The Price of Selfishness in Network Capacity

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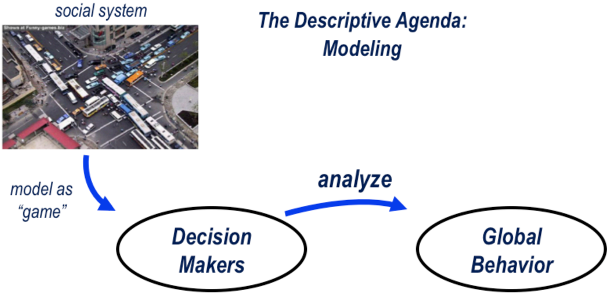
Setting the Scene

- Capacity of network of noisy channels $=$
  Network coding capacity of corresponding network
  $\Rightarrow$ Study network coding capacity
- Achieving network coding capacity may require
  - Central control
  - Full network knowledge
- Typical MANET
  - No central controller
  - Incomplete network knowledge
- Goal: Understand optimal *achievable* performance
  $\Rightarrow$ Study best performance of independent users

Tool: Game Theory

Osborne and Rubinstein, 1994: Game theory is a family of "... tools designed to help us understand the phenomena that we observe when decision-makers interact."

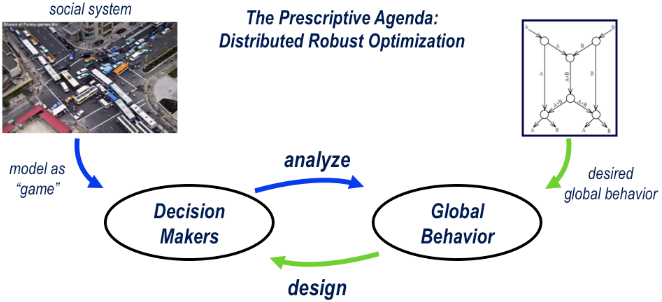


GOALS:

- Model socio-cultural environment
- Explain and predict experimental or observational data

Tool: Game Theory

Recently: Game theory also useful for engineering design

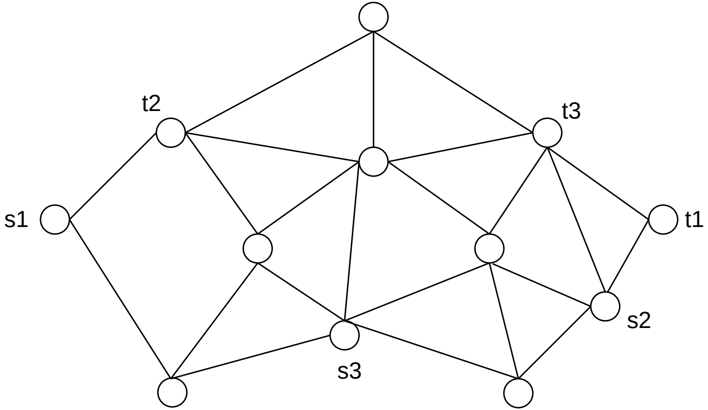


GOAL:

- Establish a metric for global performance
- Design local cost functions to encourage good global performance

Multiple Unicast Problem

Multiple unicast flows in shared wireless network
Possible transmissions indicated by edges of graph

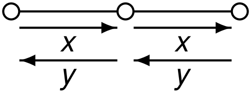


Cost of solution = # transmissions per packet (steady state)

Reverse Carpooling

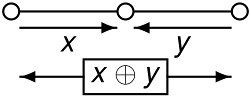
Limited form of network coding

Opportunity for network coding arises when
two unicasts traverse same node in opposite directions



