r,F}(R, h)$
- (not necessarily unique) NP-optimal choice of F:

$$F^*(t) = \Delta + E_{sp}(R, h_{\min}) - E_{sp}(R + t, h_{\min})$$

- Closed-form expressions for corresponding error exponents $E_{\mathrm{i}}(R, h, F^*)$ and $E_{\emptyset}(R, h, F^*)$
- Yields m.i. thresholding rule $F^*(t) = \Delta$ when $h_{\min} = 0$

Universality

- **Proposition.** Assume that R, $h_{\min}$, $h_{\max}$, Δ and λ are such that

$$\begin{aligned} |\overline{R}^{conj}(h_{\max}) - R|^{+} \quad &\leq \Delta \quad \leq C(h_{\min}) - R, \\ -\underline{E}'_{sp}(R, h_{\max}) \quad &\leq \lambda \quad \leq \frac{1}{-\underline{E}'_{sp}(R+\Delta, h_{\max})}. \end{aligned}$$

Then the Forney incorrect-message and erasure exponents:

$$E_{sp}(R,h) + \Delta \quad \text{and} \quad E_{sp}(R+\Delta, h),$$

are achieved uniformly over $h \in [h_{\min}, h_{\max}]$ using the penalty function $F(t) = \Delta + \lambda |t|^{+}$.

Relative Minimax

- The minimax approach is pessimistic when $h_{\min} = 0$
- Choose "reference exponents" $\alpha(h)$ and $\beta(h)$
- Find F that optimizes tradeoff between the *relative exponents*

$$E_{\text{i}}(R, h, F) - \alpha(h) \quad \text{and} \quad E_{\emptyset}(R, h, F) - \beta(h)$$

- Example: Forney reference exponents

$$\alpha(h) = E_{sp}(R, h) + \Delta \quad \text{and} \quad \beta(h) = E_{sp}(R + \Delta)$$

- Then

$$F^*(t) = \Delta + E_{sp}(R, h_{\max}) - E_{sp}(R + t, h_{\max})$$

Multiple-Access Channels

- Decoding with erasures is an open problem for MAC
- Define collection of nondecreasing functions $F_{\mathcal{A}}(t)$, $\mathcal{A} \subseteq \mathcal{K}$
- Example (generally suboptimal):

$$F_{\mathcal{A}}(t) = \Delta\mathsf{R}(\mathcal{A}) + \lambda|t|^{+}$$

- Seek NP-optimal F

Universal Decision Rule

- Test **all possible subsets** of input terminals:

$$g_F(\mathbf{y}) =$$

$$\begin{cases} (\hat{\mathcal{K}}, \hat{m}_{\hat{\mathcal{K}}}) & : \text{ if } \forall \mathcal{A} \subseteq \hat{\mathcal{K}} \ : \ \overset{\circ}{I}^{(|\mathcal{A}|+1)}(\mathbf{x}(\hat{m}_{\mathcal{A}}); \mathbf{y}\mathbf{x}(\hat{m}_{\hat{\mathcal{K}}\setminus\mathcal{A}})) > \mathsf{R}(\mathcal{A}) + \\ & \quad \max_{\{i_k \neq \hat{m}_k \ \forall k \in \mathcal{A}\}} F_{\mathcal{A}} \left(\overset{\circ}{I}^{(|\mathcal{A}|+1)}(\mathbf{x}(i_{\mathcal{A}}); \mathbf{y}\mathbf{x}(\hat{m}_{\hat{\mathcal{K}}\setminus\mathcal{A}})) - \mathsf{R}(\mathcal{A}) \right) \\ (\emptyset, \emptyset) & : \text{ else} \end{cases}$$

where

$$\begin{aligned} \overset{\circ}{I}^{(K)}(\mathbf{x}_1; \mathbf{x}_2; \cdots; \mathbf{x}_K) &= \text{empirical m.i. between } K \text{ r.v.}'\text{s} \\ &\triangleq -\frac{1}{2} \log \det \left[\rho(\mathbf{x}_j, \mathbf{x}_k)\right]_{j,k=1}^{K} \end{aligned}$$

- Define *pseudo sphere-packing exponents* $E_{psp,\mathcal{A}}(\mathsf{R}(\mathcal{A}), h)$ for all $\mathcal{A} \subseteq \mathcal{K}$
- NP-optimal choice of $F_{\mathcal{A}}$:

$$F^*_{\mathcal{A}}(t) = \Delta_{\mathcal{A}} + E_{psp,\mathcal{A}}(\mathsf{R}(\mathcal{A}), h_{\min}) - E_{psp,\mathcal{A}}(\mathsf{R}(\mathcal{A}) + t, h_{\min})$$

- Closed-form expressions for corresponding error exponents