Optimizing Wireless Network Performance in Random Environments

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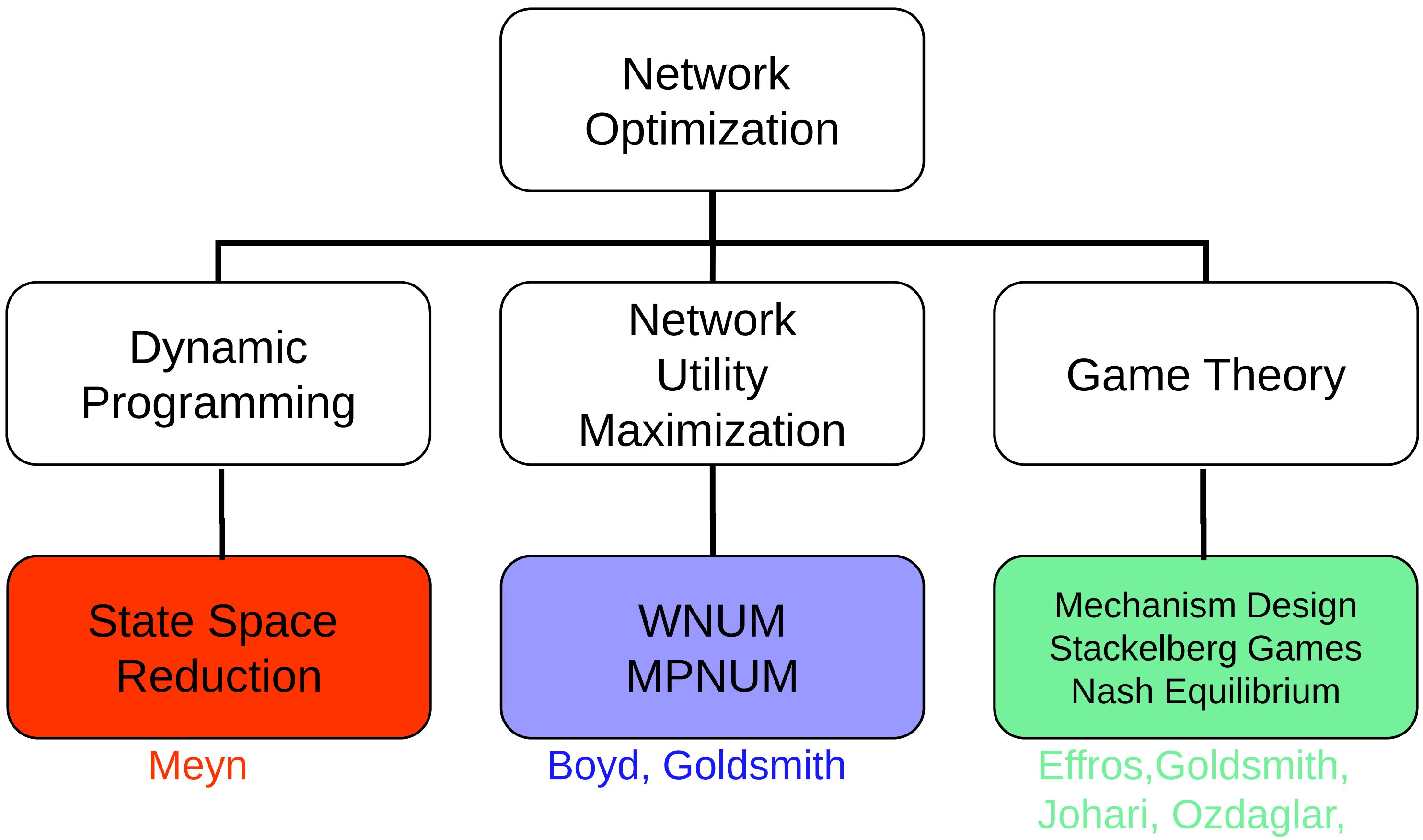
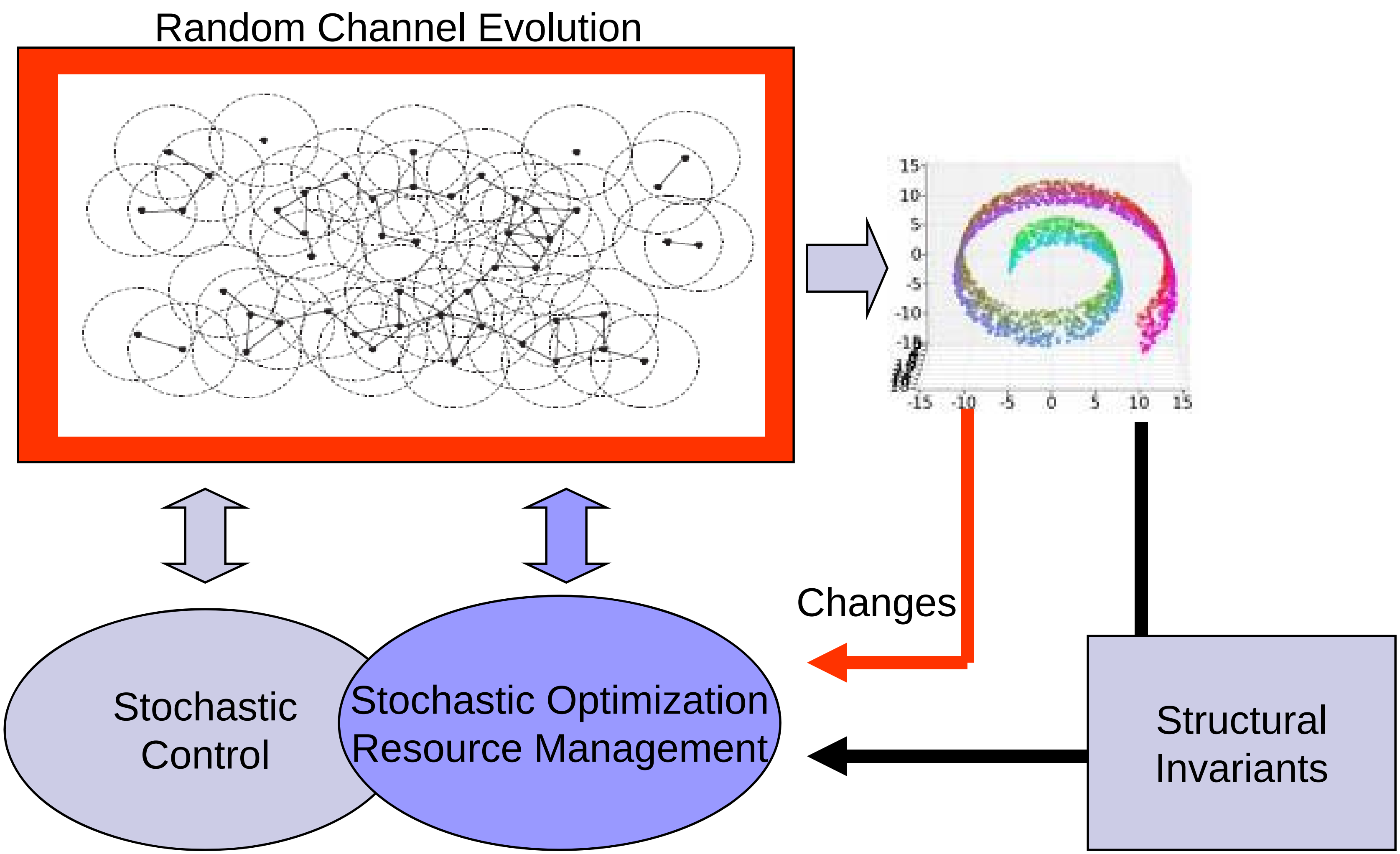
Approaches to Network Optimization*



