On network capacity and the power of side information

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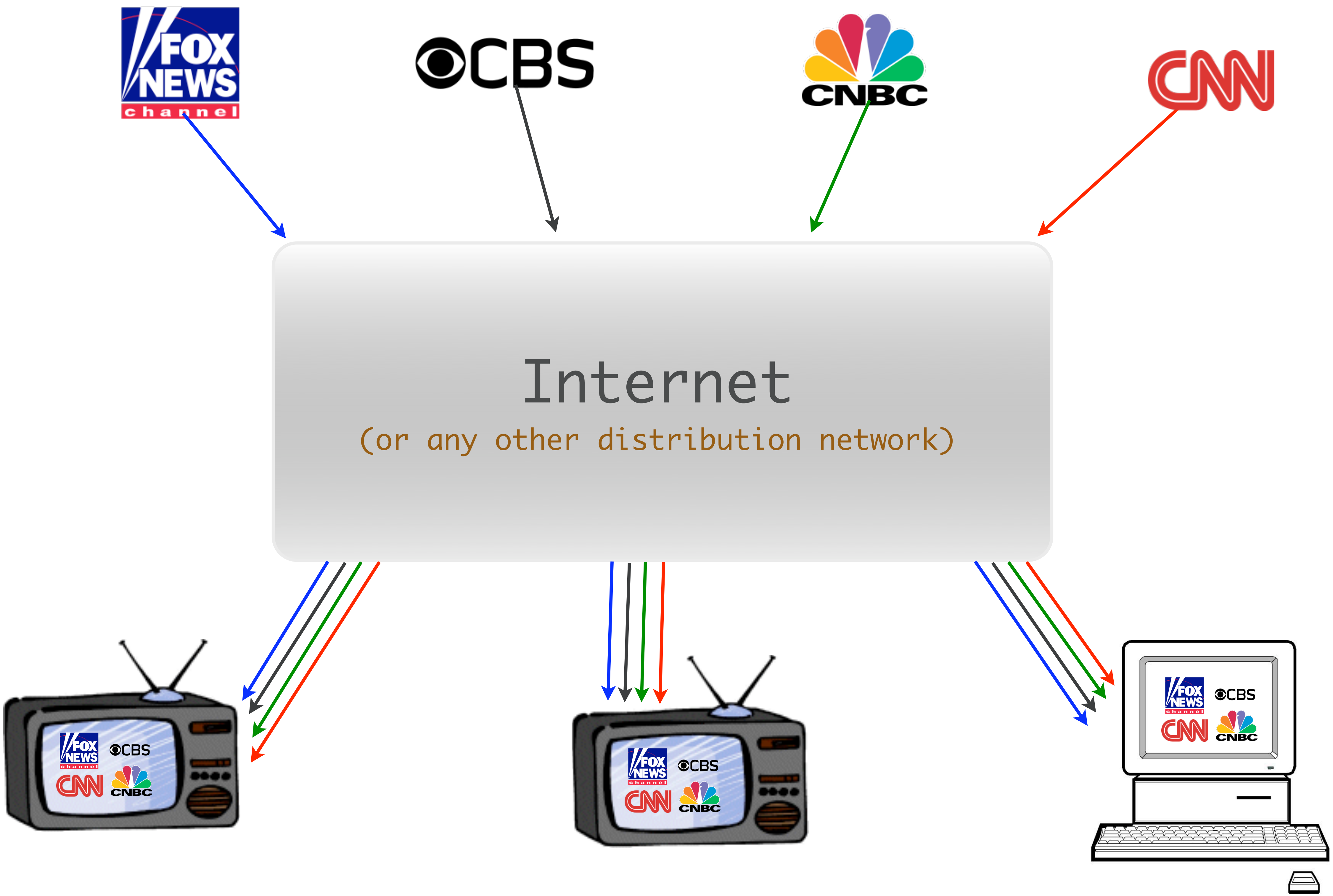
Motivation



