Universal Decoding in MANETs

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Unreliable and Uncertain Links

- Unknown channel law $p(y|x_1, x_2, x_3)$
- Decision space includes *erasure* option (acknowledging unreliable transmission)
- Need universal encoders and decoders
- Capacity region and error exponents
- Status Quo: universal codes *without* erasure option; Forney's work on list/erasure decoding for *fixed* channels

Universal Decoding

- Unknown channel law $p(y|x)$
- Constant-composition codes (codewords $\mathbf{x}(m)$ have type p_X)
- Type-I exponent for outputting **incorrect message**
- Type-II exponent for **erasures**
- What is the (**Neyman-Pearson**-like) decoding rule that optimizes the fundamental tradeoff between these exponents?
- Status Quo: Csiszár and Körner (1981) and Merhav and Feder (2007) proposed suboptimal schemes for list size ≤ 1

Achievements

- Derived asymptotic NP rule for single-user channel
- This rule coincides with Forney's rule in some rate regime, and therefore cannot be improved
- Derived asymptotic NP rule for multiple-access channel

Single-User Channels

- Universal variable-size list **decoding rule**

$$g_F(\mathbf{y}) = \begin{cases} \hat{m} & : \text{ if } I(\mathbf{x}(\hat{m}); \mathbf{y}) > R + \max_{i \neq \hat{m}} F(I(\mathbf{x}(i); \mathbf{y}) - R) \\ \emptyset & : \text{ else.} \end{cases}$$

  variation on Maximum Mutual Information decoding rule, parameterized by nondecreasing function F

- Special case: $F(t) = \Delta + \lambda\, |t|^+$, $\lambda \geq 1$ (Csiszár and Körner)

- Incorrect-message and erasure exponents

$$E_{\mathrm{i}}(R, p_X, p_{Y|X}, F) \triangleq \liminf_{N \to \infty} -\frac{1}{N} \log \mathbb{E}[N_{\mathrm{i}}]$$

$$E_{\emptyset}(R, p_X, p_{Y|X}, F) \triangleq \liminf_{N \to \infty} -\frac{1}{N} \log Pr[\mathcal{E}_{\emptyset}]$$

Error Exponents

- Sphere-packing exponent for channel $p_{Y|X} \in \mathscr{W}$:

$$E_{sp}(R, p_X, p_{Y|X}) \triangleq \min_{\tilde{p}_{Y|X} : I(p_X, \tilde{p}_{Y|X}) \leq R} D(\tilde{p}_{Y|X} \| p_{Y|X} \mid p_X)$$

- *Modified random coding exponent* for channel $p_{Y|X}$:

$$\begin{aligned} E_{r,F}(R, p_X, p_{Y|X}) \quad &\triangleq \quad \min_{\tilde{p}_{Y|X}} [D(\tilde{p}_{Y|X} \| p_{Y|X} \mid p_X) \\ &\quad + |\tilde{F}(I(p_X, \tilde{p}_{Y|X}) - R)|^{+}] \end{aligned}$$

- (not necessarily unique) NP-optimal choice of F:

$$F^*(t) \triangleq \Delta + E_{sp}(R, p_X, \mathscr{W}) - E_{sp}(R + t, p_X, \mathscr{W})$$

- Simple expressions for corresponding error exponents

Universality

- **Proposition.** Assume that R, p_X, $\mathscr{W}$, Δ and λ are such that

$$
\begin{aligned}
|\overline{R}^{conj}(p_X, \mathscr{W}) - R|^+ &\leq \Delta &\leq I(p_X, \mathscr{W}) - R, \\
-\underline{E}'_{sp}(R, p_X, \mathscr{W}) &\leq \lambda &\leq \frac{1}{-\underline{E}'_{sp}(R+\Delta, p_X, \mathscr{W})}.
\end{aligned}
$$

Then the Forney incorrect-message and erasure exponents:

$$E_{sp}(R, p_X, p_{Y|X}) + \Delta \quad \text{and} \quad E_{sp}(R + \Delta, p_X, p_{Y|X}),$$

are achieved uniformly over $p_{Y|X} \in \mathscr{W}$ using the penalty function $F(t) = \Delta + \lambda |t|^+$.

