Local Dynamics for Topology Formation

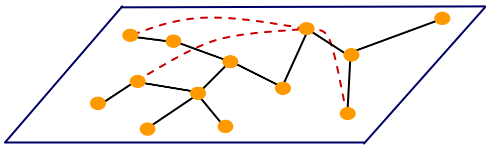
Ramesh Johari (Stanford)

Joint work with Esteban Arcaute (Stanford) and Shie Mannor (McGill)

Local Dynamics for Topology Formation

FLOWS ACHIEVEMENT

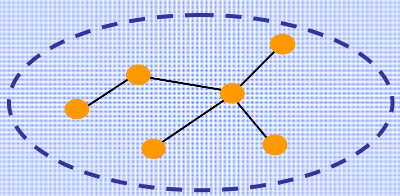
STATUS QUO



Goal: Find efficient topology for routing in an ad hoc network

Our prior work presented an approach based on **network formation games**, *but dynamics required global info.*

NEW INSIGHTS

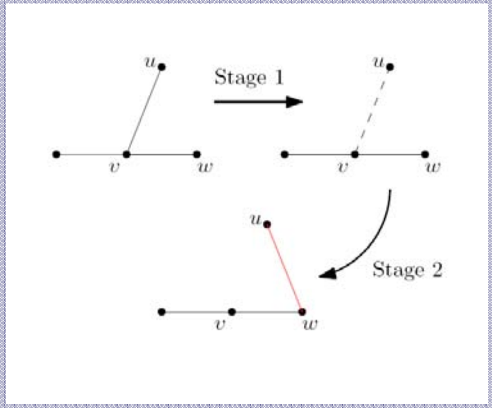


Nodes only need information about a small neighborhood around them to ensure convergence to efficient topologies

(E. Arcaute, R. Johari, S. Mannor)s

MAIN RESULT:

Simple, 2-stage local dynamics yield efficient topologies



HOW IT WORKS:

At each stage of the dynamics,
a node only sees its local neighborhood

Stage 1: Select one or more edges to break
Stage 2: Select a node with which to form a link

Link is formed if target node agrees

Idea: Decentralized, local action yields improvement in global objective function

ASSUMPTIONS AND LIMITATIONS:

Significant generalization from prior work, but still assume that link formation cost is sufficiently high to eliminate redundant links

IMPACT

This work makes several strides beyond our earlier results:

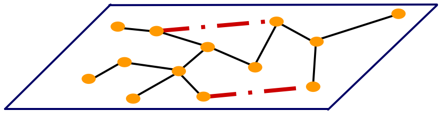
--nodes only need to know about their immediate neighborhood

--cost model is generalized to include several cases of interest

Key contribution: Dynamics for convergence in network formation game models are rare.

This line of work provides an important class of such dynamics.

NEXT-PHASE GOALS



Extend results (and cost structure) to allow for redundancy in converged topology

Simple dynamics using only local information can yield efficient topologies

Motivation

- If MANETs try to build a network topology (for routing, distributed computation or control, etc.), they suffer from a lack of global information.
- What *local* link formation dynamic leads to a good global topology?
- We propose an approach using the theory of *network formation games*:

  View nodes as self-interested agents who negotiate to form links with each other

The model

- V = set of nodes
- E = set of (bidirectional) links that nodes form with each other; let G = (V, mE) denote the graph
- c_u = "routing cost" for node u
- $d(u, v; G)$ = path cost ($\sum c_w$) along shortest path between u and v in G
- $e(u ; G)$ = number of edges incident on u in G

The model

- Cost to a node:

$$C(u\ ; G) = \sum_{v \neq u} d(u, v; G) + \beta\ e(u\ ; G)$$

  where g(·) is a nonnegative, increasing function, and $\beta > 0$ is a cost for link formation

[Also: assume cost is very large if graph is disconnected]