An example where all users want everything

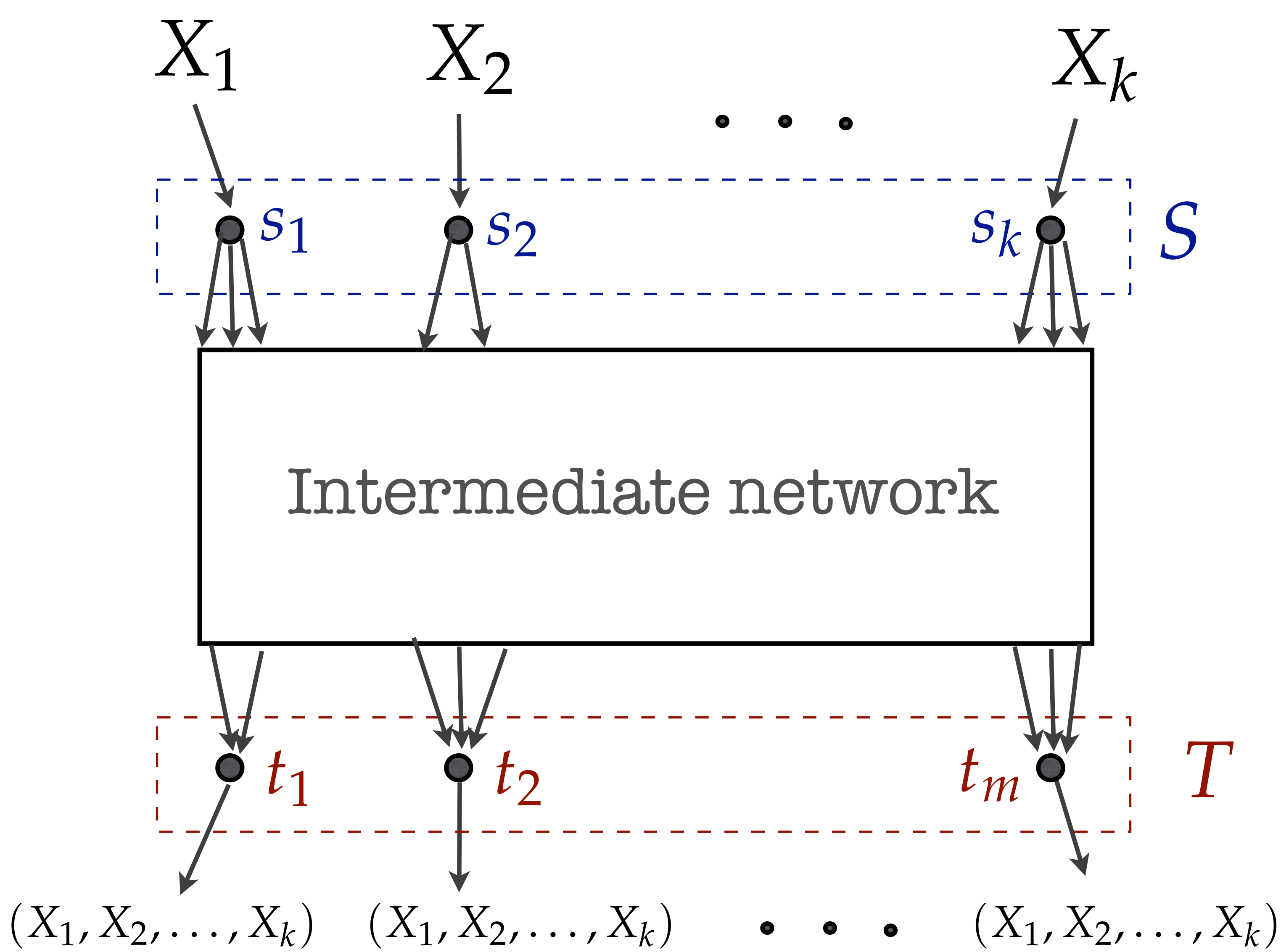
The multicast model for networks

- Directed Graph : $\mathcal{N} = (V, E)$

Vertices $V = \{1, 2, \ldots, n\}$

Edges $E = \{(1,2), (2,3), \ldots, (n-1, n)\}$

- Source nodes : $S = (s_1, s_2, \ldots, s_k)$
- Sink nodes : $T = (t_1, t_2, \ldots, t_m)$
- *Error-free* links, *capacities* $(C_e : e \in E)$



Basic question: Given $(C_e : e \in E)$, what demands are *feasible* ?

[Ho et al, 2006] :

Demands are feasible with capacities $(C_e : e \in E)$ iff

$$\sum_{e \in \Gamma_o(C)} C_e \geq H(X_{S \cap C} | X_{S \setminus C})$$

for all $C \subseteq V$.

