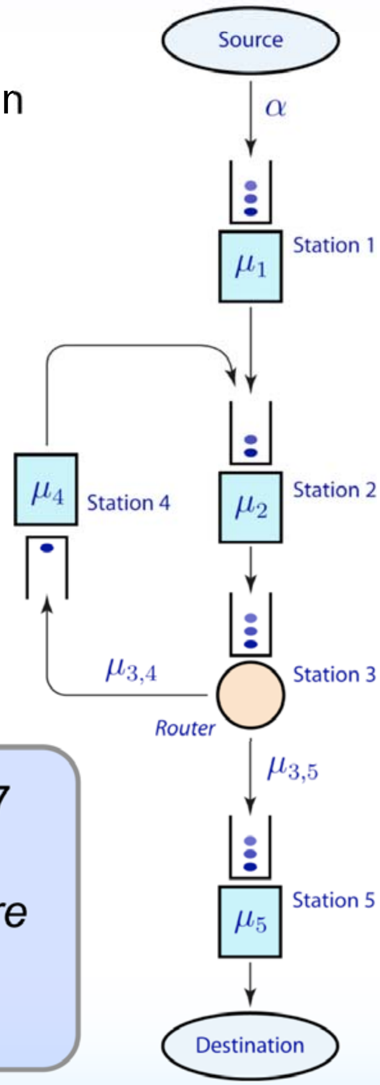
Optimizing MaxWeight For Routing

Wei Chen and Sean P. Meyn

ECE & CSL University of Illinois

MaxWeight: Issues Raised at July Meeting

Routing requires *information.* In the MaxWeight policy, this information is obtained through queue length values. This can lead to irrational behavior when information is scarce.



Issues addressed < 7/07

- Why does MW work?
- Understanding of important structure led to h-MW policies
- Performance evaluation and approximate optimality
- Analysis based on CTCN

Issues addressed > 7/07

- Improving robustness using logarithmic perturbation
- Specialization to routing
- Investigation of special structure for relaxations, h-MW policy
- Simulation studies

Example: (and Leith, 2007 Subramanian, submitted). *MaxWeight or Backpressure routing will send packets upstream*!

MaxWeight can be improved once it is better understood

Context of the reported work

Upper bounding techniques	
Novel techniques	
New application of old techniques	
Unconsummated union – applicable "layers"	
Application layer	
Transport layer	
Routing layer	
Scheduling/queuing layer	
Physical layer	
Dynamics	
Arbitrary movement	*Possible extension*
Random waypoint	
No mobility	
Capacity-achieving techniques and architectures	
Yes, new techniques	
No, old techniques	

Assumptions	
Realistic	
Unrealistic but **robust**	
Overhead, feedback	
Ignored	
Considered	
Topologies	
Canonical problems	
Infinite / scaling law	
Finite *but arbitrary*	
Other considerations	
Low latency as well as optimal throughput	
Proposed network coding application: Relaxations are required to reduce complexity	
Adaptation	
Mobility a potential extension	

Work shows how important global information can be used *if available.*
Generally, amount of global information required for approximate optimization is low

Optimizing MaxWeight for Routing

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ACHIEVEMENT DESCRIPTION

STATUS QUO

What is the state of the art and what are its limitations?

MW routing *inflexible with respect to performance improvement*

MW corresponds to *h-myopic*, with *h* quadratic. ***Key geometric property of quadratic identified by Meyn prior to July meeting.***

NEW INSIGHTS

KEY NEW INSIGHTS:

- New perturbation technique:

$$\tilde{x}_i = x_i \log(1 + x_i/\theta)$$
$$h(x) = h_0(\tilde{x})$$

- Application to routing & refinements for decentralization
- Heavy traffic optimality
- Taylor series approximation gives interpretation as adaptive MaxWeight - Diagonal matrix adapts to varying congestion

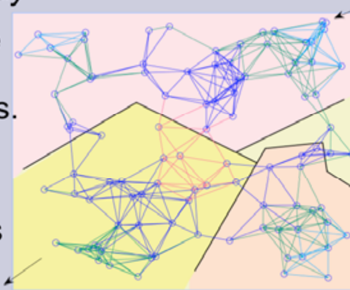
MAIN RESULT:

h-myopic policy is universally stabilizing

Application to policy synthesis for approximately optimal performance (delay or backlog) in heavy traffic, with log regret

Numerical study underway

Investigate performance and feasibility: 100 nodes, multiple arrivals. Only wireline models investigated to-date.



Excellent performance as predicted by theory

Decentralized implementation appears feasible.

HOW IT WORKS:

Step 1: Estimation of network cuts

Step 2: Estimation of congestion on either side

Step 3: Choice of h_0 - piecewise quadratic

Special case: Single dominant destination gives h_0 quadratic function of workload, cost, and effective cost w.r.t. *workload relaxation*

END-OF-PHASE GOAL

- Decentralized implementation, use of consensus algorithms
- Wireless models: Apply D. Shah's insights on maxproduct convergence
- Full analysis of multiple bottlenecks
- Integration with Network Coding projects: *Can we code around network hot-spots*?

