Information Theory for Mobile Ad-Hoc Networks (ITMANET)

Thrust I

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(and everyone!)
Application Metrics and Network Performance

Capacity and Fundamental Limits

Upper Bound

Lower Bound

Constraints

Models and Dynamics

Degrees of Freedom

Layerless Dynamic Networks

New Paradigms for Upper Bounds

Application Metrics

Constraints

End-to-End Performance and Network Utility

Utility = U(C, D, E)

Energy/SNR

New MANET Theory

Fundamental Limits of Wireless Systems
Goals

- Characterize the performance potential of ITMANETs.
- Objectivity: Upper bounds provide an objective assessment of:
  - How well current schemes are doing
  - Where future efforts are best spent
- Utility: To be useful, upper bounds must be
  - Concrete
  - Computable
  - Helpful
  - Practical
- Generality: We seek general upper bounds to apply to
  - Arbitrary networks
  - Arbitrary demands
- Reality: Traditional assumptions are a poor match for ITMANETs. To develop realistic upper bounds, we must master
  - Delay
  - Variability
  - Non-ergodicity
Final goal

- Final objective: Network SPICE, “SPINE”

(Simulation Program with Integrated Network Emphasis)
Traditional approaches – tools and results

• Tools:
  – Fano’s Inequality
    • The information that we want to estimate must be determined by our evidence.
  – Min-cut Max-flow Theorem
    • Information flow through a network is limited by the tightest bottleneck on the best routes.

• Results:
  – Small systems (e.g., relay channel)
  – Asymptotic results under assumptions re: structure and size (e.g., scaling laws)
Traditional approaches - results

- A simple 3-node network (Kramer et al.)

\[
\begin{align*}
R_0 & \leq \min\{I(T; Y_1|X_1), I(T, X_1; Y_2)\} \\
R_0 + R_1 & \leq I(X; Y_1|X_1) \\
R_0 + R_2 & \leq \min\{I(T, U, X_1; Y_2), I(X, X_1; Y_2)\} \\
R_0 + R_1 + R_2 & \leq I(T; Y_1|X_1) + I(X; Y_1|T, U, X_1) + I(U; Y_2|T, X_1) \\
R_0 + R_1 + R_2 & \leq I(T; Y_1|T, U, X_1) + I(T, U, X_1; Y_2) \\
R_0 + R_1 + R_2 & \leq I(T; Y_1, Y_2|X_1)
\end{align*}
\]
Traditional approaches - results

# of nodes

Year

[Graph showing exponential growth in the number of nodes from 1950 to 2010, with notable increases around the Internet and Information Theory, and a comment suggesting it’s time for another approach.]
New approaches – overview and progress

- Network equivalences
- Code type equivalences
- Conflict graph presentation
- Hierarchical analyses
Network equivalences – original results
Network equivalences - progress
• Goal: Show that restricting the code type at all nodes of the network does not reduce the space of achievable results.
Conflict graph representation – connecting elements through codes

- A vertex represents a configuration of the network in terms of codes among components of the network
  - An edge between two vertices if the two configuration cause conflict (i.e. They cannot be served simultaneously).

- Stable Set: no edge connecting any pair of vertices in the set

Koetter, Medard et al.
Analog network coding

Channel outputs

relay

\[ Y_3 = h_{13}X_1 + h_{23}X_2 + Z_3 \]

node 4

\[ Y_4 = h_{14}X_1 + h_{24}X_2 + h_{34}X_3 + Z_4 \]

node 5

\[ Y_5 = h_{15}X_1 + h_{25}X_2 + h_{35}X_3 + Z_5 \]

Channel input at the relay

\[ X_{3,n} = f_n(Y_{3,n-1}, \ldots, Y_{3,1}) \]

Goldsmith, Medard, et al.
Scaling laws using multiple access

Squarelet acts as a component and interconnections occur among components, albeit in a multiple access fashion.

- Source-destination pairs relay traffic over dense squarelets.
- Induces virtual multiple antenna multiple access and broadcast channels.
- For the best communication scheme

\[ \rho^* (n) = O\left(n^{1-\frac{\alpha}{2} + \varepsilon}\right) \]

for \( \varepsilon > 0 \) arbitrarily small and for any \( 2 < \alpha < 3 \).

- Thus scheme is order optimal for \( 2 < \alpha < 3 \), so that decomposition is not detrimental in an order sense.

Shah, Gupta et al.
Achievements Overview

New bounding techniques

- **Goldsmith**: Interference channel with cognitive user, “asymmetric” cooperation
- **Shah**: Multiple access decomposition for constructive scaling laws
- **Koetter, Effros, Medard**: Equivalence classes of networks based on ability of a channel or a building block to emulate arbitrary channels

Code construction

- **Zheng**: Error exponents unequal error protection, embedded control messages to reduce overhead
- **Koetter, Effros**: Matroidal codes

Networking and optimization

- **Moulin**: Covert channel by timing information
- **Koetter**: Likelihood forwarding, relay information before decoding

Combinatorial Tools

- **Goldsmith, Medard, Katabi**: Generalized joint relaying, combine symbols in PHY, bits, or network layer
- **Medard, Koetter**: Network coding capacity based on the notion of conflict graphs
Thrust Synergies: An Example

**Thrust 1**
Upper Bounds

**Shah:** multiple access decomposition for constructive scaling laws

**Koetter, Effros, Medard:** Equivalence classes of networks based on ability of a channel or a building block to emulate arbitrary channels

**Goldsmith, Medard, Katabi:** Generalized joint relaying, combine symbols in PHY, bits, or network layer

**Koetter:** likelihood forwarding, relay information before decoding

**Thrust 2**
Layerless Dynamic Networks

**Thrust 3**
Application Metrics and Network Performance

**Medard, Koetter:** network coding capacity based on the notion of conflict graphs

\[ (C^*, D^*, E^*) \]
Roadmap

• Generalized equivalence results beyond point–to-point multiple access and broadcast components
• Derive bounds in cases where equivalence fails
• Conflict graphs for soft constraints such as multiple access constraints
• Schedule creation from conflict graph (thrust 3 interaction)
• Explore code-type equivalence conjecture
• Investigate code design implications of restricted code types
Publications


- V. Doshi, D. Shah, M. Médard, S. Jaggi, “Distributed Functional Compression through Graph Coloring”, *Data Compression Conference (DCC)*, 2007, Snowbird, UT.


- Y. Liang, A Goldsmith, M. Effros, "Distortion Metrics of Composite Channels with Receiver Side Information".