sources already present outside C

sum of rates on the outgoing links from a subset C of V

sources present in C

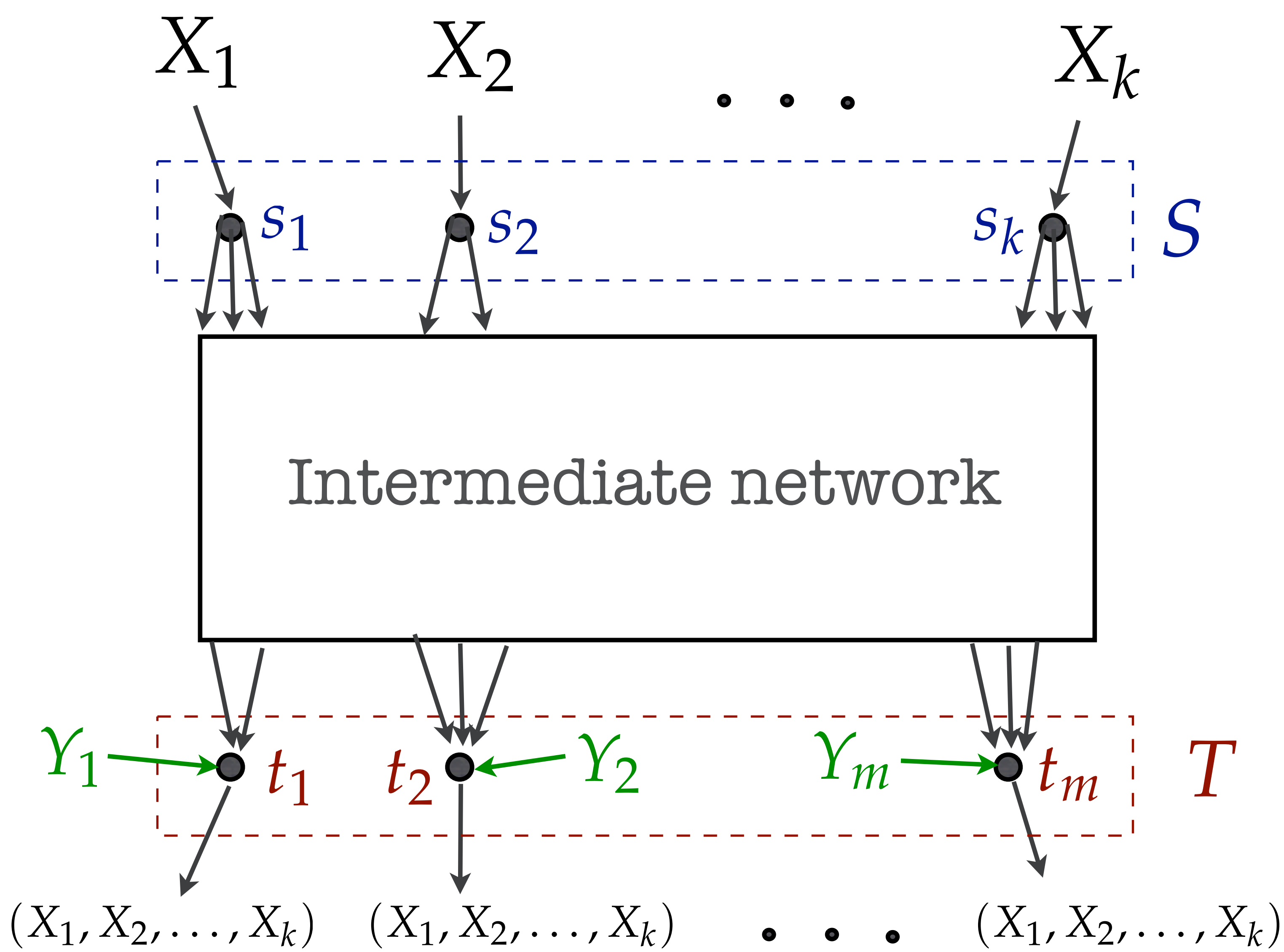
- Linear codes suffice for these rates

Problem with this model: Often, the sinks have some extra information already available, or may not want all the sources present in the network.

This work:

Generalizations of the above result to multicast with side information

Side information at sink nodes



- Source nodes : $S = (s_1, s_2, \ldots, s_k)$
- Sink nodes : $T = (t_1, t_2, \ldots, t_m)$
- Side information at sink nodes : $Y_1, Y_2, \ldots, Y_m$

Result 1 :

Demands are feasible with capacities $(C_e : e \in E)$ *iff*

$$\underbrace{\sum_{e\in\Gamma_o(C)} C_e}_{\text{sum of rates on the outgoing links from a subset } C \text{ of } V} \geq H(X_{S\cap C}|X_{S\setminus C}, Y_{T\setminus C})$$

for all $C \subseteq V$.