COMMUNITY CHALLENGE

- Un-consummated union challenge: Integrate coding and resource allocation
- Generally, solutions to complex decision problems should offer insight

Algorithms for dynamic routing: Visualization and Optimization

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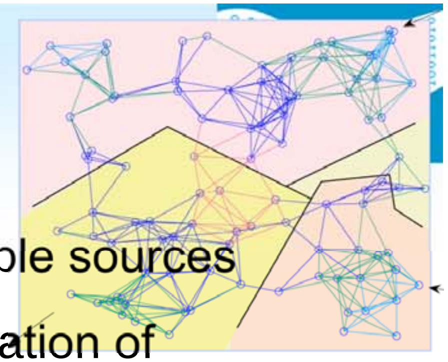
Simulations for Single Traffic Stream

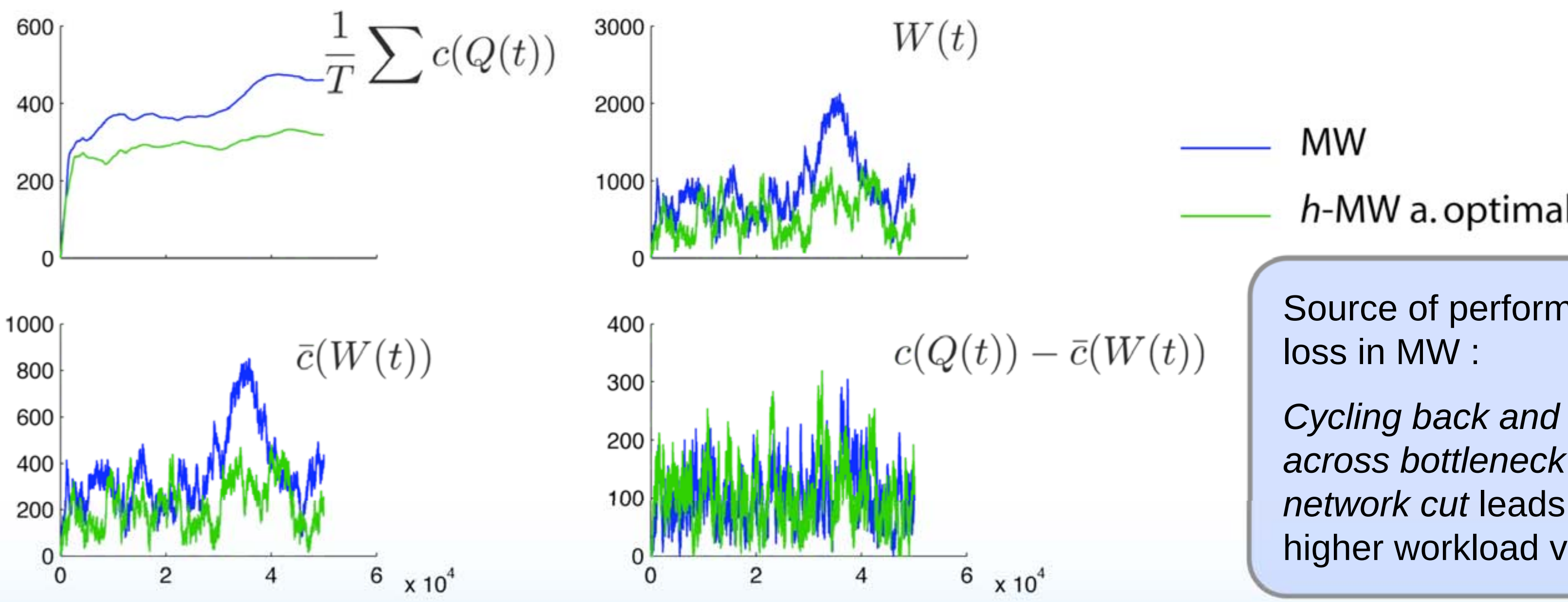


- Network approximately 100 nodes. Single destination, multiple sources
- MaxWeight compared to policy based on logarithmic perturbation of

$$h_0(x) = c(x) = \sum x_i \qquad\qquad h_0(x) = \hat{J}(w) + \frac{b}{2}[c(x) - \bar{c}(w)]^2$$

Greedy — *Approximation of DP solution*

- Simulation for high load: 50% improvement over greedy, 25% over MW



Source of performance loss in MW :

Cycling back and forth across bottleneck network cut leads to higher workload values

Performance improves for functions *h* that more closely approximate DP solution

Summaries and challenges

CONCLUSIONS: Alignment of workload vector and ℓ_1 cost leads to vastly simplified analysis and implementation

Logarithmic perturbation gives universally stabilizing policies.

