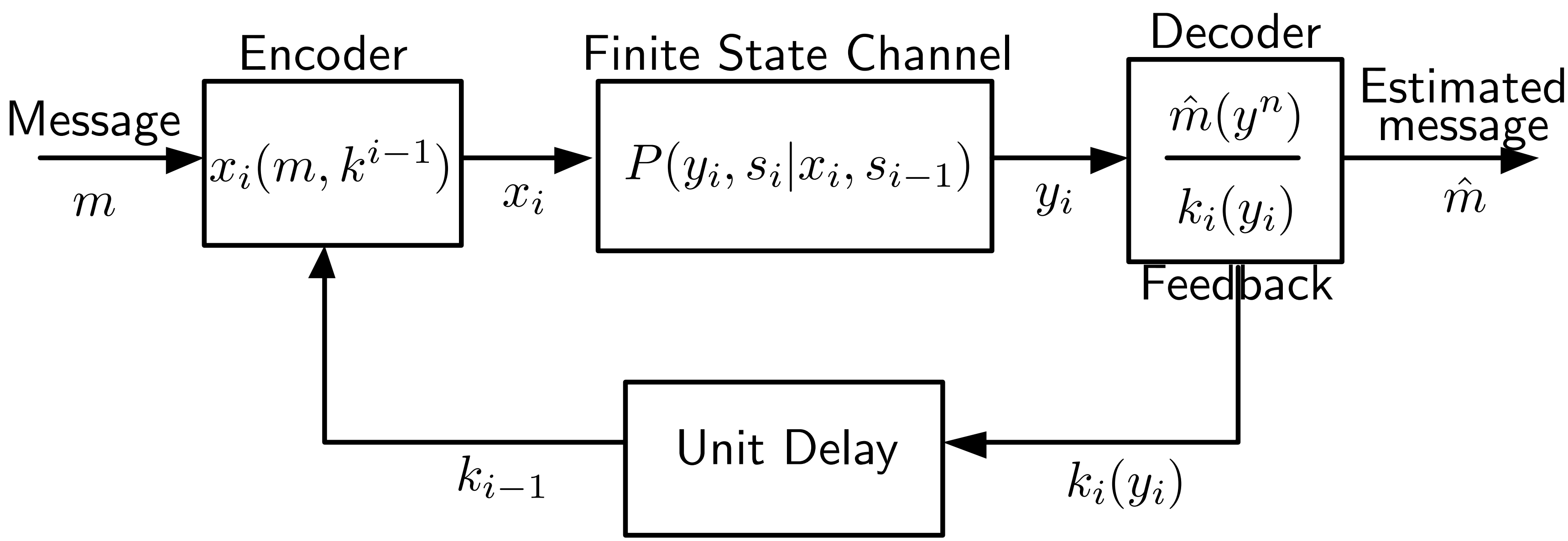
Communication Setting



Finite State Channel(FSC) property: $p(y_i, s_i|x^i, s^{i-1}, y^{i-1}) = p(y_i, s_i|x_i, s_{i-1})$

Question

What is the capacity of channel in this setting?

Answer

Theorem 1: [Gallager] For FSC without Feedback

$$\lim_{n\to\infty} \frac{1}{n} \max_{p(x^n)} I(X^n; Y^n) \geq C \geq \lim_{n\to\infty} \frac{1}{n} \max_{p(x^n)} \min_{s_0} I(X^n; Y^n | s_0)$$

and if FSC is indecomposable then the bounds are equal.

Theorem 2: [This poster] For FSC with Feedback

$$\lim_{n\to\infty} \frac{1}{n} \max_{p(x^n || k^{n-1})} I(X^n \to Y^n) \geq C_{FB} \geq \lim_{n\to\infty} \frac{1}{n} \max_{p(x^n || k^{n-1})} \min_{s_0} I(X^n \to Y^n | s_0)$$

and if FSC is indecomposable without Intersymbol Interference (ISI) then the bounds are equal.