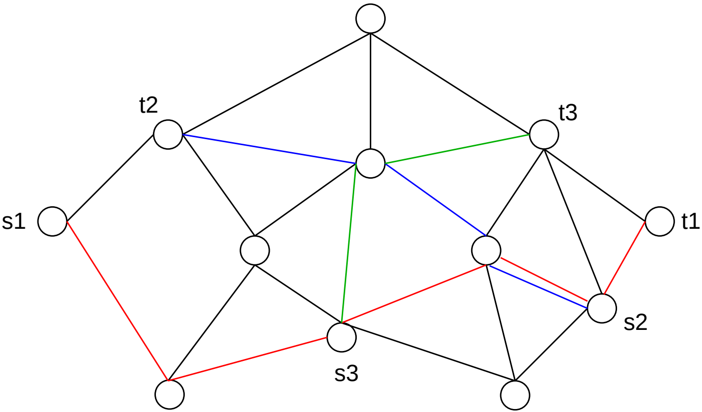
Without network coding, 4 transmissions are required

With network coding, 3 transmissions are required



The Network Coding Game

Optimal design *possible* but *not feasible* for MANETs



Players = unicasts

Global cost = # transmissions per packet (steady state)

GOAL: Design local costs to encourage globally good behavior

Formal Set-Up

Players:	$\{(s_1, t_1), \ldots, (s_n, t_n)\}$
Action profile:	$a = (a_1, \ldots, a_n) \in \mathcal{A}$
	$a_i =$ sequence of nodes from $s_i \to t_i$
Global cost:	$C : \mathcal{A} \to \mathbb{R}$
	$C(a) =$ total transmission (w/reverse carpooling)
Local cost:	$J_i : \mathcal{A} \to \mathbb{R}$, $i \in \{1, \ldots, n\}$
	Design opportunity
Design goals:	J_i depends only on information "local" to (s_i, t_i)
	Global cost is low

Measuring Global Cost

Players independently seek low local costs

Equilibrium: No unilateral change improves local cost

$$a \in \mathcal{A} \text{ s.t. } J_i(a_i, a_{-i}) = \min_{a_i'} J_i(a_i', a_{-i})$$

Properties

- Equilibrium $\not\Rightarrow$ optimal cost
- Optimal cost $\not\Rightarrow$ equilibrium
- Equilibria not equally good

Game Theory Tools

- Learning algorithms give convergence to an equilibrium
- Equilibrium selection (for some $J_i(a)$ & learning envir.)

Stability vs. Mobility:

- Rate of mobility affects time available for convergence
- Time available for convergence should inform learning goal

Measuring Global Cost

$$\begin{aligned} \mathcal{E}(G) &= \{a \in \mathcal{A} : a \text{ is an equilibrium for game } G\} \\ a^* &= \arg\min_{a \in \mathcal{A}} C(a) \end{aligned}$$

Price of Anarchy: (Worst case, worst cost equilibrium)

$$POA = \sup_{G} \max_{a \in \mathcal{E}(G)} \frac{C(a)}{C(a^*)}$$

Price of Stability: (Worst case, best cost equilibrium)

$$POS = \sup_{G} \min_{a \in \mathcal{E}(G)} \frac{C(a)}{C(a^*)}$$

Cost Function 1

Global Cost

$$J_i(a) = C(a) = \sum_i \left[N^{>}(a_i) + \frac{1}{2} N^{=}(a_i) \right]$$

where

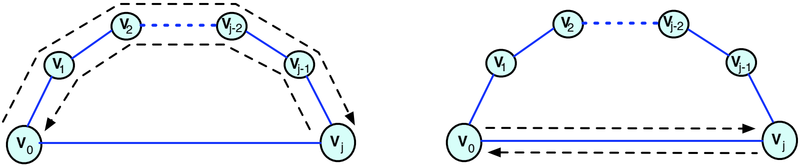
$$N^{*}(a_i) = \#\text{ of transmissions in unicast } i \text{ in * direction}$$

Equilibria exist:

$$a^* \in \arg\min_{a \in \mathcal{A}} C(a)$$

is an equilibrium

$$POA = \infty \qquad POS = 1$$



Nash Equilibrium

Optimal

Cost Function 2

Wonderful Life Cost

$$J_i(a) = C(a) - C(a_i^0, a_{-i}) = N^{>}(a_i)$$

Equilibria exist:

$$\begin{array}{rrcl} & J_i(a_i', a_{-i}) & \leq & J_i(a_i'', a_{-i}) \\ \Leftrightarrow & C(a_i', a_{-i}) - C(a_i^0, a_{-i}) & \leq & C(a_i'', a_{-i}) - C(a_i^0, a_{-i}) \\ \Leftrightarrow & C(a_i', a_{-i}) & \leq & C(a_i'', a_{-i}). \end{array}$$

$$a^* \in \arg\min_{a \in \mathcal{A}} C(a) \text{ is an equilibrium}$$

$$POA = \infty \qquad POS = 1$$

Optimal POS. Only *local* information. Poor POA.