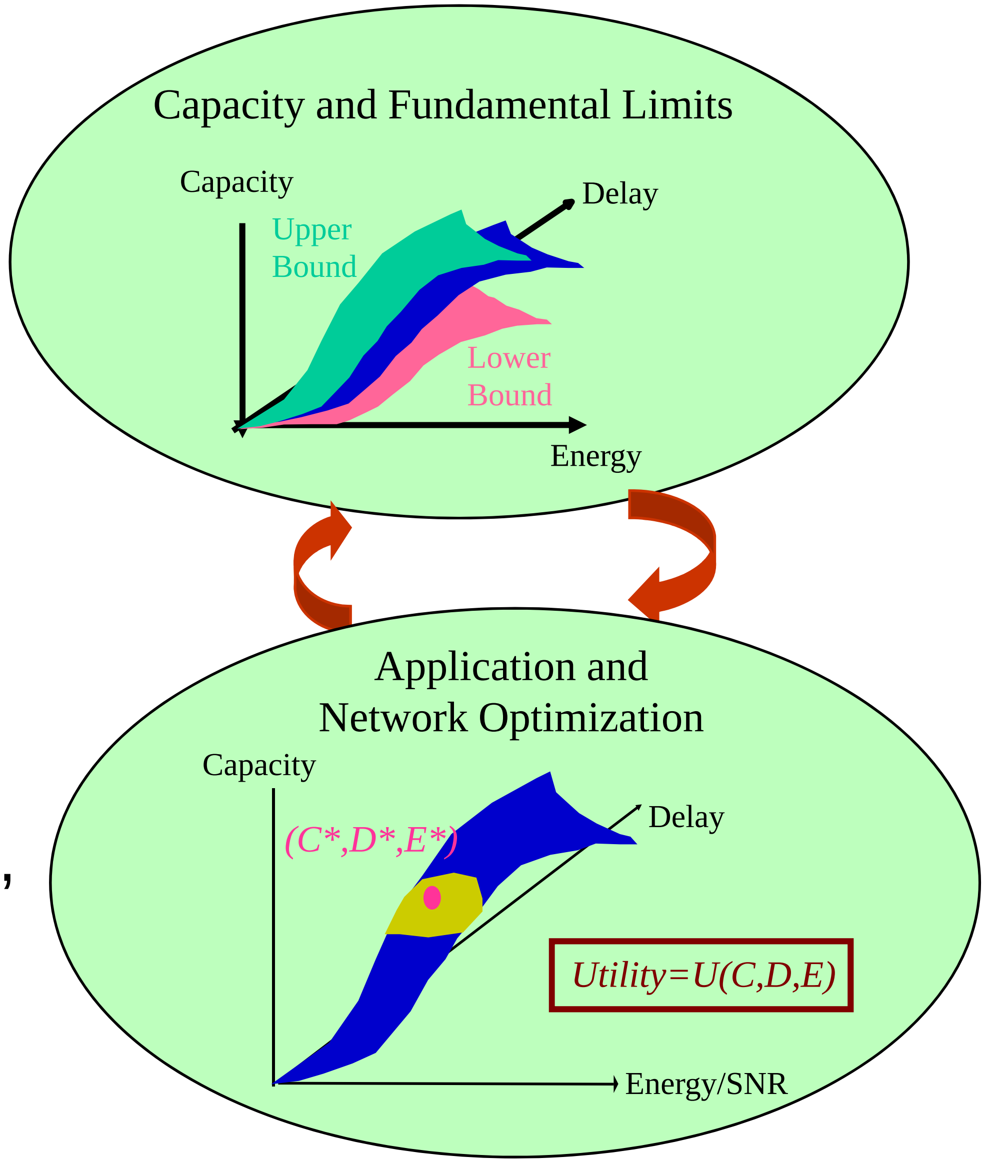
*Most prior work is for wired/static networks