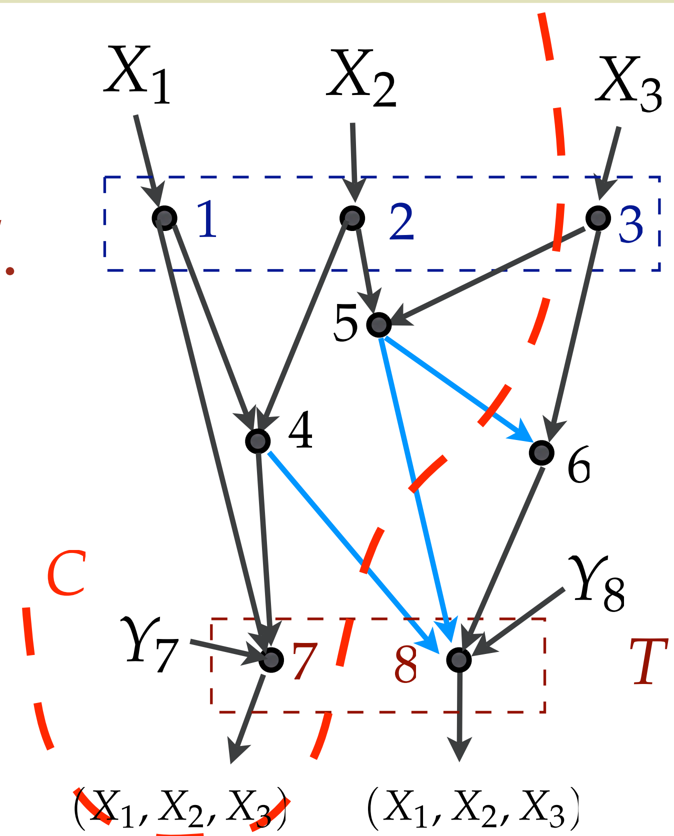
side information present outside C

sources already present outside C

sources present in C

- Linear codes suffice for achieving these rates.

- Expression similar to the result for Multicast without side information given in *[Ho et al, 2006]*.



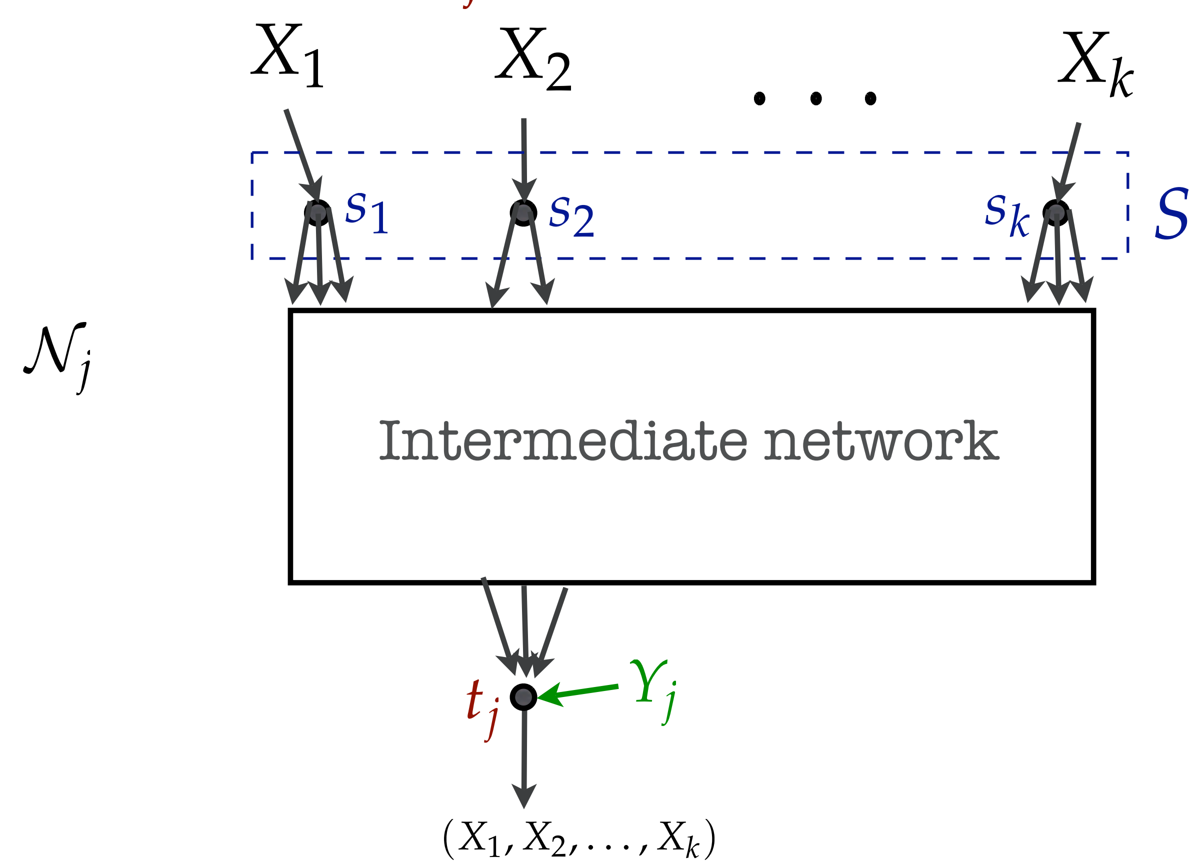
<u>Converse:</u>

- Cutset bound

  (Cover and Thomas, 1991)

