# Stochastic NUM:

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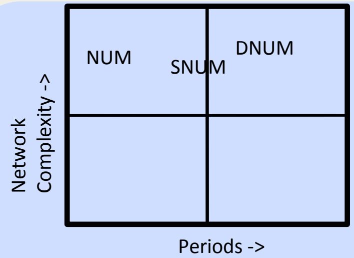
## STATUS QUO




NUM assumes quasi-static environment, does not take PHY-layer statistics into account.

Single period model.

Iteration vs. clock time

Average constraints?

## NEW INSIGHTS

Extend NUM to time varying, random environments

Multi-period (finite) models

Average constraints (Power, Delay)

Policies that optimally allocate resources over time or states.

On-line optimization algorithms as methods to describe properties of networks

## ACHIEVEMENT DESCRIPTION

### SNUM

Current

- single link cases
- multiple link case

**Planned directions**

- OFDM-A
- multi-casting
- adaptive routing & shadowing
- Stochastic dynamics

**How it works:** Maximize expected utility function subject to instantaneous and average resource constraints. Yields resource policies that optimize performance and allocate resources over distribution or time.

maximize $E[U(r)]$

subject to

$E[Q(r,G,a,b,..] = q$

$E[P]=p$

$T(a,G) < 0$

$r>0;\ S>0$

## END-OF-PHASE GOAL

Optimal methods of optimally trading off metrics:

- Delay
- Rate
- Power
- Admission "costs"

## COMMUNITY CHALLENGE

***Generalization of SNUM solution algorithms may suggest fundamental trade-offs or limits***

**A framework for incorporating channel and other time varying, random phenomena into NUM**

# NUM – Background

## NUM Model

$$\begin{array}{ll} \max_r & U(r) \\ \text{s.t.} & \\ & Ar \leq c \\ & r \geq 0 \end{array}$$

Typical U(r)

$$U(r) = \begin{cases} \frac{r^{1-\alpha}}{1-\alpha} & \alpha > 0 \quad \alpha \neq 1 \\ \ln(r) & \alpha = 1 \end{cases}$$

## NUM Quantities

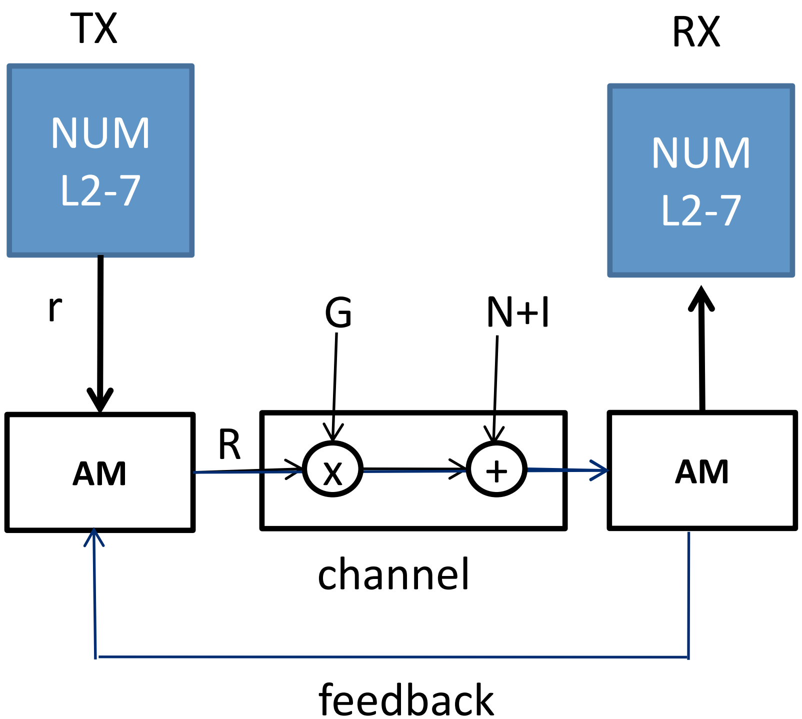
- Concave utility functions U(r)
  - Model upper layer protocols
- Source rates r
- Routing matrix A
- Fixed link capacities c

