# An Equivalence Theory of Network Capacity

Ralf Koetter, Michelle Effros, & Muriel Médard

ITMANET – FLoWS

December 6, 2007

# Outline

Network Capacities:

- Known for limited collection of (mostly small) networks
- Scaling laws $\Rightarrow$ coarse behaviors / strategies (not capacities) assuming many homogeneous nodes

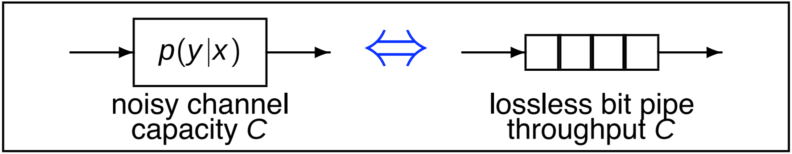
Why?

- Finding achievable rate regions is difficult!
  - Often optimized one network at a time
  - Don't know how to do "network" relaying
  - Don't know how to deal with interference
- Finding upper bounds is difficult!
  - Very few tools (e.g., Fano's, cut-set bounds)
  - Cutset bounds are often very loose.

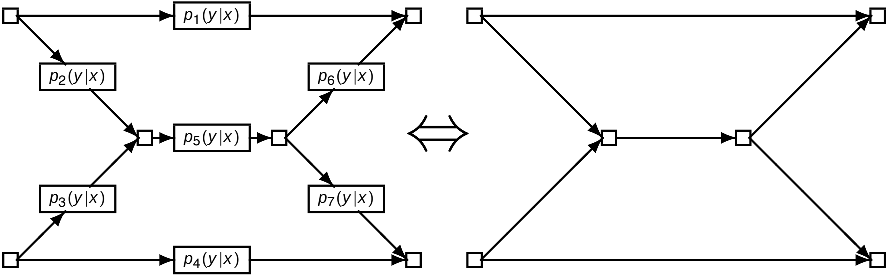
Goal: Systematic method for characterizing capacity limits

# Central Insight

Network equivalence ...

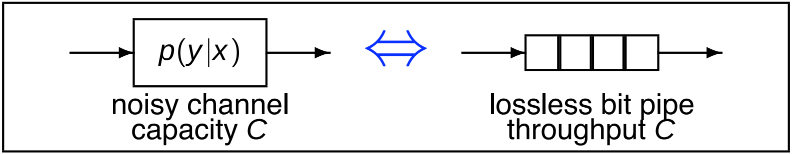



... could turn noisy networks into noiseless networks!



# Central Insight

Network Equivalence Implications




- Reduces complexity: channel statistics $\Rightarrow$ capacities
- Allows use of existing tools
  - Component capacities
  - Network coding
- Highlights combinatoric core of network capacity problem

# FLoWS Achievements

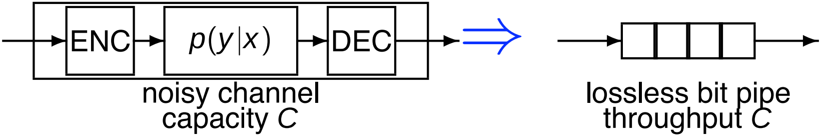
- Link equivalence
- Generalizations in progress
  - Broadcast
  - Multiple access

  including
  - Examples ($\Rightarrow$ equivalences)
  - Counter-examples ($\Rightarrow$ upper bounds)

# FLoWS Achievements

## How it Works – Link Equivalence

### Shannon Theory



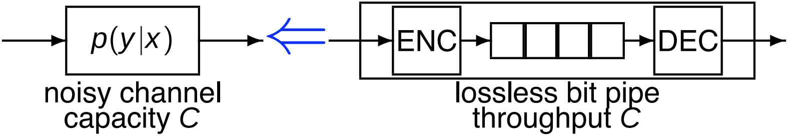

Using a good channel encoder & channel decoder
we can make the noisy channel virtually indistinguishable
from the noiseless bit pipe with throughput $C$

EASY – [Shannon, 1948]

# FLoWS Achievements

## How it Works – Link Equivalence

### Dual to Shannon Theory



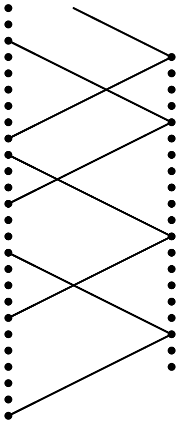

Using a good source encoder & source decoder
we can make the noiseless bit pipe virtually indistinguishable
from the noisy channel with transition probability $p(y|x)$.

