Universal Decoding in MANETs

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Unreliable and Uncertain Links

- Unknown channel law $p(y|x_1, x_2, x_3)$
- Decision space includes *erasure* option
  (acknowledging unreliable transmission)
- Need universal encoders and decoders
- Capacity region and error exponents
- Status Quo: universal codes *without* erasure option;
  Forney’s work on list/erasure decoding for *fixed* channels
Universal Decoding

- Unknown channel law $p(y|x)$
- Constant-composition codes (codewords $x(m)$ have type $p_X$)
- Type-I exponent for outputting incorrect message
- Type-II exponent for erasures
- What is the (Neyman-Pearson-like) decoding rule that optimizes the fundamental tradeoff between these exponents?
- Status Quo: Csiszár and Körner (1981) and Merhav and Feder (2007) proposed suboptimal schemes for list size $\leq 1$
Achievements

- Derived asymptotic NP rule for single-user channel

- This rule coincides with Forney’s rule in some rate regime, and therefore cannot be improved
- Derived asymptotic NP rule for multiple-access channel
• Universal variable-size list **decoding rule**

\[ g_F(y) = \begin{cases} \hat{m} & : \text{if } I(x(\hat{m}); y) > R + \max_{i \neq \hat{m}} F(I(x(i); y) - R) \\ \emptyset & : \text{else} \end{cases} \]

... variation on Maximum Mutual Information decoding rule, parameterized by nondecreasing function \( F \)

• Special case: \( F(t) = \Delta + \lambda |t|^+ \), \( \lambda \geq 1 \) (Csizárr and Körner)

• Incorrect-message and erasure exponents

\[
E_i(R, p_X, p_{Y|X}, F) \triangleq \lim \inf_{N \to \infty} - \frac{1}{N} \log \mathbb{E}[N_i] \\
E_{\emptyset}(R, p_X, p_{Y|X}, F) \triangleq \lim \inf_{N \to \infty} - \frac{1}{N} \log \mathbb{P}[E_{\emptyset}]
\]
**Error Exponents**

- Sphere-packing exponent for channel $p_{Y|X} \in \mathcal{W}$:
  \[ E_{sp}(R, p_{X}, p_{Y|X}) \triangleq \min_{\tilde{p}_{Y|X}: I(p_{X}, \tilde{p}_{Y|X}) \leq R} D(\tilde{p}_{Y|X} \parallel p_{Y|X} \mid p_{X}) \]

- **Modified random coding exponent** for channel $p_{Y|X}$:
  \[ E_{r,F}(R, p_{X}, p_{Y|X}) \triangleq \min_{\tilde{p}_{Y|X}} [D(\tilde{p}_{Y|X} \parallel p_{Y|X} \mid p_{X}) + |\tilde{F}(I(p_{X}, \tilde{p}_{Y|X}) - R)|^+] \]

- (not necessarily unique) NP-optimal choice of $F$:
  \[ F^*(t) \triangleq \Delta + E_{sp}(R, p_{X}, \mathcal{W}) - E_{sp}(R + t, p_{X}, \mathcal{W}) \]

- Simple expressions for corresponding error exponents
• **Proposition.** Assume that $R$, $p_X$, $\mathcal{W}$, $\Delta$ and $\lambda$ are such that

$$|\overline{R}^{\text{conj}}(p_X, \mathcal{W}) - R|^+ \leq \Delta \leq I(p_X, \mathcal{W}) - R,$$

$$-E_{sp}'(R, p_X, \mathcal{W}) \leq \lambda \leq \frac{1}{-E_{sp}'(R + \Delta, p_X, \mathcal{W})}.$$

Then the Forney incorrect-message and erasure exponents:

$$E_{sp}(R, p_X, p_{Y|X}) + \Delta \quad \text{and} \quad E_{sp}(R + \Delta, p_X, p_{Y|X}),$$

are achieved uniformly over $p_{Y|X} \in \mathcal{W}$ using the penalty function $F(t) = \Delta + \lambda |t|^+$. 


Example: Compound Binary Symmetric Channel
Multiple-Access Channels

- Decoding with erasures is an open problem for MAC
- Define collection of nondecreasing functions $F_A(t)$, $A \subseteq K$
- Example (generally suboptimal):
  \[
  F_A(t) = \Delta R(A) + \lambda |t|^+ 
  \]
- Seek NP-optimal $F$
Universal Decision Rule

- Test all possible subsets of input terminals:

\[
g_F(y, u) = \begin{cases} 
(K, m_K) : & \text{if } \forall A \subseteq K : I^{(|A|+1)}(x(m_A); yx(m_K \setminus A)|u) > R(A) + \\
\max_{\{i_k \neq m_k \forall k \in A\}} F_A(I^{(|A|+1)}(x(i_A); yx(m_K \setminus A)|u) - R(A)) \\
(\emptyset, \emptyset) & : \text{else}
\end{cases}
\]

where

\[
u = \text{time-sharing sequence}
\]

\[
I^{(K)}(x_1x_2\cdots x_K) = \text{empirical m.i. between K r.v.'s}
\]

\[
= D(p_{x_1x_2\cdots x_K} \parallel p_{x_1}p_{x_2}\cdots p_{x_K})
\]
• Define the pseudo sphere-packing exponent

\[ E_{\text{psp}}(R, p_{XU}) \triangleq \min_{A \subseteq K} E_{\text{psp}, A}(R(A), p_{XU}) \]

where

\[ E_{\text{psp}, A}(R(A), p_{XU}) \triangleq \min_{\tilde{p}_{XU} \in \mathcal{M}(p_{XU})} \min_{p_{Y|XU}} \min_{\tilde{p}_{Y|XU} \in \mathcal{M}(p_{Y|XU})} \min_{\tilde{p}_{Y|XU} : \hat{I}_{\tilde{p}_{Y|XU}} \geq I_{\tilde{p}_{Y|XU}}(X_A; Y_{X\setminus A} | U) \leq R(A)} D(\tilde{p}_{Y|XU} \tilde{p}_{XU} \| p_{Y|X} p_{XU}). \]

• NP-optimal choice of \( F_A \):

\[ F_A^*(t) = \Delta_A + E_{\text{psp}, A}(R(A), p_{XU}) - E_{\text{psp}, A}(R(A) + t, p_{XU}) \]

• Simple expressions for corresponding error exponents