**But,**

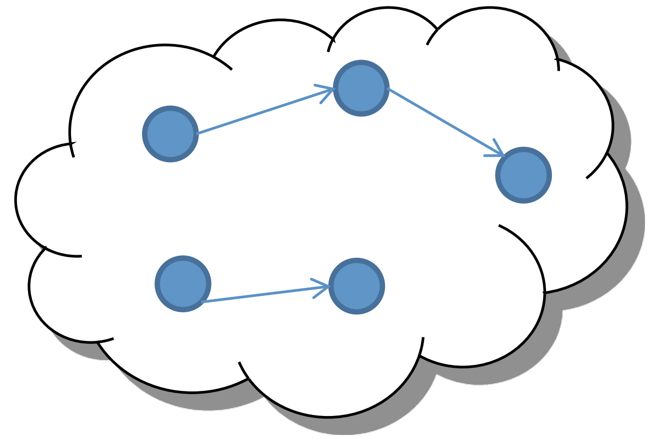
- Single Period
  - Traffic
  - Link states
- Not tied to random channel

# Stochastic NUM

**Link**




**Network**



Channel State $G \in \mathcal{R}^{nxn} \sim p(G)$
Noise $N \in \mathcal{R}^{n}$
Interference $I \in \mathcal{R}^{n}$

# Stochastic NUM

**Formally**

$$\begin{array}{ll} \max & \mathbf{E}[\sum U(r_i(G))] \\ \text{s.t.} & \\ & \mathbf{E}[A(G)r_i(G)] \leq \mathbf{E}[R_l(G,S)] \\ & \mathbf{E}[S_l(G)] \leq \bar{S} \\ & S_l(G) \geq 0 \\ & f(r(G),R(G)) = 0 \end{array}$$

**Comments**

- Channel state G revealed
- Seek policies
  - Source rate r(G)
  - Link Rate R(G,S(G))
- Subject to constraints
  - Queue stability
  - Average Tx Power
  - State constraints a.s.
- State based routing A(G)

# Method of Solution: FROEC

- Dual Decomposition

$$\max \quad \mathbf{E}[\sum U(r_i(G))] + \lambda_1 \mathbf{E}[A(G) r_i(G) - R_i(G,S)] + \lambda_2 \mathbf{E}[S_i(G) - \bar{S}]$$

- Realization based approach
  - Iterative unbiased estimates of Lagrange multipliers
  - Converges in E[] and in Probability
  - Learns $p(G) \Rightarrow \lambda$
- Price Interpretations for Lambdas
  - Trade offs in average “ link utilization” vs. average power
  - Trade offs in state, G, based backpressure, Tx power, capacity using estimates of lambda

# Single Link – Protocol Control

- Upper layer protocols know channel G and command link.

$$\begin{array}{ll} \max & \mathbf{E}[U(r(G))] \\ \text{s.t.} & \\ & r(G) = R(G, S(G)) \\ & \mathbf{E}[S(G)] \leq \bar{S} \\ & S_i(G) \geq 0 \end{array}$$

- Results:

$$R(G, S(G)) = \alpha \mathcal{W}(\theta);\ \mathcal{W} \text{ Lambert function and } \theta = \frac{[-\frac{KG}{N\lambda}]^{\frac{1}{\alpha}}}{\alpha}$$

$$\frac{S(G)}{\bar{S}} = \frac{-1}{[\alpha \mathcal{W}(\theta)]^{\alpha} \bar{S} \lambda} - \frac{N}{K(BER_{\text{target}}) \bar{S} G}.$$

- Where we assume

$$\begin{aligned} R(G, S(G)) &= \text{f( SNR) where f is concave e.g.} \\ &= \log\left(1 + K(BER) \frac{S(G)G}{N)}\right) \end{aligned}$$

