Local Dynamics for Topology Formation

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This work makes several strides beyond our earlier results:

--nodes only need to know about their immediate neighborhood
--cost model is generalized to include several cases of interest

Key contribution: Dynamics for convergence in network formation game models are rare. This line of work provides an important class of such dynamics.

**MAIN RESULT:**
Simple, 2-stage local dynamics yield efficient topologies

**HOW IT WORKS:**
At each stage of the dynamics, a node only sees its local neighborhood
Stage 1: Select one or more edges to break
Stage 2: Select a node with which to form a link
Link is formed if target node agrees

Idea: Decentralized, local action yields improvement in global objective function

**ASSUMPTIONS AND LIMITATIONS:**
Significant generalization from prior work, but still assume that link formation cost is sufficiently high to eliminate redundant links

Extend results (and cost structure) to allow for redundancy in converged topology

Simple dynamics using only local information can yield efficient topologies
Motivation

• If MANETs try to build a network topology (for routing, distributed computation or control, etc.), they suffer from a lack of global information.

• What *local* link formation dynamic leads to a good global topology?

• We propose an approach using the theory of *network formation games*:

  View nodes as self-interested agents who negotiate to form links with each other
The model

- $V =$ set of nodes
- $E =$ set of (bidirectional) links that nodes form with each other; let $G = (V, mE)$ denote the graph
- $c_u =$ “routing cost” for node $u$
- $d(u, v; G) =$ path cost ($\sum c_w$) along shortest path between $u$ and $v$ in $G$
- $e(u ; G) =$ number of edges incident on $u$ in $G$
The model

• Cost to a node:

\[ C(u \ ; \ G) = \sum_{v \neq u} d(u, v; G) + \beta e(u \ ; \ G) \]

where \( g(\cdot) \) is a nonnegative, increasing function, and \( \beta > 0 \) is a cost for link formation

[ Also: assume cost is very large if graph is disconnected ]

• So nodes tradeoff *connectivity* to other nodes, against *cost* of maintaining edges

• For tractability, we assume that \( \beta \) is sufficiently large so that equilibria are *trees*
The game

- We consider a network formation game where nodes declare the other nodes they wish to connect to.
- Links are formed if two nodes wish to connect to each other.
- Stability concept: pairwise Nash stable equilibrium
  - No unilateral deletion of links is profitable
  - No bilateral formation of a link is profitable
Efficiency

• We are interested in minimizing the *total routing cost* across the network

• *Observation 1:*
  All stars centered at a node $u$ of minimum $c_u$ are efficient

• *Observation 2:*
  All trees are pairwise Nash stable

• *A priori,* our equilibrium concept does not select good topologies!
Local myopic dynamics

• We define a simple dynamic that can select good equilibria

• Let $\ell > 1$ be given

• We consider a dynamic where each node can only “see” the topology within its $\ell$-neighborhood: the set of nodes within $\ell$ hops or less

• Important generalization from earlier work, which required global view of the topology
Local myopic dynamics

• *At each round:* Select an active node $u$.

$u$ performs two consecutive deviations in a round (called *stages*) with nodes in his $\ell$-neighborhood, to minimize its cost at the end of the round.

All other nodes minimize their cost stage by stage.

⇒ *At each round, the selected node is granted “one-step look-ahead”*

• This “look-ahead” allows the node to create a favorable *intermediate* state

• In our model, this guides the topology to efficient equilibria
Assume that, for all \( u \), there holds \( c_u = \Theta(1) \), and that the initial topology is connected.

Then:

- the dynamics converge almost surely; and
- all fixed points of the dynamics:
  1. have constant diameter; and
  2. are pairwise Nash stable.

Note: Constant diameter implies constant efficiency ratio, so these dynamics select efficient equilibria!
Discussion

• The results are stronger when all costs are identical: For more general connectivity cost structures (including the Jackson-Wolinsky connections model), such dynamics converge to efficient equilibria.

• Proof technique uses a mapping to “tree formation games”, where cost to nodes is very large when the graph is not a tree.

In this model, our dynamics are just a local best response dynamics!

Can prove there exists a potential for these best response dynamics.
Discussion

• Of note:
  Our results have been generalized to include models where cost is *exponential* in distance between nodes; this is the *connections model* of Jackson and Wolinsky

• Main limiting assumption:

  *The link formation cost is assumed to be very large.*
  We have studied an alternate model where nodes incur a cost only for the traffic that passes *through* them, and contract with each other to form links.
  In this model, the link formation cost does not need to be large to get related convergence results.