Information Theory for Mobile Ad-Hoc Networks (ITMANET): The FLoWS Project

Distributed Optimization Methods for General Performance Metrics with Quantized Information and Local Constraints

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Distributed Control and Optimization Methods for Wireless Networks

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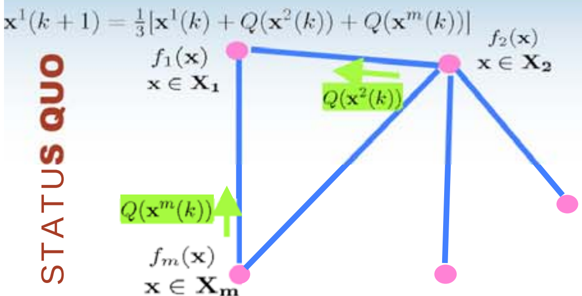
FLoWS

STATUS QUO



Two shortcomings of distributed optimization literature in designing algorithms for wireless networks:

-Existing distributed optimization methods assume that agents can transmit and process continuous values without taking into account physical layer constraints

- Most work focuses on distributing performance measures. There is no systematic methodology for designing algorithms that can distribute general local constraints on decision variables.

NEW INSIGHTS

Our earlier work on subgradient methods and consensus policies can be combined with specific quantization rules and local projections leading to algorithms that can operate in the presence of:
- **communication bandwidth and storage constraints**
- **local constraints on decisions**
- **time-varying connectivity**

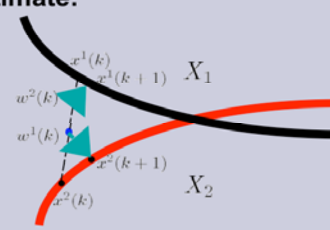
ACHIEVEMENT DESCRIPTION

MAIN RESULTS:

- **We developed a distributed optimization algorithm for optimizing general performance measures in the presence of communication bandwidth constraints.**
- **We provided a performance analysis that show the dependence on the available bandwidth.**
- **We developed a constrained consensus algorithm, and provided convergence and convergence rate results.**

HOW THEY WORK:

- **Quantized optimization algorithm: Each user maintains an estimate of the optimal solution. At each step, he**
  - **combines his estimate with that of his neighbors,**
  - **performs a subgradient step using his local performance measure, and**
  - **quantizes the resulting estimate.**
- **Constrained consensus algorithm: Each user asynchronously combines local estimates and projects the resulting estimate onto his local constraint set**



ASSUMPTIONS-LIMITATIONS:

- **We considered a particular quantization scheme that leads to a convergent algorithm.**
- **We have not modeled the effect of the channel noise.**
- **The optimization algorithm is unconstrained.**

END-OF-PHASE GOAL

•We will explore effects of different quantization schemes and channel noise on the performance of the algorithms.

•We will combine the constrained consensus policy with the multi-agent optimization algorithm.

COMMUNITY CHALLENGE

Design of optimization algorithms that address the challenges and constraints associated with large-scale time-varying networks

Distributed control and optimization algorithms in the presence of quantization effects and local constraints

Motivation

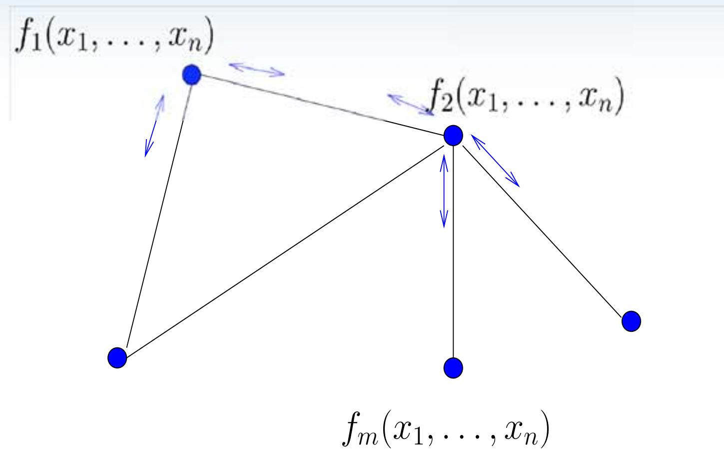
- Increasing interest in distributed optimization and control of ad hoc wireless networks, which are characterized by:
  - Lack of centralized control and access to information
  - Time-varying connectivity
- Control-optimization algorithms deployed in such networks should be:
  - Distributed relying on local information
  - Robust against changes in the network topology
- Most focus in this area has been on the canonical **consensus problem**
- Goal: Given initial values of agents, establish conditions under which through local interactions and computations, agents asymptotically agree upon a common value
- Examples:
  - Control of moving vehicles (UAVs): alignment of the heading angle
  - Information processing in sensor networks: computing averages of initial local observations (i.e., consensus on a particular value)
- Existing work:
  - No optimization of different objectives corresponding to multiple agents
  - Assumes complete (unquantized) information available
  - No constraints on agent values

