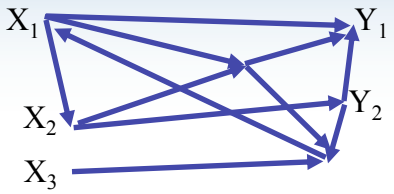
On Matroidal Solutions for Network Coding.

A. Cohen, M. Effros, S. ElRouayheb and R. Koetter.

FLoWS

STATUS QUO



Network coding capacities are well-understood for some demands. Little theory is developed for general demands:

- Negative results.
- Suboptimal bounds.
- Network scaling laws.

NEW INSIGHTS

Given an arbitrary solution for an arbitrary network, we try to show that there exists an equivalent (in terms of rates) matroidal solution. This may result in:

- A better understanding on which tools to apply. The set over which a solution is sought can be much smaller and more structured.
- Conclusions regarding very general networks.
- Usage of known results from matroid theory.

ACHIEVEMENT DESCRIPTION

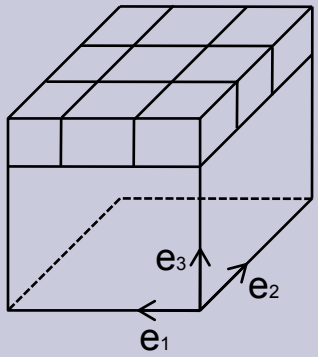
CONJECTURE:

Any arbitrary network solution can be approximated by a matroidal solution (i.e., with integer-valued entropies) over a possibly larger alphabet size.

Consequence: We can limit the search to matroidal solutions, where the entropy of any subset of the edges is an integer.

HOW IT WORKS:

- Show that any multivariate distribution over finite alphabets can be approximated by a *dyadic distribution,* with an excess entropy of at most 2 bits regardless of the alphabet size.
- Extend the result to show that all marginal distributions can also be made dyadic, with a negligible excess entropy as the alphabet size increases.
- Dyadic distributions translate to *rational entropies*.
- Since the conditional entropies are also approximated with an arbitrary precision, the resulting solution is also implementable.



Note: *the conjecture above aims at arbitrary random vectors, independent of any network setting or topology.*

ASSUMPTIONS AND LIMITATIONS:

- Assuming noiseless links (common network coding setting).
- Large alphabet size: large block length, hence a large delay.
- Assuming each source is i.i.d. and memoryless operations in the nodes.

END-OF-PHASE GOAL

- **Complete the proof of the discussed conjecture.**
- **Extend the result to non-i.i.d. sources and non-memoryless nodes.**
- **Discuss concrete examples where the existence of a matroidal solution solves a problem or changes our perspective on it.**
- **Extend our understanding of subclasses of matroids and their relation to network solutions.**

COMMUNITY CHALLENGE

- ***Generalize known random coding techniques to matroidal solutions.***
- ***Apply the results to different networking problems, such as noisy networks.***

We may limit our attention (and the toolbox we use) to matroidal solutions.