Directed Information a powerful tool for finding capacity of wireless channels with feedback

Feedback and Directed Information in Wireless Networks
A. Goldsmith (joint with H. Permuter and T. Weissman)

Role of feedback links in capacity of wireless channels and networks not well understood.

Feedback often modelled in terms of imperfect/quantized CSI at the transmitter, but not clear CSI is the best thing to send on a FB channel. Insight and appropriate coding strategies are largely absent.

Directed information is a dual for mutual information on finite-state Markov channels with feedback.

The source-channel separation theorem holds for time-invariant deterministic feedback.

If channel state known at encoder and decoder, then feedback does not increase capacity.

For finite-state Markov channels with feedback, capacity is based on directed information rather than mutual information.

\[
\lim_{n \to \infty} \frac{1}{n} \max_{p(x^n|y^n)} I(X^n \to Y^n) \geq C_{FB} \geq \lim_{n \to \infty} \frac{1}{n} \max_{p(x^n|y^n)} \min_{p(y^n)} I(X^n \to Y^n|S^n)
\]

End-of-phase goal:
- Determine if directed information can be used to obtain capacity of finite state broadcast channels
- Investigate application of directed information to general wireless networks with feedback

Graduate Level:
Extensions for wireless networks.

Prize level:
Capacity results for multihop networks with noisy and delayed feedback

Assumptions and limitations:
- The feedback is the channel output with unit delay and no noise. Bounds are only tight for indecomposable channels.

Achievement description:

Main result:
- For finite-state Markov channels with feedback, capacity is based on directed information rather than mutual information.

How it works:
- Directed information \( I(X \to Y) \) is the natural extension of mutual information for channels with output feedback.

\[
I(X^n \to Y^n) \triangleq \sum_{\ell=1}^{\infty} I(X^{\ell}; Y_{\ell}|X^{\ell-1})
\]

Many techniques from Gallager's capacity proof for FSC without FB can be used in our proof, but not all.