Diversity-Multiplexing Tradeoffs in MIMO Relay Channels

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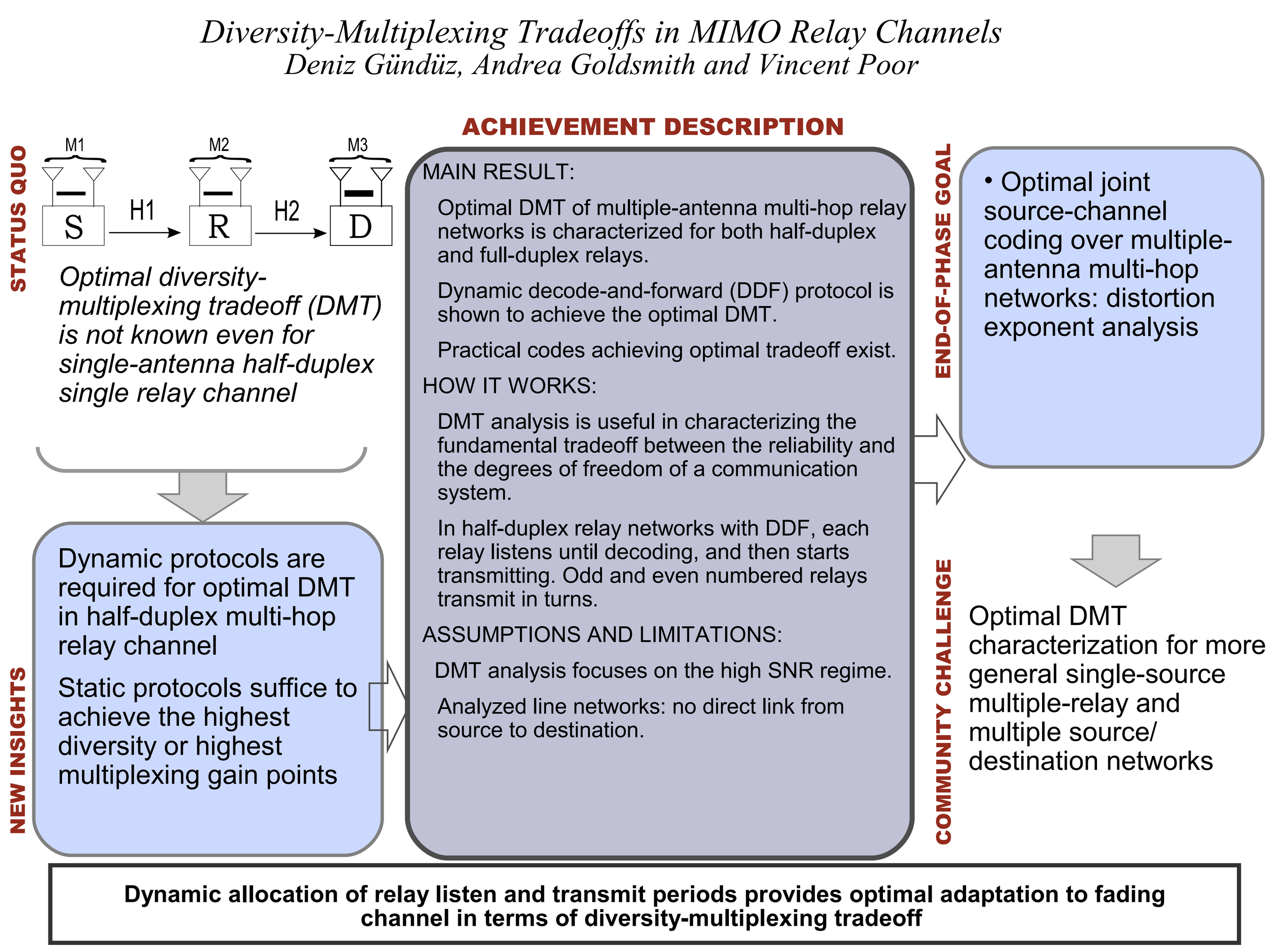
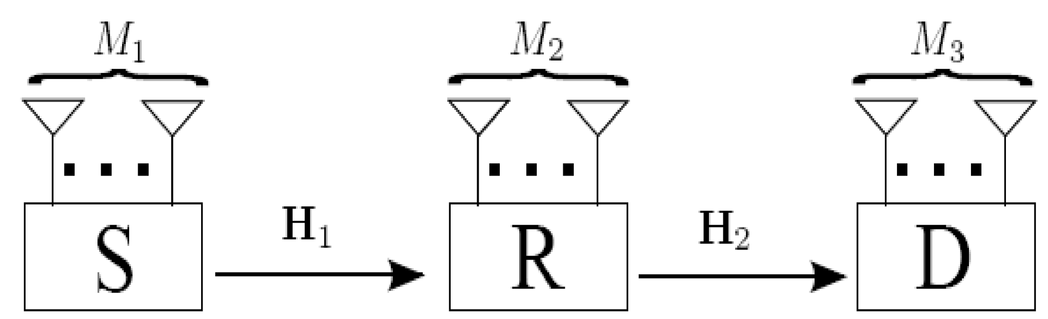