For large θ, Taylor series allows interpretation of policy as *adaptive* MW

Performance improvement over MW as expected in simulations

Simulations verify that tighter approximations to the DP solution results in better performance

SCIENTIFIC FOUNDATIONS

Stochastic Lyapunov theory combined with relaxation techniques based on workload to approximate DP solution

HOW BAD IS THE REAL WORLD?

The real world is very bad. Without attention to bottleneck network cuts, a decentralized routing algorithm will create inefficiency through cycling.

PERFORMANCE? Only stability has been established for logarithmic perturbation, though results from simulation studies give optimism.

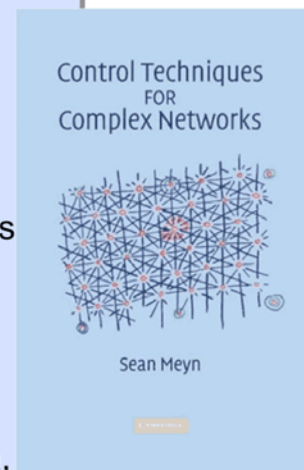
CAN WE LEARN? Less learning is needed in routing models than first anticipated. Key is the location of bottleneck links. How can this information be shared? Coordination with Ozdgalar and Shah will likely bridge this gap

CAN WE CODE? With the identification of dynamic bottlenecks, it is then evident where the capacity region can be improved

Largest current research bottleneck concerns
learning dynamic bottleneck location and workload

References

- S. P. Meyn. Sequencing and routing in multiclass queueing networks. Part I: Feedback regulation. 40(3):741–776, 2001.
- S. P. Meyn. Sequencing and routing in multiclass queueing networks. Part II: Workload relaxations. 42(1):178–217, 2003.
- S. P. Meyn. Stability and asymptotic optimality of generalized MaxWeight policies. Under revision for SIAM J. Control Opt. (Preliminary version to appear at the 46th IEEE Conference on Decision and Control, December 2007).
- W. Chen and S. P. Meyn Optimizing MaxWeight For Routing. In preparation.
- S. P. Meyn. Control Techniques for Complex Networks. Cambridge University Press 2007.



References in on-going research

- *Iterative Scheduling Algorithms*, M. Bayati, B. Prabhakar, D. Shah and M. Sharma, Proceedings of IEEE Infocom 2007
- *Distributed Subgradient Methods for Multi-agent Optimization* Angelia Nedic and Asuman Ozdaglar. Preprint 2007.
- *Polynomial Complexity Algorithms for Full Utilization of Multi-hop Wireless Networks* Atilla Eryilmaz, Asuman Ozdaglar and Eytan Modiano. Preprint 2007.

Optimizing MaxWeight: From July Meeting

FLoWS

STATUS QUO

What is the state of the art and what are its limitations?

Static routing: *ignores dynamics*

MW routing: *inflexible with respect to performance improvement*

Subramanian & Leigh 2007*: MW can be irrational*

NEW INSIGHTS

KEY NEW INSIGHTS:

MW **=** *h-myopic* for a fluid model, with *h* quadratic

Fluid model: $\frac{d}{dt}q(t) = \Delta_u(q(t))$	h-myopic policy: $\arg\min_u \langle \nabla h(x), \Delta_u(x) \rangle$

Key geometric property of quadratic is identified:

$$\frac{\partial}{\partial x_i} h(x) = 0 \qquad \text{whenever } x_i = 0$$

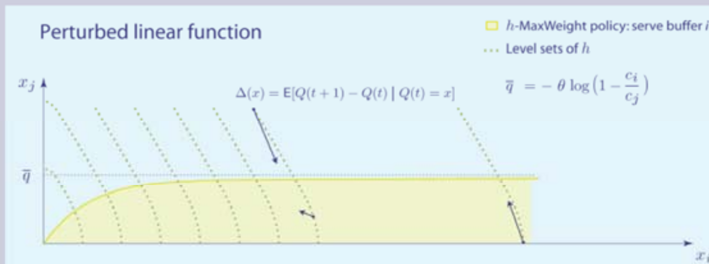
Leads to broad new classes of policies

ACHIEVEMENT DESCRIPTION

MAIN RESULT:

Perturbation technique to generate functions with appropriate geometry

Application to policy synthesis for approximately optimal performance (delay or backlog) in heavy traffic, with logarithmic regret



HOW IT WORKS:

Key analytical tool is Lyapunov theory for Markov processes: The function *h* satisfies Condition (V3) of Meyn & Tweedie 1992; An exponentiated version satisfies (V4)

For approximate optimality, *workload relaxation* Relaxation also provides tool for visualization of high dimensional dynamics. Optimal solutions evolve in region containing *monotone region* for the effective cost.

END-OF-PHASE GOAL

- Decentralized implementation: Policy can be designed to use available information.
- Adaptation - on-line policy improvement
- Full analysis of multiple bottlenecks
- Integration with Network Coding projects: *Can we code around network hot-spots*?

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