Innovation

"Potentially Wonderful Life Cost"

$$J_i(a) = \Phi(a) - \Phi(a_i^0, a_{-i})$$

for Φ arbitrary.
Equilibria exist:

$$\begin{array}{rrcl} & J_i(a_i', a_{-i}) & \leq & J_i(a_i'', a_{-i}) \\ \Leftrightarrow & \Phi(a_i', a_{-i}) - \Phi(a_i^0, a_{-i}) & \leq & \Phi(a_i'', a_{-i}) - \Phi(a_i^0, a_{-i}) \\ \Leftrightarrow & \Phi(a_i', a_{-i}) & \leq & \Phi(a_i'', a_{-i}) \end{array}$$

$$a^* \in \arg\min_{a \in \mathcal{A}} \Phi(a) \text{ is an equilibrium}$$

GOAL: Design Φ to improve POA subject to constraint on POS

Potentially Wonderful Life Cost

Example:

$$\Phi(a) = \sum_{i=1}^{n} |a_i| + (\alpha - 1)C(a)$$

$$\begin{aligned}
\text{where } |a_i| &= \text{\# of transmissions in unicast } i \text{ using solution } a \\
\alpha - 1 &= \text{non-negative Lagrangian constraint}
\end{aligned}$$

Potentially Wonderful Life Cost:

$$\begin{aligned}
J_i(a_i, a_{-i}) &= \Phi(a) - \Phi(a_i^0, a_{-i}) \\
&= N^{\leq}(a_i) + \alpha N^{>}(a_i)
\end{aligned}$$

Potentially Wonderful Life Cost

Potentially Wonderful Life Cost:

$$J_i(a_i, a_{-i}) \quad = \quad N^{\leq}(a_i) + \alpha N^{>}(a_i)$$

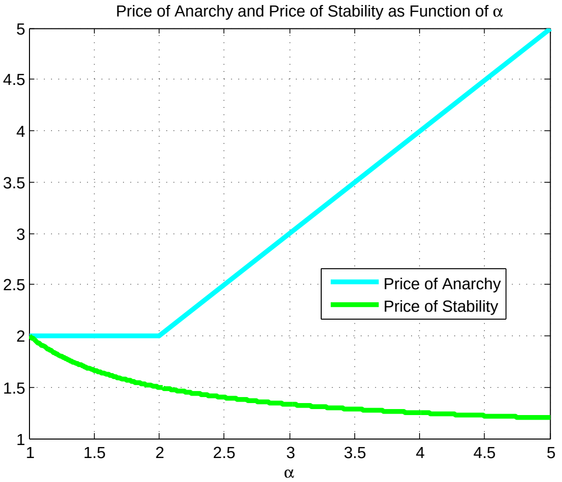
Price of Anarchy

$$POA = \begin{cases} 2 & \alpha \in [1, 2] \\ \alpha & \alpha \in (2, \infty) \end{cases}$$

Price of Stability

$$POS = \frac{\alpha + 1}{\alpha}$$

Potentially Wonderful Life Cost



Optimality

Can any local cost achieve lower POA?

$$J_i(a_i, a_{-i}) = N^{\leq}(a_i) + \alpha N^{>}(a_i) \quad \Rightarrow \quad POA \geq 2$$

$$J_i \text{ does not depend on network structure} \quad \Rightarrow \quad POA \geq 2$$

Can any local cost achieve better tradeoff (POA vs. POS)?

No. Given tradeoff optimal.