Example: Compound Binary Symmetric Channel

Multiple-Access Channels

- Decoding with erasures is an open problem for MAC
- Define collection of nondecreasing functions $F_{\mathcal{A}}(t)$, $\mathcal{A} \subseteq \mathcal{K}$
- Example (generally suboptimal):

$$F_{\mathcal{A}}(t) = \Delta \mathsf{R}(\mathcal{A}) + \lambda |t|^{+}$$

- Seek NP-optimal F

Universal Decision Rule

- Test **all possible subsets** of input terminals:

$$g_F(\mathbf{y},\mathbf{u}) = \begin{cases} (\hat{\mathcal{K}}, \hat{m}_{\hat{\mathcal{K}}}) & : \text{ if } \forall \mathcal{A} \subseteq \hat{\mathcal{K}} \; : \; \overset{\circ}{I}^{(|\mathcal{A}|+1)}(\mathbf{x}(\hat{m}_{\mathcal{A}}); \mathbf{y}\mathbf{x}(\hat{m}_{\hat{\mathcal{K}}\setminus\mathcal{A}})|\mathbf{u}) > \mathsf{R}(\mathcal{A}) + \\ & \quad \max_{\{i_k \neq \hat{m}_k \, \forall k \in \mathcal{A}\}} F_{\mathcal{A}} \left(\overset{\circ}{I}^{(|\mathcal{A}|+1)}(\mathbf{x}(i_{\mathcal{A}}); \mathbf{y}\mathbf{x}(\hat{m}_{\hat{\mathcal{K}}\setminus\mathcal{A}})|\mathbf{u}) - \mathsf{R}(\mathcal{A}) \right) \\ (\emptyset, \emptyset) & : \text{ else} \end{cases}$$

where

$$\begin{aligned} \mathbf{u} &= \text{time} - \text{sharing sequence} \\ \overset{\circ}{I}^{(K)}(\mathbf{x}_1\mathbf{x}_2\cdots\mathbf{x}_K) &= \text{empirical m.i. between } K \text{ r.v.'s} \\ &= D(p_{\mathbf{x}_1\mathbf{x}_2\cdots\mathbf{x}_K} \| p_{\mathbf{x}_1} p_{\mathbf{x}_2} \cdots p_{\mathbf{x}_K}) \end{aligned}$$

- Define the *pseudo sphere-packing exponent*

$$E_{psp}(\mathsf{R}, p_{\mathsf{X}U}) \triangleq \min_{\mathcal{A}\subseteq\mathcal{K}} E_{psp,\mathcal{A}}(\mathsf{R}(\mathcal{A}), p_{\mathsf{X}U})$$

where

$$E_{psp,\mathcal{A}}(\mathsf{R}(\mathcal{A}), p_{\mathsf{X}U}) \triangleq \min_{\tilde{p}_{\mathsf{X}U}\in\mathcal{M}(p_{\mathsf{X}U})} \min_{p_{Y|\mathsf{X}}} \min_{\tilde{p}_{Y|\mathsf{X}U}\,:\,\mathring{I}_{\tilde{p}_{\mathsf{X}U}\,\tilde{p}_{Y|\mathsf{X}U}}(\mathsf{X}_{\mathcal{A}};Y\mathsf{X}_{\mathcal{K}\setminus\mathcal{A}}|U)\le\mathsf{R}(\mathcal{A})} D(\tilde{p}_{Y|\mathsf{X}U}\,\tilde{p}_{\mathsf{X}U}\|p_{Y|\mathsf{X}}\,p_{\mathsf{X}U}).$$

- NP-optimal choice of $F_{\mathcal{A}}$:

$$F^*_{\mathcal{A}}(t) = \Delta_{\mathcal{A}} + E_{psp,\mathcal{A}}(\mathsf{R}(\mathcal{A}), p_{\mathsf{X}U}) - E_{psp,\mathcal{A}}(\mathsf{R}(\mathcal{A})+t, p_{\mathsf{X}U})$$

- Simple expressions for corresponding error exponents