Optimization requires models and tools



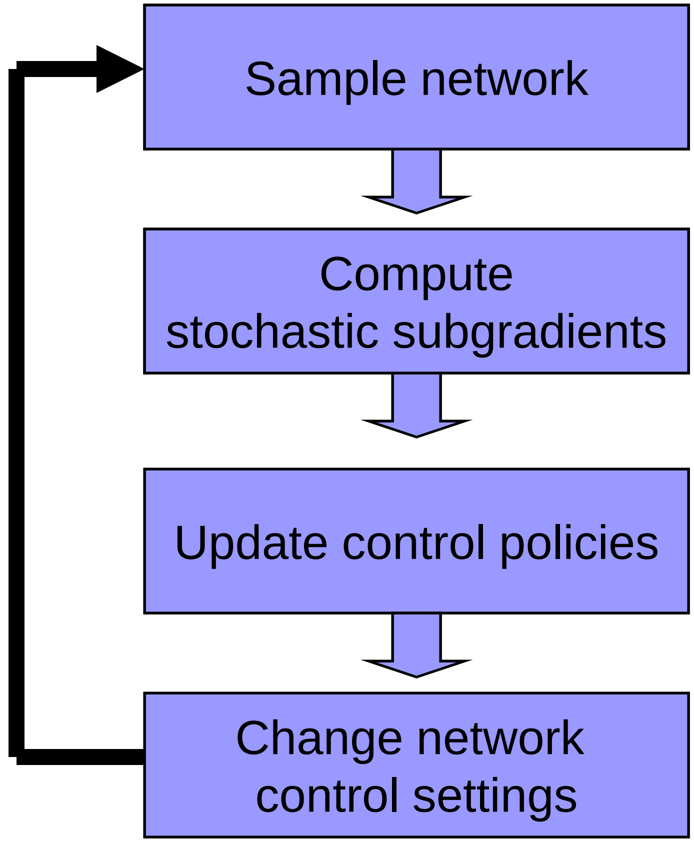
Models come from Thrusts 1 and 2

- Time-varying capacity regions define underlying link rates
- Currently incorporating generalized relaying at PHY into model
- When models unknown, need online learning



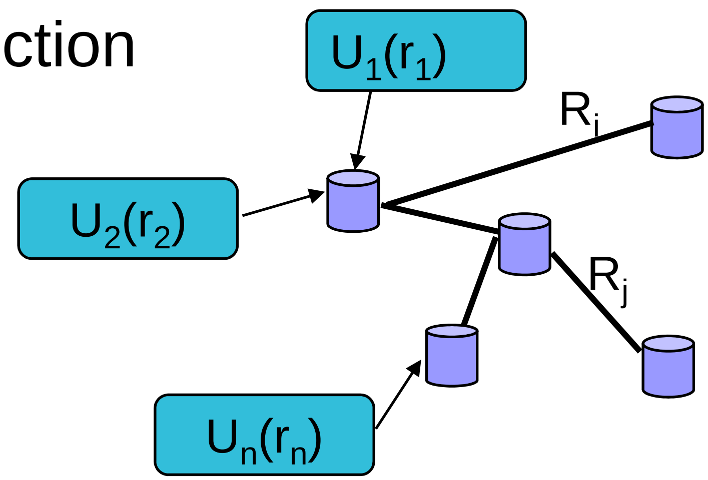
On-Line Learning

- Samples network
- Iteratively finds best control policies
- Application of stochastic approximation



Network Utility Maximization

- Maximizes a network utility function
- Assumes
  - Steady state
  - Reliable links
  - Fixed link capacities