Introduction

- Relays are commonly used in wireless networks for connectivity, coverage extension and/or for improved reliability.
- Fundamental capacity limits are unknown even for a single relay channel
- Our focus: Characterizing the tradeoff between rate gain through multiplexing versus the robustness gain through diversity, i.e., the diversity-multiplexing tradeoff (DMT)
- Optimal DMT of half-duplex relay channels is open
- Consider a multiple antenna multi-hop system in which the source transmission can only be received by the relay terminal

System Model



• First consider a two-hop multiple antenna relay channel

• Source, relay and destination has M_1, M_2 and M_3 antennas, respectively.

• Quasi-static Rayleigh fading channels:

$$\mathbf{Y}_i = \sqrt{\frac{SNR}{M_i}}\mathbf{H}_i\mathbf{X}_i + \mathbf{W}_i,$$

•Define, for i=1,2,

$$M_i^* \triangleq \min\{M_i, M_{i+1}\}.$$

Diversity-Multiplexing Tradeoff

- For increasing SNR, consider a family of codes and say that they achieve a multiplexing gain of r, if the rate R(SNR) satisfies

$$\lim_{SNR\to\infty}\frac{R(SNR)}{\log(SNR)} = r.$$

- The diversity gain d of this family is

$$d = -\lim_{SNR\to\infty}\frac{\log P_e(SNR)}{\log(SNR)}$$

in which P_e(SNR) is the error probability.

- For a MIMO system with M1 transmit and M2 receive antennas and sufficiently long codewords, the optimal DMT curve $d_{M1,M2}(r)$ is given by the piecewise-linear function connecting the points (k,d(k)), k=0,..,min(M1,M2), where

d(k)=(M1-k)(M2-k)

DMT of Full-duplex Relay channels

• Assume the relay can receive and transmit simultaneously

• The DMT $d^f_{M\_1,M\_2,M\_3}(r)$ of an (M1,M2,M3) full-duplex system is characterized by

$$d^f_{M_1,M_2,M_3}(r) = \min\{d_{M_1,M_2}(r), d_{M_2,M_3}(r)\}$$

• Achieved by decode-and-forward relaying with block Markov structure

• Follows easily since DF achieves capacity

Static Protocols for Half-duplex Relaying

•The source transmits during the first aT channel uses, 0<a<1.

•The relay tries to decode the message and forwards over the remaining (1-a)T channel uses.

•Call this: *decode-and-forward with fixed allocation* (fDF)

•The DMT of the half-duplex (M_1,M_2,M_3) relay channel with fixed time allocation a is

$$d^{fDF}_{M_1M_2M_3}(r) = \min\left\{d_{M_1,M_2}\left(\frac{r}{a}\right), d_{M_2,M_3}\left(\frac{r}{1-a}\right)\right\}$$

Dynamic Decode-and-Forward (DDF) Protocol for Half-duplex Relaying

• DDF (Azarian et al.) achieves the best known DMT for half-duplex relay channels, yet short of the upper bound

• We show its optimality in multi-hop relay channels:

$$d^{DDF}(r) = \inf_{(\boldsymbol{\alpha_1},\boldsymbol{\alpha_2})\in\tilde{\mathcal{O}}_2}\sum_{i=1}^{2}\sum_{j=1}^{M_i^*}(2j-1+|M_i-M_{i+1}|)\alpha_{i,j}$$