Directed Information and Causal Conditioning

Directed Information [Massey90]

$$I(X^n \to Y^n) \triangleq \sum_{i=1}^{n} I(X^i; Y_i | Y^{i-1})$$

$$I(X^n; Y^n) = \sum_{i=1}^{n} I(X^n; Y_i | Y^{i-1})$$

Causal Conditioning Distribution[Kramer98, Massey05]:

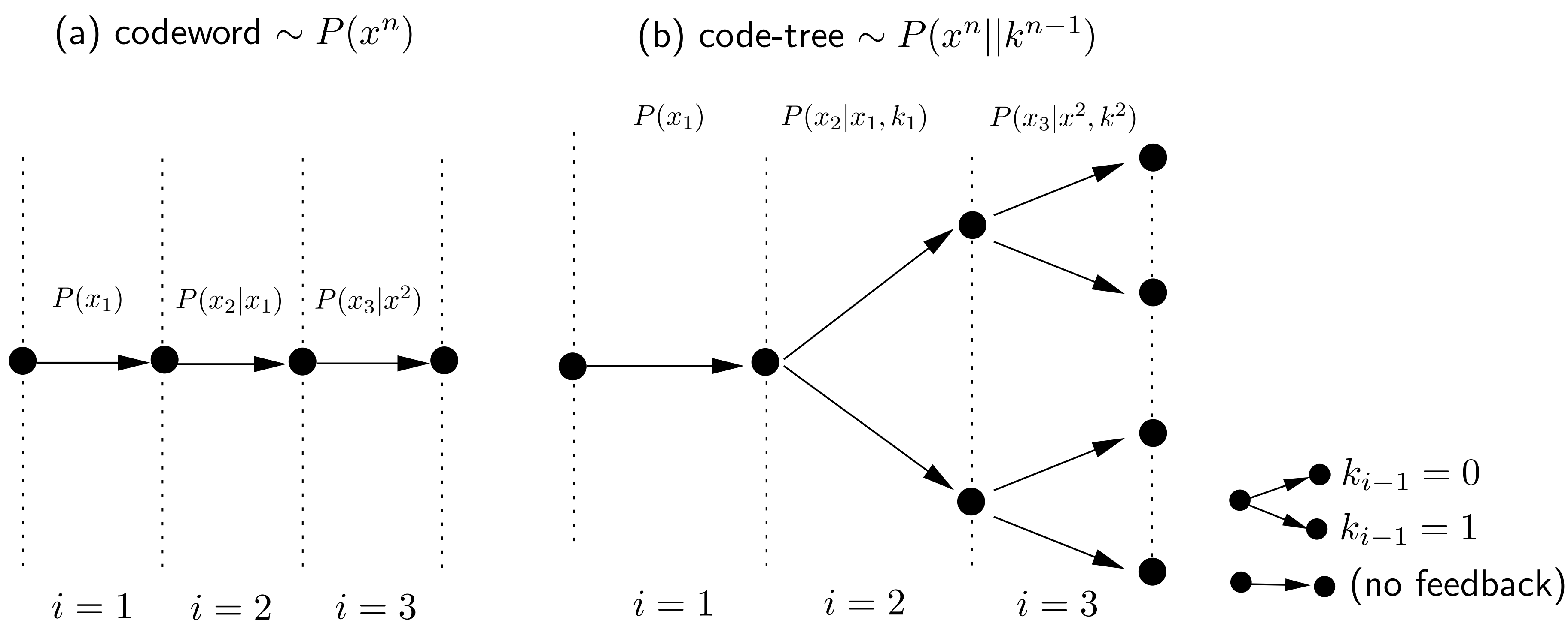
$$p(y^n || x^{n-1}) \triangleq \prod_{i=1}^{n} p(y_i | x^{i-1}, y^{i-1})$$

$$p(y^n | x^n) = \prod_{i=1}^{n} p(y_i | x^n, y^{i-1})$$

Causal conditioning distribution

$$P(x^n||k^{n-1}) \triangleq \prod_{i=1}^{n} P(x_i|x^{i-1}, k^{i-1})$$

An example for binary feedback



Few lemmas

Lemma 1:

$$p(x^n, y^n) = p(x^n || y^{n-1}) p(y^n || x^n)$$
$$p(x^n, y^n) = p(x^n) p(y^n | x^n)$$