HARD –
Must show that there is never anything to be gained
by working about the noisy channel capacity

# FLoWS Achievements

## How it Works – Link Equivalence

### Method of Types



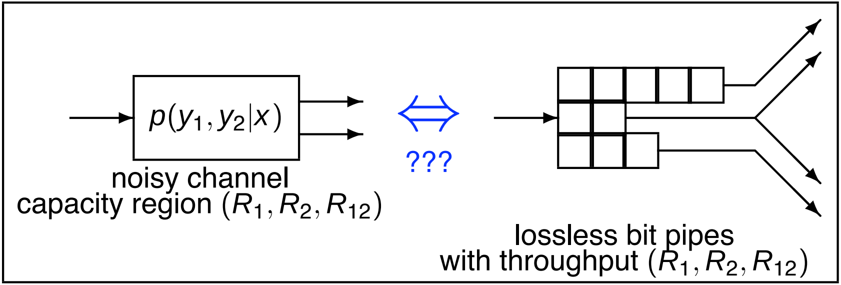
- Draw $Y^n$ vectors
- Each covers $\sim 2^{nH(X|Y)}$ typical $X^n$ vectors
- There are $\sim 2^{nH(X)}$ typical $X^n$ vectors
- Therefore $\sim 2^{nI(X;Y)}$ $Y^n$ values suffice
- Encode typical $X^n$ to jointly typical $Y^n$

Used $\sim C = I(X;Y)$ bits per sample.
Mimics noisy channel *on typical strings*.
Noisy channel can't do anything with atypical cases.

# FLoWS Achievements

## How it Works – Generalizations

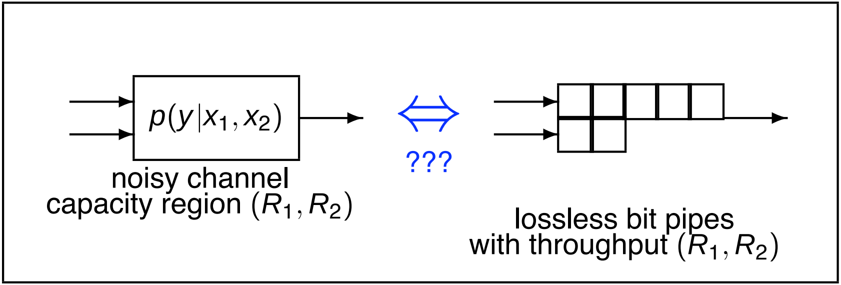
### Broadcast



# FLoWS Achievements

## How it Works – Generalizations

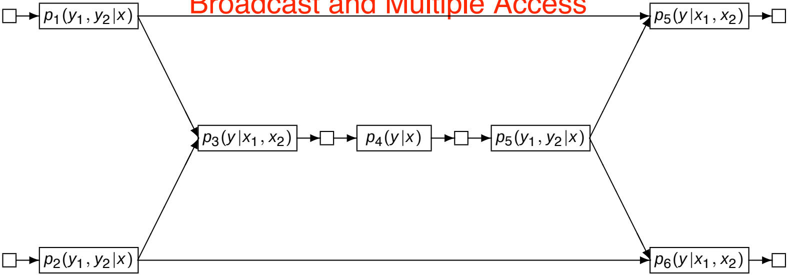
### Multiple Access



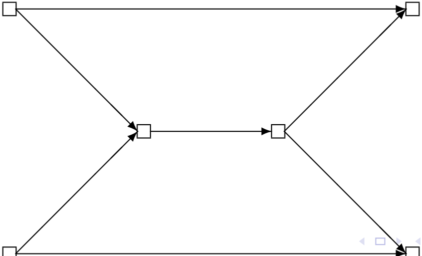
# FLoWS Achievements

## How it Works – Generalizations

## Broadcast and Multiple Access




⇕??



# FLoWS Achievements

## How it Works – Generalizations

When equivalences fail ...

... the result is upper bounds.

- Find the smallest possible bit pipes with which you can mimic the noisy channels.
- The capacity of the resulting noiseless network is an upper bound on the capacity of the noisy network.

# FLoWS Achievements

## Assumptions and Limitations

- Incomplete understanding
  of broadcast and multiple access cases
- Assumes links are memoryless and discrete
- Requires solution of combinatoric network coding problem
  (high complexity for large networks)
- Metrics other than capacity may not be preserved
  (e.g., error exponents)

# End-of-Phase Goal

- More complete understanding of equivalences / hierarchies for broadcast and multiple access channels.
- Bounding networks where equivalence can't be established.

# Community Challenges

- Graduate level – Identify additional equivalences and hierarchies.
- Prize level – Understand limits of capacity ordering as a practical intellectual tool.