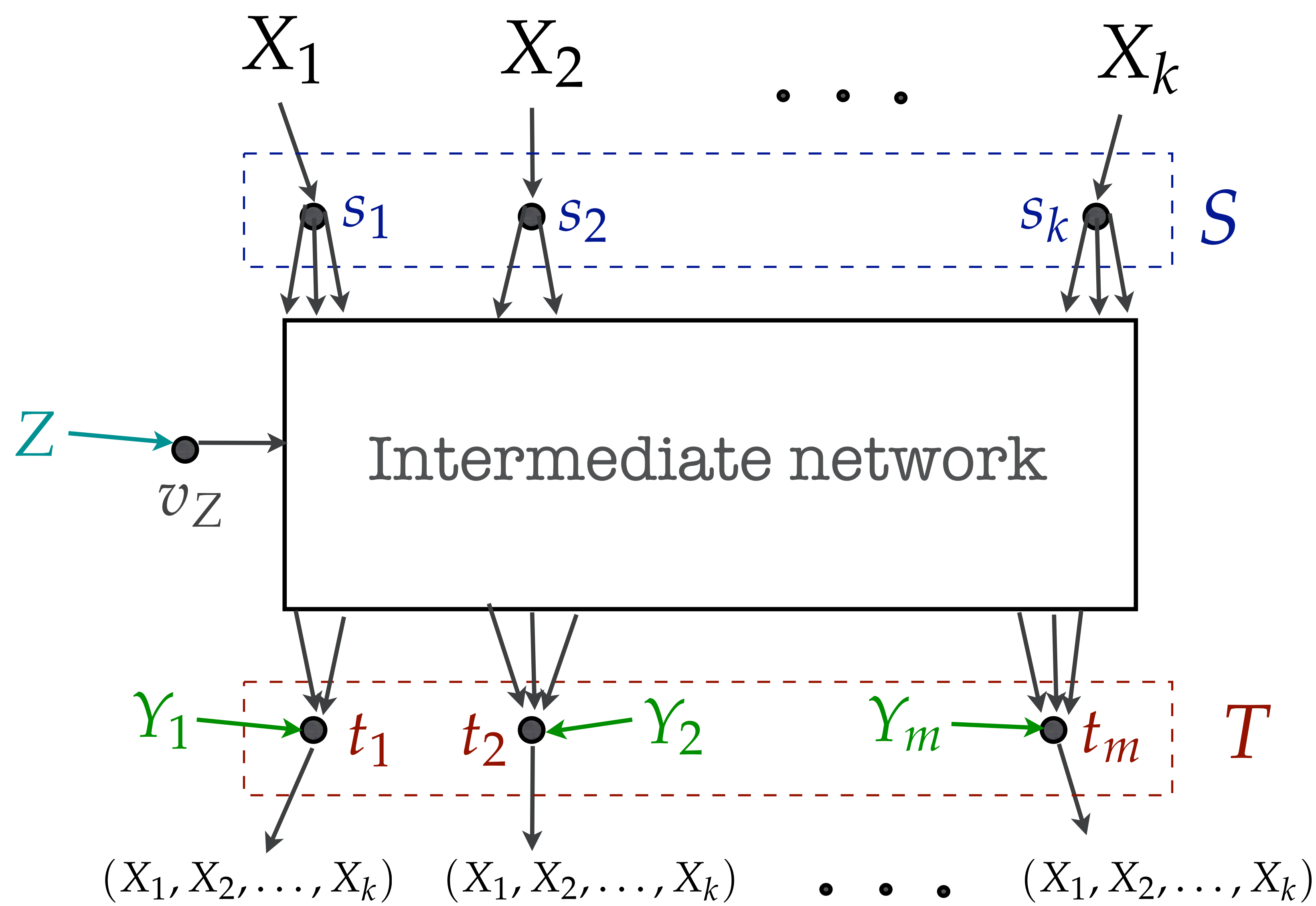
Achievability:

- network $\mathcal{N}_j$ - delete all sinks except t_j



- pure multicast network with sources $(X_1, X_2, \ldots, X_k, Y_j)$
- random linear codes for $\mathcal{N}_j$ as in [Ho et al, 2006]
  - need only input and output rates at each node
  - good for each $\mathcal{N}_j$ => good for $\mathcal{N}$
- can be done for all rates in the claimed region