- Dynamics are only in the queues

flow k

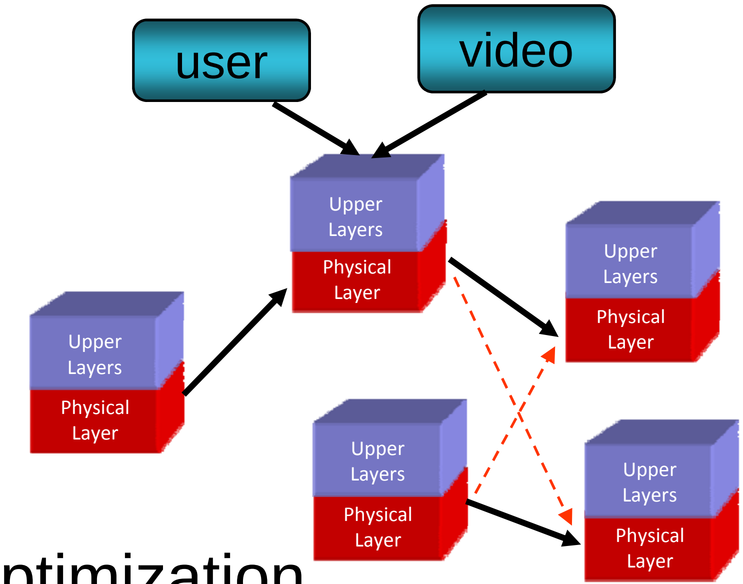
$$\max \quad \sum U_k(r_k)$$

$$s.t. \qquad Ar \leq R$$

routing

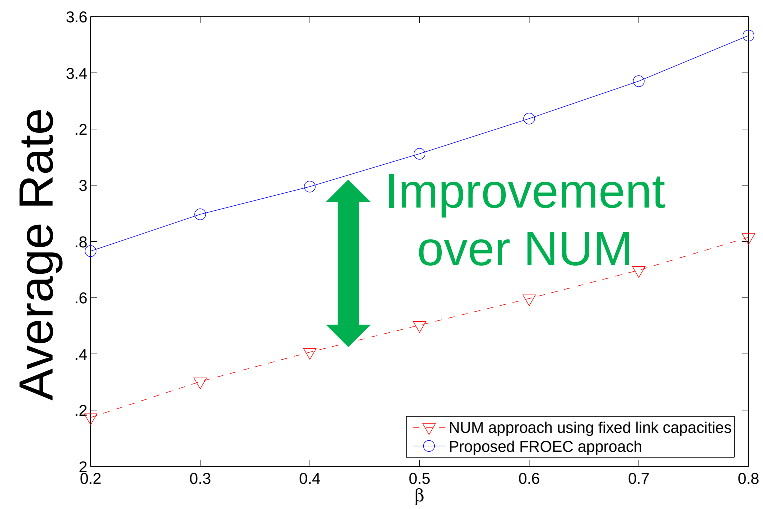
Fixed link capacity

Wireless NUM



- Extends NUM to wireless networks
  - Random lossy links
  - Error recovery mechanisms
  - Network dynamics
- Network control as stochastic optimization

$$\begin{aligned} &\max && E[\textstyle\sum U(r_m(G))] \\ &\textbf{st} && E[r(G)] \le E[R(S(G),G)] \\ & && E[S(G)] \le \overline{S} \end{aligned}$$



- Next steps
  - Adaptive PHY layer and reliability
  - Existence convergence properties
  - Channel estimation errors

Beyond WNUM

- WNUM Limitations
  - Adapts to channel and network dynamics
  - Cross-layer optimization of PHY and higher layers
  - *But* limited to elastic traffic flows

- MPNUM extends WNUM
  - Traffic can have defined start and stop times
  - Traffic QoS metrics can me met
  - General capacity regions can be incorporated
  - Multiple time periods explicitly captured