Lemma 2:

$$\sum_{x^n} p(x^n || y^{n-1}) = 1$$
$$\sum_{x^n} p(x^n) = 1$$

Lemma 3:

$$|I(X^n \to Y^n) - I(X^n \to Y^n | S)| \leq H(S)$$
$$|I(X^n; Y^n) - I(X^n; Y^n | S)| \leq H(S)$$

Proof for the case of feedback

1. Randomly chosen code-tree, with distribution $\prod_{i=1}^{n} p(x_i|x^{i-1}, y^{i-1}) = p(x^n||y^{n-1})$.

2. Maximum likelihood decoder: $\arg\max p(y^n|m) = \arg\max p(y^n||x^n)$

3. Show that if

$$R < \sum_{y^n} \sum_{x^n} p(x^n||y^{n-1}) \cdot p(y^n||x^n) \ln \frac{p(y^n||x^n)}{\sum_{x^n} p(x^n||y^{n-1}) p(y^n||x^n)},$$

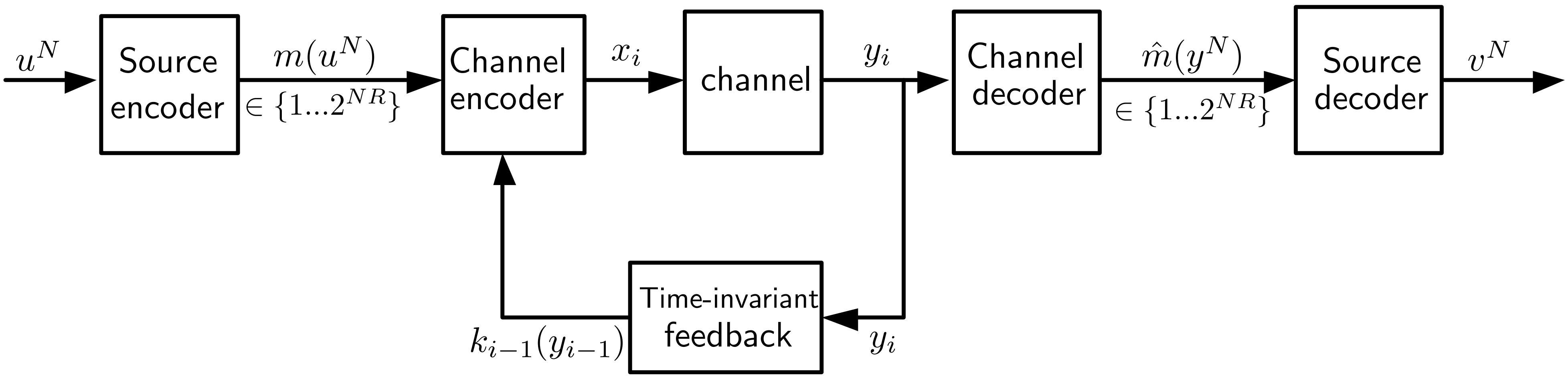
   then $P_e \to 0$.

4. show that

$$\sum_{y^n} \sum_{x^n} p(x^n||y^{n-1}) \cdot p(y^n||x^n) \ln \frac{p(y^n||x^n)}{\sum_{x^n} p(x^n||y^{n-1}) p(y^n||x^n)} = I(X^n \to Y^n)$$

New insights

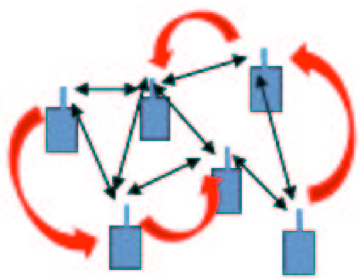
- Source-channel coding separation is optimal for any stationary and ergodic source and for any channel with time-invariant deterministic feedback,



- Feedback does not increase the capacity of a connected FSC when the state of the channel is known both at the encoder and the decoder.

Next Step: Wireless Network

- We will extend the capacity results to multi users settings. Existence of feedback can simplify communication schemes in wireless network!



- Based on the point-to-point channel the MAC has been solved.

- Significant progress has been done on broadcast channel.

- We will investigate the capacity of multihop networks with noisy and delayed feedback.