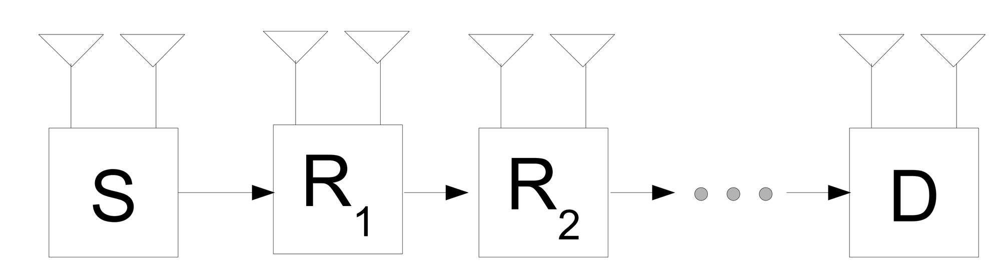
where

$$\tilde{\mathcal{O}}_2 \triangleq \Big\{(\boldsymbol{\alpha_1},\boldsymbol{\alpha_2}) \in \mathcal{R}^{M_1^*+} \times \mathcal{R}^{M_2^*+}\Big| \alpha_{i,1} \geq \cdots \geq \alpha_{i,M_i^*} \geq 0, r > \frac{S_1(\boldsymbol{\alpha_1})S_2(\boldsymbol{\alpha_2})}{S_1(\boldsymbol{\alpha_1})+S_2(\boldsymbol{\alpha_2})}\Big\}$$

and

$$S_i(\boldsymbol{\alpha_i}) \triangleq \sum_{j=1}^{M_i^*}(1-\alpha_{i,j})^+$$

Multiple Relay Networks

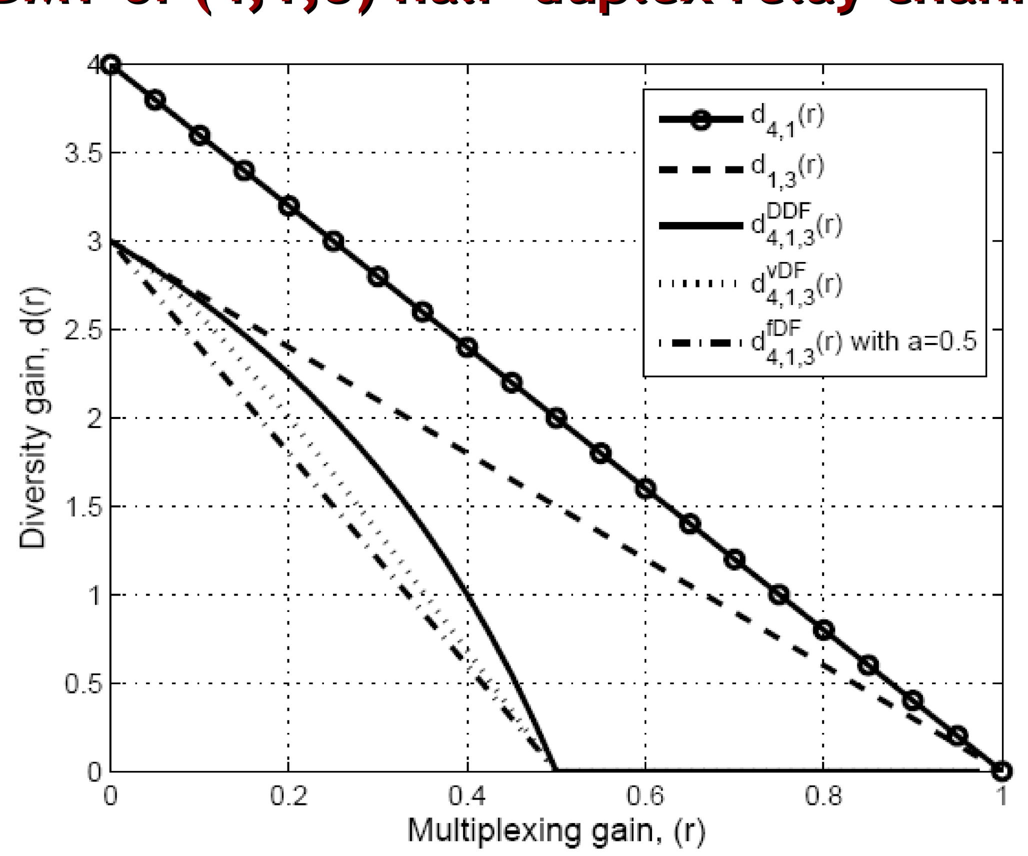


• In the case of multiple full-duplex relays, results easily extend and the DMT is dominated by the hop with the minimum diversity gain.

