The Capacity Region of Large Wireless Networks

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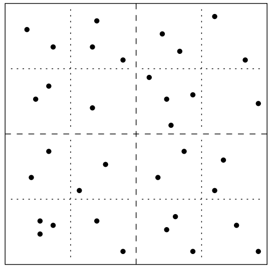
Model



- n nodes $V(n)$ placed uniformly at random on $[0, \sqrt{n}]^2$
- $y_v[t] = \sum_{u \in V(n)\setminus\{v\}} h_{u,v}[t] x_u[t] + z_v[t]$
- Additive Gaussian noise $z_v[t]$
- $h_{u,v}[t] = r_{u,v}^{-\alpha/2} \exp(\sqrt{-1}\theta_{u,v}[t])$
- Path loss exponent $\alpha > 2$
- $\{\theta_{u,v}[t]\}_{u,v}$ i.i.d. uniform over $[0, 2\pi)$
- Fast fading: $\{\theta_{u,v}[t]\}_t$ stationary ergodic
- Slow fading: $\{\theta_{u,v}[t]\}_t$ constant
- Full CSI at all nodes

Main Result

- Traffic matrix $\lambda \in \mathbb{R}^{n\times n}$
- Capacity region $\Lambda(n) \subset \mathbb{R}^{n\times n}$ (set of all achievable λ)
- Partition $[0, \sqrt{n}]^2$ into square-grids
- Grid at level ℓ has spacing $2^{-\ell}\sqrt{n}$
- $\{V_{\ell,i}(n)\}_{i=1}^{4^\ell}$ are the nodes in squares at level ℓ

Define

$$\Lambda_G(n) \triangleq \Big\{\lambda \in \mathbb{R}_+^{n\times n} : \sum_{u\in V_{\ell,i}(n)} \sum_{v\notin V_{\ell,i}(n)} (\lambda_{u,v} + \lambda_{v,u}) \leq g_\alpha(4^{-\ell} n)$$
$$\forall \ell \in \{1, \ldots, L(n)\} \cup \{\log(n)\}, i \in \{1, \ldots, 4^\ell\}\Big\},$$

where

$$L(n) \triangleq \tfrac{1}{2}\log(n)\big(1 - \log^{-1/2}(n)\big), \quad g_\alpha(r) \triangleq \begin{cases} r^{2-\min\{3,\alpha\}/2} & \text{if } r \geq 1, \\ 1 & \text{else.}\end{cases}$$

Theorem. *Under either fast or slow fading, for any $\alpha > 2$, $\varepsilon > 0$,*

$$\Omega(n^{-\varepsilon})\Lambda_G(n) \subseteq \Lambda(n) \subseteq O(n^{\varepsilon})\Lambda_G(n)$$

with probability $1 - o(1)$ as $n \to \infty$.

Examples

Multiple Classes of Source-Destination Pairs

- K classes of source-destination pairs
- Pairs in class i randomly chosen at distance $\Theta(n^{\beta_i})$ for some fixed $\beta_i \in [0, 0.5]$
- Source nodes in class i generate traffic at rate $\lambda_i(n)$

$$\Longrightarrow \quad \lambda_i^*(n) = \Theta(n^{\beta_i(2-\min\{3,\alpha\})\pm\varepsilon})$$

$\Rightarrow$ $K = 1$ and $\beta_i = 0.5$ recovers uniform random source-destination pairing

$\Rightarrow$ Compare to random source-destination pairing with $\tilde{n} \triangleq n^{2\beta_i}$

Traffic Variation with Source-Destination Separation

- Pair nodes uniformly at random into source-destination pairs
- $\lambda_{u,v} = \rho(n)\max\{1, r_{u,v}\}^\beta$ for constant $\beta \in \mathbb{R}$

$$\Longrightarrow \quad \rho^*(n) = \begin{cases} \Theta(n^{(2-\beta-\min\{3,\alpha\})/2\pm\varepsilon}) & \text{if } \beta \geq -\min\{3,\alpha\}, \\ \Theta(n^{1\pm\varepsilon}) & \text{else.}\end{cases}$$

$\Rightarrow$ For $\beta \geq -\min\{3,\alpha\}$ long distance communication is the bottleneck

$\Rightarrow$ For $\beta < -\min\{3,\alpha\}$ short distance communication is the bottleneck

Sources with Multiple Destinations

- K classes of source nodes
- Each source node in class i has $\Theta(n^{\beta_i})$ destination nodes for some constant $\beta_i \in [0, 1]$
- Generates independent traffic at the same rate $\lambda_i(n)$ for each of its destinations

$$\Longrightarrow \quad \lambda_i^*(n) = \Theta(n^{1-\beta_i-\min\{3,\alpha\}/2\pm\varepsilon})$$

$\Rightarrow$ $\beta_i = 0$ recovers uniform random source-destination pairing

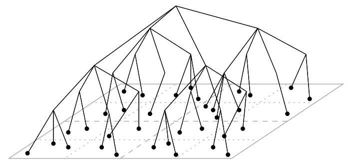
$\Rightarrow$ Long distance communication is always the bottleneck

$\Rightarrow$ For a fixed source node, time sharing between its destinations is optimal (but not across source nodes)

Communication Scheme

- Two layer communication scheme
- Top or routing layer: routes data over a tree graph
- Bottom or physical layer: provides tree abstraction
- Achieves the entire capacity region (in scaling sense)

Routing Layer



- Construct a tree $G = (V_G, E_G)$
- $V(n) \subset V_G$ are the leaves of G
- Intermediate nodes in G "represent" nodes in $V_{\ell,i}(n)$
- Hierarchy induced by nesting of grid structure

In the routing layer, messages are transmitted between $u, v \in V(n) \subset V_G$ by routing them over G.

Physical Layer

- The physical layer provides the tree abstraction G
- To send a message along an edge $e \in E_G$ towards the root, the message is "distributed" over the wireless network
- To send a message along an edge $e \in E_G$ away from the root, the message is "concentrated" over the wireless network
- This distribution/concentration is performed using cooperative communication ($\alpha \in (2, 3]$) or multi-hop communication ($\alpha > 3$).