Side information at a non-sink nodes

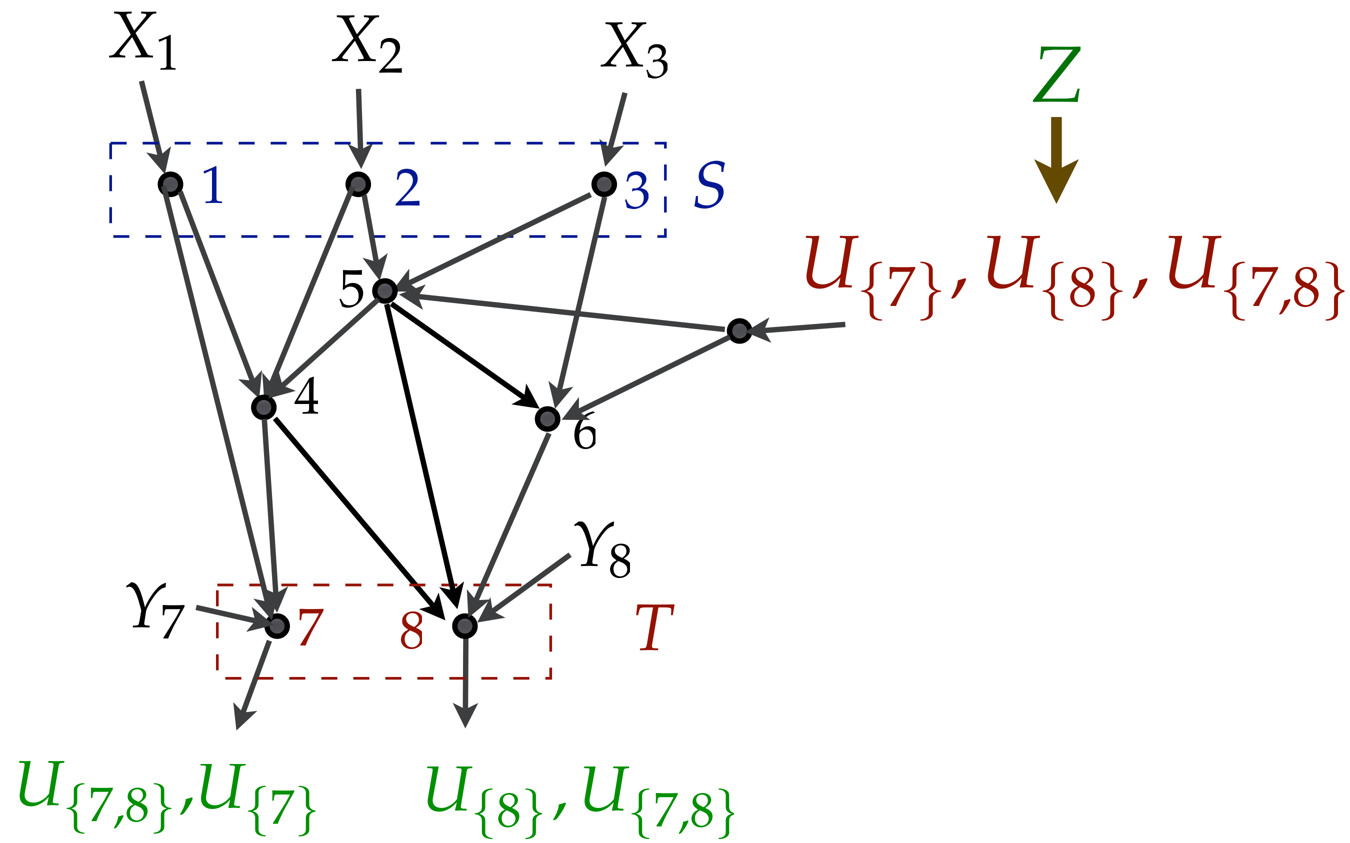


- Source nodes : $S = (s_1, s_2, \ldots, s_k)$
- Sink nodes : $T = (t_1, t_2, \ldots, t_m)$
- Side information at sink nodes : $Y_1, Y_2, \ldots, Y_m$
- Side information Z at a non-sink node v_Z