Multi-Agent Optimization Model

- Consider a network with node set
  $V = \{1, \ldots, m\}$
- Agents want to cooperatively solve the problem

$$\min_{x \in X} \sum_{i=1}^{m} f_i(x)$$

- Function $f_i(x) : R^n \to R$ is a performance measure known only by node *i*



- Agents update and exchange information at discrete times $t_0, t_1, \ldots$
- Agent *i* information state is denoted by $x^i(k) \in R^n$ at time *tk*

Agent *i* Update Rule:($X = R^n$)

$$x^i(k+1) = \sum_{j=1}^{m} a^i_j(k) x^j(k) - \alpha^i(k) d^i(k)$$

$a^i_j(k)$ are weights, $\alpha^i(k)$ is stepsize, $d^i(k)$ is subgradient of f_i at $x^i(k)$

Time-varying communication is modeled by matrix $A(k)$ [columns $a^i(k)$]

Assumptions

Assumption (Weights)

- There exists a scalar $0 < \eta < 1$ s.t. for all i, $a_i^i(k) \geq \eta$ for all $k \geq 0$. If $a_j^i(k) > 0$, then $a_j^i(k) \geq \eta$.

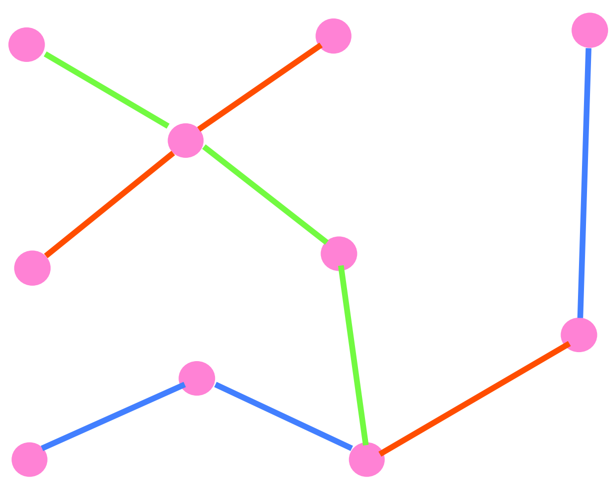
•(Doubly Stochastic Weights)

(a) $\sum_{j=1}^{m} a_j^i(k) = 1$ for all i and k.

(b) $\sum_{i=1}^{m} a_j^i(k) = 1$ for all j and k.

Assumption (Information Exchange)

- Agent *i* influences any other agent infinitely often – connectivity
- Agent *j* send his information to neighboring agent *i* within a bounded time interval – bounded intercommunications



Linear Dynamics and Transition Matrices

- Compact representation of agent local-update relation: for $k \geq s$

$$
\begin{aligned}
x^i(k+1) &= \sum_{j=1}^{m} [\Phi(k,s)]_j^i x^j(s) \\
&- \sum_{r=s}^{k-1} \left(\sum_{j=1}^{m} [\Phi(k,r+1)]_j^i \alpha^j(r) d_j(r) \right) - \alpha^i(k) d_i(k).
\end{aligned}
$$

where $\Phi(k,s)$ are transition matrices from time s to k:

$$\Phi(k,s) = A(s)A(s+1)\cdots A(k-1)A(k) \qquad \text{for all } k \geq s$$

- We analyze convergence properties of the distributed method by establishing:
  - Convergence of transition matrices (shown in July meeting)
  - Convergence of stopped “subgradient updates”

Convergence of Transition Matrices

Proposition: Let weights rule, connectivity, and bounded intercommunication interval assumptions hold:

- The limit $\bar{\Phi}(s) = \lim_{k\to\infty} \Phi(k,s)$ exists for each s.
- The limit matrix $\bar{\Phi}(s)$ has identical columns and the columns are stochastic, i.e.,
$$\bar{\Phi}(s) = \phi(s)e',$$
where $\phi(s) \in R^m$ is a stochastic vector for each s.
- For every i, $[\Phi(k,s)]_i^j$, $j = 1, ..., m$, converge to the same limit $\phi_i(s)$ as $k \to \infty$ with a geometric rate, i.e., for all i, j and all $k \geq s$,
$$\left|[\Phi(k,s)]_i^j - \phi_i(s)\right| \leq 2\,\frac{1+\eta^{-B_0}}{1-\eta^{B_0}}\,\left(1-\eta^{B_0}\right)^{\frac{k-s}{B_0}}$$
where η is the lower bound on weights, B is the intercommunication interval bound, and $B_0 = (m-1)B$.

Main Convergence Result

Proposition: Let the subgradients of *fi* be uniformly bounded by a constant *L*. Then for every *i*, the averages $\hat{x}^i(k)$ of estimates $x^i(0), \ldots, x^i(k-1)$ are such that

$$f(\hat{x}^i(k)) \leq f^* + \frac{m\ dist^2(y(0), X^*)}{2\alpha k} + \alpha L^2 \left(\frac{C}{2} + 2mC_1\right)$$

where $f = \sum_i f_i$, f^* is the optimal value, and X^* is the optimal set of the problem, $y(0) = \frac{1}{m}\sum_i x^i(0)$, $C = 1 + 8mC_1$ and

$$C_1 = 1 + \frac{m}{1 - (1 - \eta^{B_0})^{\frac{1}{B_0}}} \frac{1 + \eta^{-B_0}}{1 - \eta^{B_0}},$$

with η minimal weight, $B_0 = (m-1)B$, B intercommunication bound.