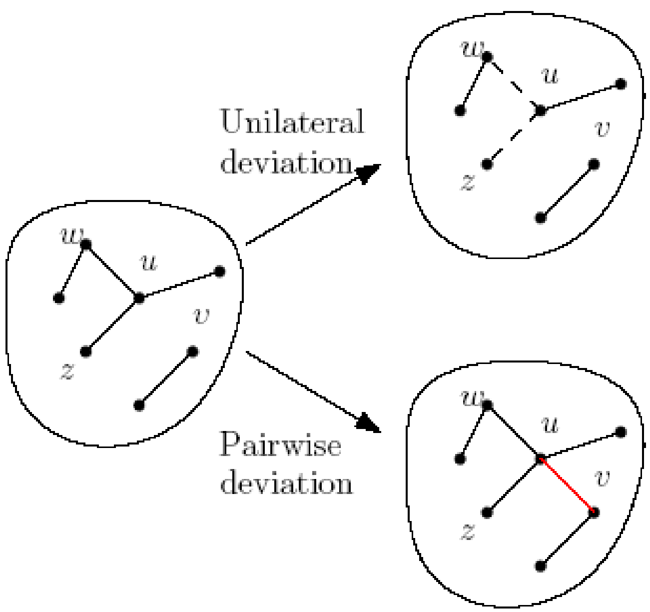
- So nodes tradeoff *connectivity* to other nodes, against *cost* of maintaining edges
- For tractability, we assume that β is sufficiently large so that equilibria are *trees*

The game

- We consider a *network formation game* where nodes declare the other nodes they wish to connect to
- Links are formed if two nodes wish to connect to each other
- Stability concept: *pairwise Nash stable* equilibrium

-No unilateral deletion of links is profitable

-No bilateral formation of a link is profitable



Efficiency

- We are interested in minimizing the *total routing cost* across the network
- *Observation 1:*
  All stars centered at a node u of minimum c_u are efficient
- *Observation 2:*
  All trees are pairwise Nash stable
- *A priori*, our equilibrium concept does not select good topologies!

Local myopic dynamics

- We define a simple dynamic that can *select* good equilibria
- Let $\ell > 1$ be given
- We consider a dynamic where each node can only "see" the topology within its ℓ-*neighborhood*: the set of nodes within ℓ hops or less
- Important generalization from earlier work, which required global view of the topology

Local myopic dynamics

- *At each round:*
  Select an active node u.

  u performs two consecutive deviations in a round (called *stages*) with nodes in his ℓ-neighborhood, to minimize its cost at the end of the round.

  All other nodes minimize their cost stage by stage.

  $\Rightarrow$ At each round, the selected node is granted *"one-step look-ahead"*
- This "look-ahead" allows the node to create a favorable *intermediate* state
- In our model, this guides the topology to efficient equilibria

Theorem

Assume that, for all u, there holds $c_u = \Theta(1)$, and that the initial topology is connected.

Then:

- *the dynamics converge almost surely; and*
- *all fixed points of the dynamics:*
    1. *have constant diameter; and*
    2. *are pairwise Nash stable.*

Note: Constant diameter implies constant efficiency ratio, so **these dynamics select efficient equilibria!**

Discussion

- The results are stronger when all costs are identical: For more general connectivity cost structures (including the *Jackson-Wolinsky connections model*), such dynamics converge to *efficient* equilibria

- Proof technique uses a mapping to "tree formation games", where cost to nodes is very large when the graph is not a tree

  In this model, our dynamics are just a local *best response dynamics!*

  Can prove there exists a potential for these best response dynamics

Discussion

- Of note:
  Our results have been generalized to include models where cost is *exponential* in distance between nodes; this is the *connections model* of Jackson and Wolinsky

- Main limiting assumption:

  The link formation cost is assumed to be very large.
  We have studied an alternate model where nodes incur a cost only for the traffic that passes *through* them, and contract with each other to form links.
  In this model, the link formation cost does not need to be large to get related convergence results.