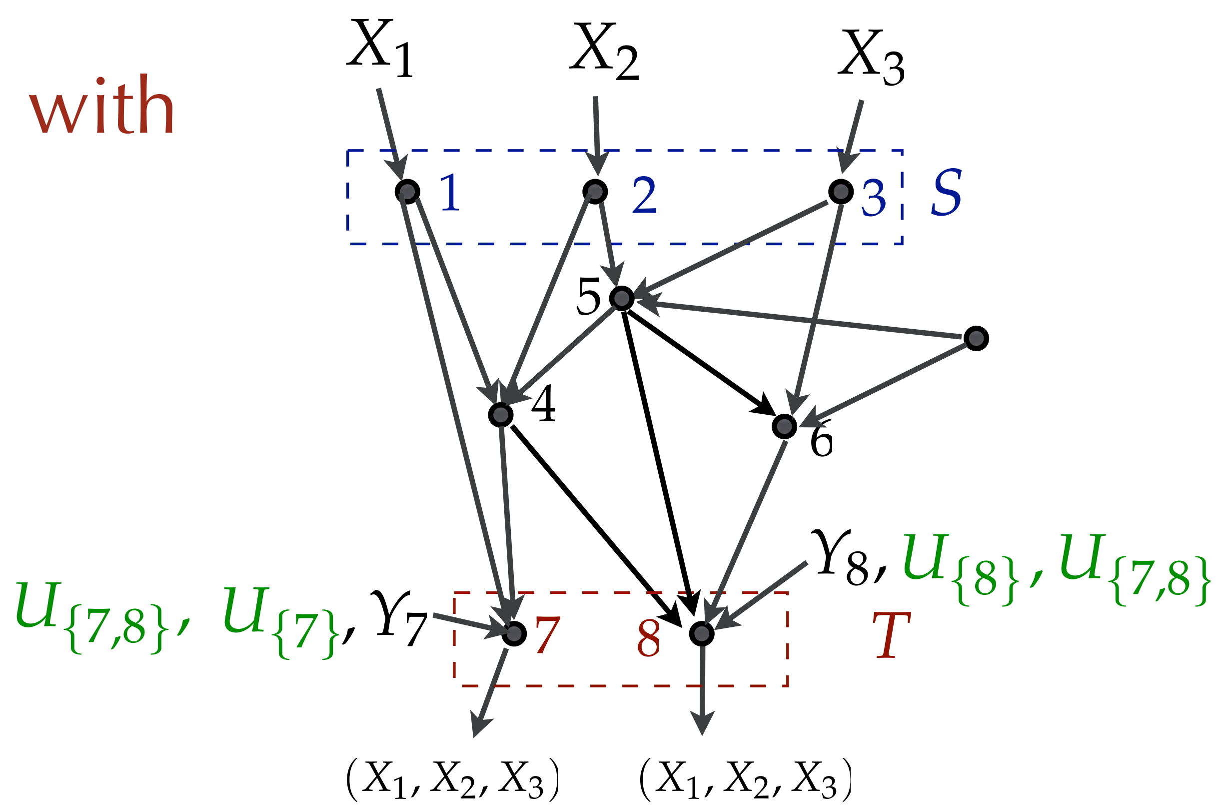
Coding scheme

- Separate codewords for each subset of the sink nodes.