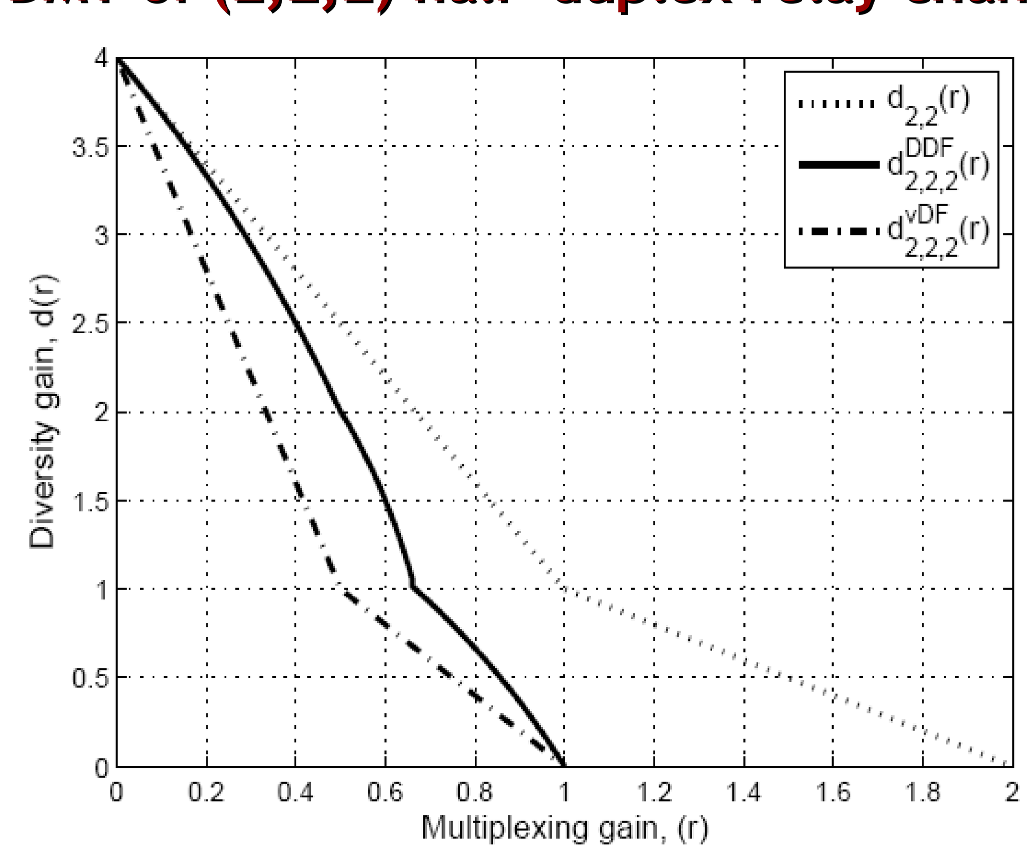
•For half-duplex relays, odd and even numbered relays transmit in turn. We show that DDF is DMT optimal here as well.

•DMT is dominated by the two consecutive hops with the minimum diversity gain.

DMT of (4,1,3) half−duplex relay channel



DMT of (2,2,2) half−duplex relay channel



Conclusions

• Characterized diversity-multiplexing tradeoff of MIMO multi-hop relay channels for both full-duplex and half-duplex relays

•For full-duplex relays, decode-and-forward protocol achieves the optimal DMT, which is simply the minimum of the DMTs of all links.

•For half-duplex relays,dynamic decode-and-forward protocol, in which the relay listens until decoding and then forwards, achieves the optimal DMT, which is no longer a piecewise-linear function of the multiplexing gain.

Insights and Future Work

Insights

• Dynamic time allocation protocols that adapt to the unknown channel state are required for optimal performance

•Static protocols suffice to achieve the highest diversity or highest multiplexing gain points

Future work

• Analyze optimal joint source-channel coding over multiple-antenna multi-hop networks: distortion exponent analysis

• Characterization of DMT for more complicated half-duplex relay networks