- Estimates are per iteration
- Capture tradeoffs between accuracy and computational complexity

Quantized Method

FLoWS

- When agents can send only a finite number of bits
- Agent *i* estimate is given by

$$x^i(k+1) = \sum_{j=1}^{m} a_j^i(k) x_Q^j(k) - \alpha^i(k) d^i(k)$$

$$x_Q^i(k+1) = \lfloor x^i(k+1) \rfloor$$

$\lfloor \cdot \rfloor$ represents rounding down to the nearest integer multiple of $1/Q$

- The consensus may not converge when rounding to the nearest integer [Kashyap, Basar, and Srikant 06]
- The quantization effects can be viewed as errors in the original process

$$x_Q^i(k+1) = x^i(k+1) - e^i(k), \qquad e^i(k) = x^i(k+1) - \lfloor x^i(k+1) \rfloor$$

- The error is bounded and it turns out it decreases to zero

Convergence Result for the Quantized Method

Proposition: Let the subgradients of *fi* be uniformly bounded by a constant *L*. Then for every *i*, the averages $\hat{x}_Q^i(k)$ of estimates $x_Q^i(0), \ldots, x_Q^i(k-1)$ are such that

$$f(\hat{x}_Q^i(k)) \leq f^* + \frac{m\ dist^2(y(0), X^*)}{2\alpha k} + \alpha L^2 \left(\frac{C_Q}{2} + 2mC_1 \right)$$

where f^* is the optimal value of $f = \sum_i f_i$, X^* is the optimal set,

$$y(0) = \frac{1}{m} \sum_i x_Q^i(0), \qquad C_Q = 1 + \frac{8m}{\alpha L} \left(\alpha L + \frac{\sqrt{m}}{Q} \right) C_1$$

$$C_1 = 1 + \frac{m}{1 - (1 - \eta^{B_0})^{\frac{1}{B_0}}} \frac{1 + \eta^{-B_0}}{1 - \eta^{B_0}}, \qquad B_0 = (m-1)B$$

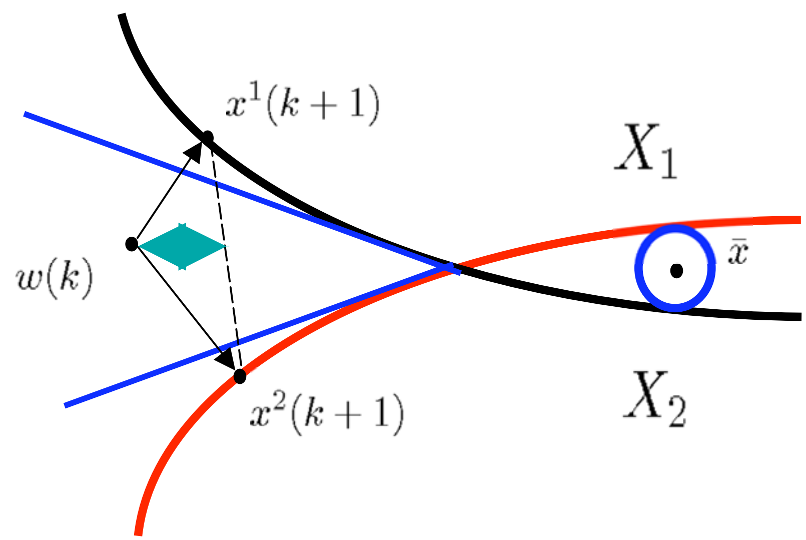
- We have $\lim_{Q \to \infty} C_Q = 1 + 8mC_1$
- The estimate reduces to the one without quantization

Constrained Consensus Policy

- Agent *i* has a nonempty closed convex constraint set X_i
- We assume that the intersection set $X = \cap_{i=1}^{m} X_i$ is nonempty
- Agent *i* updates subject to his constraint set

$$x^i(k+1) = P_{X_i}\left[\sum_{j=1}^{m} a_j^i(k)\, x^j(k)\right]$$

- We have under interior point assumption on the intersection set *X*
  - Convergence result
  - Geometric rate estimate



Conclusions

- We presented a general distributed optimization method for general performance measures
- We provided convergence analysis and convergence rate estimates
  - For unconstrained multi-agent optimization
  - For constrained consensus
- Future Work:
  - Combination of optimization and constrained consensus
  - Analysis of other quantization schemes, delays, and noise
- Papers:
  - Nedic and Ozdaglar, "Distributed Asynchronous Subgradient methods for Multi-Agent Optimization," to appear in IEEE Transactions on Automatic Control, 2007
  - Ozdaglar, "Constrained Consensus and Alternating Projections," Allerton, 2007
  - Nedic, Olshevsky, Ozdaglar, Tsitsiklis, "On Distributed Averaging Algorithms and Quantization Effects," submitted for publication, 2007