- Sequence of multicast sessions to transmit U_τ's

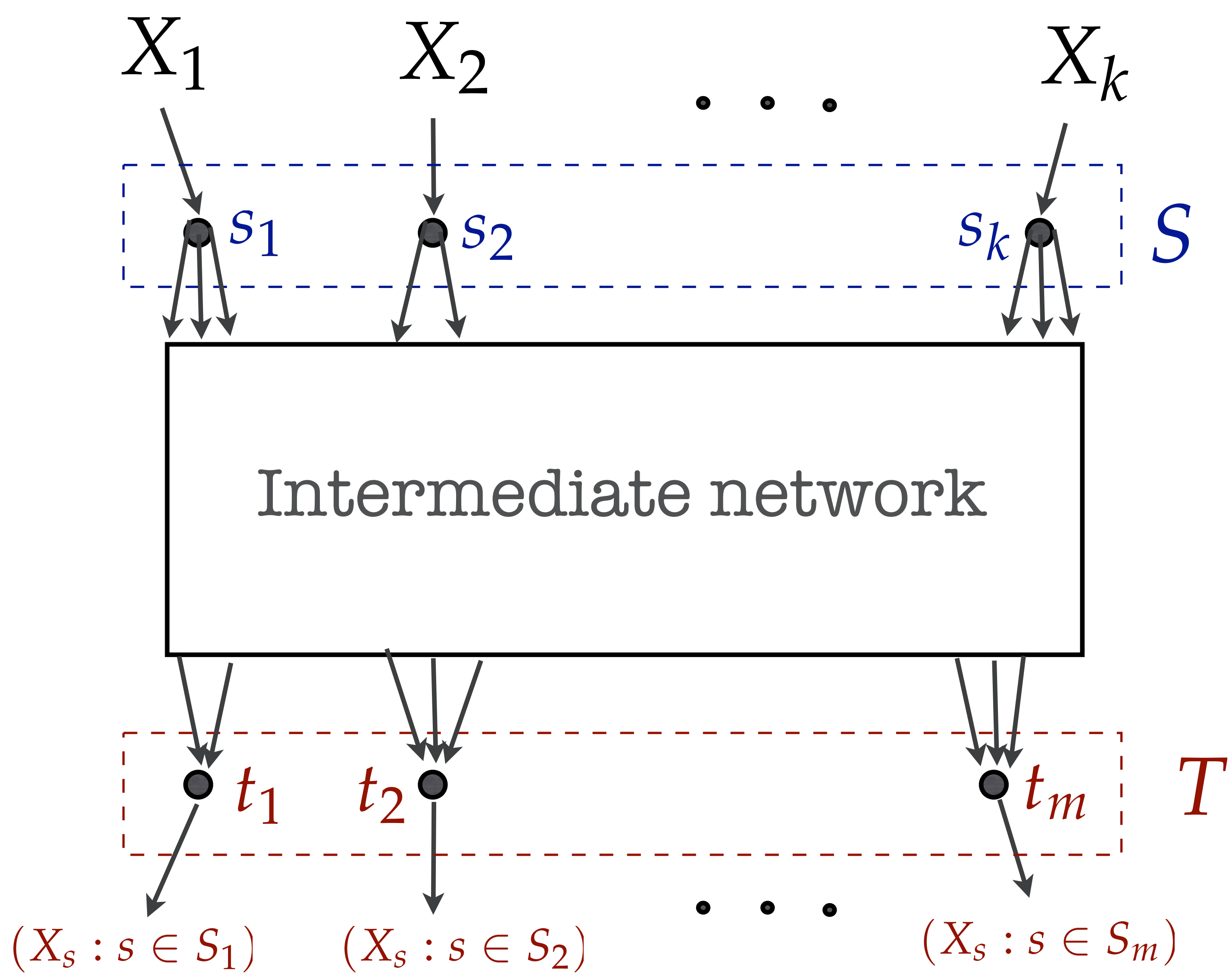


- Multicast $(X_1, X_2, \ldots, X_k)$ with U_τ's as side information



Result 2 :
An sufficient set of feasibility conditions

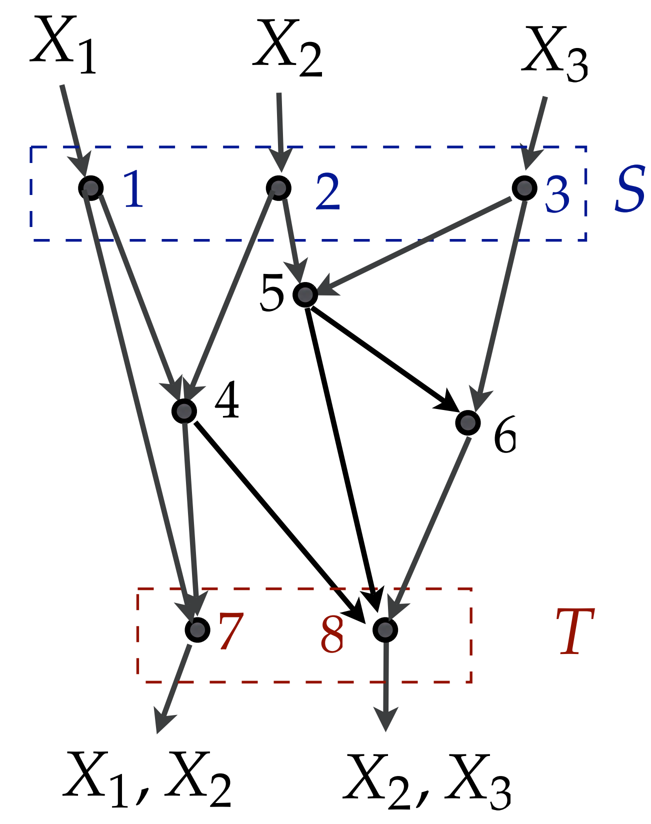
General source-demand structures



- Source nodes : $S = (s_1, s_2, \ldots, s_k)$

- Sink nodes : $T = (t_1, t_2, \ldots, t_m)$

- Demand $(X_s : s \in S_j)$ at the sink node t_j



- Fix permutation $\sigma(\cdot)$ of $(1, 2, \ldots, k)$.
- Multicast X_i's in order $\sigma(\cdot)$

Result 3 :

Demands are feasible with capacities $(C_e : e \in E)$ if

$$\sum_{e \in \Gamma_o(C)} C_e \geq \sum_{i=1}^{k} \max_{j : t_j \in T \setminus C} H(X_{\{\sigma(i)\} \cap S_j} | X_{\{\sigma(1), \sigma(2), \ldots, \sigma(i-1)\} \cap S_j})$$

for all $C \subseteq V$.

Summary

Key ideas

- *Use existing result for multicast to design coding strategies for cases where side information is present only at the sink nodes.*
- *Use the result obtained for case with side information at the sinks to design coding strategies for other cases.*

Results

- *Characterization of the entire set of feasible rates for multicast when side information is present only at the sinks*
  - Linear codes suffice
- *Sufficient condition for feasibility for*
  - Multicast when side information is present at a non-sink node
  - Networks with general source-demand structures

Future directions

- *Better coding strategies for the cases where feasibility conditions are not tight*
  - Optimal ordering of multicast sessions ?
  - Other transmission strategies ?
- *Lower bound on the rates when the feasibility conditions are not tight*
  - Bound on the difference between the lower bound and the capacity region?
- *Complexity issues*
  - When do linear codes suffice?