# Single Link – Decupled Layers (Power vs. Backlog Tradeoffs)

- Buffer between upper layers and link

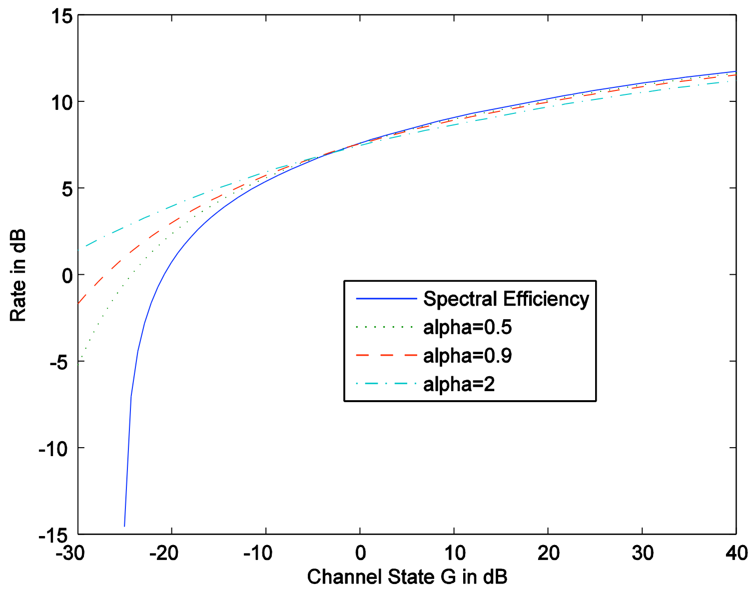
$$
\begin{aligned}
\max \quad & \mathbf{E}[U(r(G))] \\
\text{s.t.} \quad & \\
& \mathbf{E}[S_i(G)] \leq \bar{S} \\
& \mathbf{E}[r] = \mathbf{E}[R(G, S(G))] \\
& S(G) \geq 0
\end{aligned}
$$

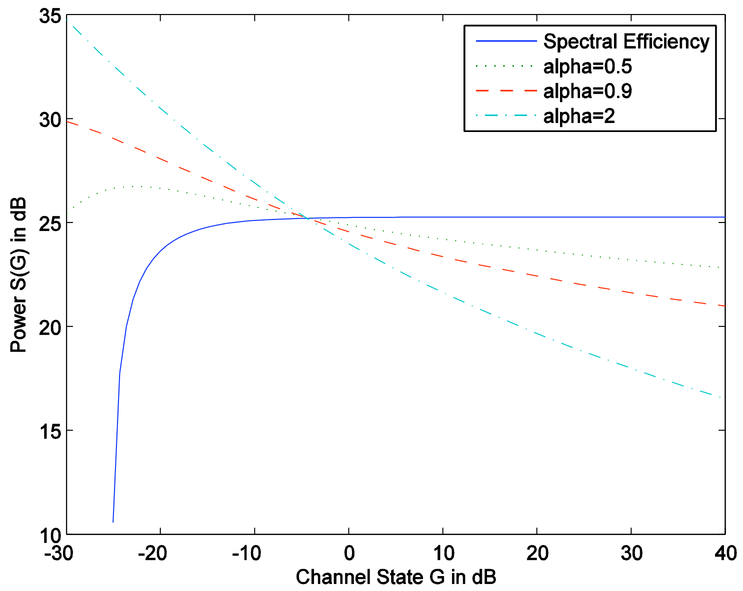
- Results:
  - FROEC: Lagrange "price" estimates trade off between backpressure and Tx power dynamically
  - Closed form solution (AM)
    - Fixed source rates r(G)=r
    - Rate adaptation

$$
R = \begin{cases} \log(\frac{\bar{S}G}{N\lambda_0}) & \bar{S}G \geq N\lambda_0 \\ 0 & \text{otherwise.} \end{cases}
$$

# Comparisons

Decoupling helps $\mathbf{E}[U(r)] \leq U(\mathbf{E}[r])$



# Interfering Links

- Dual Decomposition

$$\max \quad \mathbf{E}[\sum U(r_i(G))] + \lambda_1^T \mathbf{E}[A(G) r_i(G) - R_i(G,S)] + \lambda_2^T \mathbf{E}[S_i(G) - \bar{S}]$$

  - Solution has constant source rates (source/link decoupled)
- Set $\{\lambda_1^i\}$ marginal utility with change in average rate region. Set $\{\lambda_2^i\}$ marginal value of link power limits
- FROEC estimates trade off between backpressure on link i and Tx power on link j for a given realization at time k. $\left\{ \dfrac{\lambda_1^i(G_1 \ldots G_k)}{\lambda_2^j(G_1 \ldots G_k)} \right\}$