Multi-Period Stochastic Control of Networks

Stephen Boyd and Ekine Akuiyibo

ITMANET PI meeting 03/5–6/09

The problem and issues

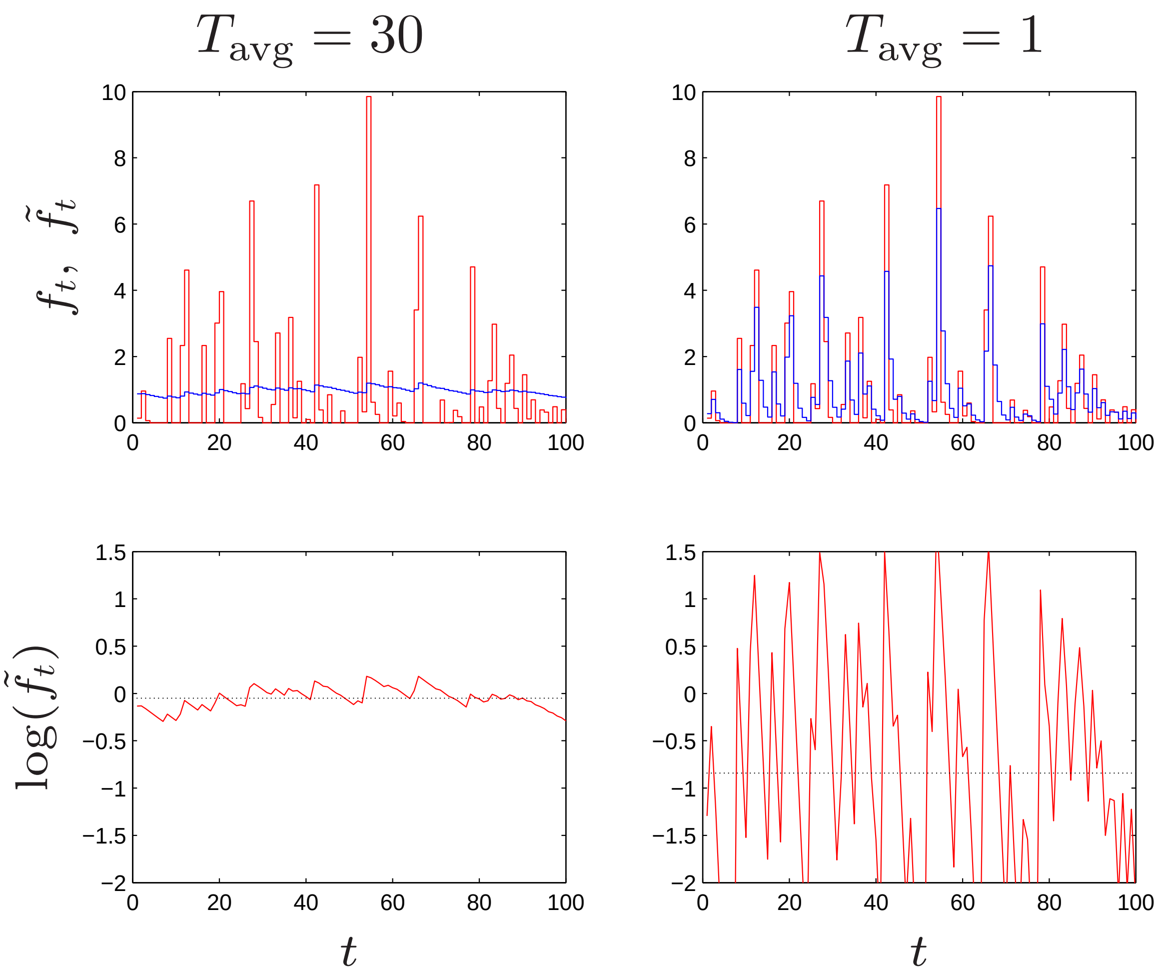
- goal: multi-period resource ($e.g.$, flow rate, power) allocation
- resources ($e.g.$, link capacities, channel states) vary randomly
- maximize utility (or minimize cost) that reflects different
  - weights (priorities)
  - desired/required target levels
  - **averaging time scales**

  for different flows
- unifies what we called before WNUM, DNUM, SNUM

Dynamic utility and averaging time scale

- f_t is flow in period t, $t = 0, 1, \ldots$
- U : **R** $\rightarrow$ **R** gives utility derived for a flow value
- $\tilde{f}_t = (1-\theta)\sum_{\tau=0}^{t} \theta^\tau f_{t-\tau}$ is (first order) **smoothed** or **averaged** flow
- $T_{\rm avg} = 1/\log(1/\theta)$ gives smoothing time scale
- average smoothed utility is $\bar{U} = \lim_{t\rightarrow\infty}(1/t)\,\mathbf{E}\sum_{\tau=0}^{t} U(\tilde{f}_\tau)$
- when U is not linear, $\bar{U}$ depends on smoothing time scale $T_{\rm avg}$
- our claim: flow utility should be judged dynamically, *i.e.*, by a **utility function** and an **averaging time scale**

Example



Utility state

- represent smoothing via linear dynamical system
- $f_t \in \mathbf{R}^n$ is vector of flows
- $x_{t+1} = \Theta x_t + (I - \Theta) f_t$, $\Theta = \mathbf{diag}(\theta)$
- $(x_t)_i = \tilde{f}_i(t)$ is smoothed flow or **utility state**
- average smoothed utility is

$$\bar{U} = \lim_{t \to \infty} (1/t)\, \mathbf{E} \sum_{\tau=0}^{t} \sum_{i=1}^{n} U((x_\tau)_i)$$

Stochastic flow control

- must have $f_t \in \mathcal{R}_t$ (rate region)
- in general, $\mathcal{R}_t$ is random process on sets; we'll assume $\mathcal{R}_t$ are IID
- stochastic flow control:

  choose f_t *as function (policy) of* $\mathcal{R}_t$, x_t, *to maximize* $\bar{U}$
- can solve in principle via DP
- can solve exactly in only a few special cases
- lots of approximate solution methods

Linear quadratic formulation

- a special case for which we can get the exact solution
- rate region is defined by (random) capacity c_t: $\mathcal{R}_t = \{z \mid \mathbf{1}^T z = c_t\}$
- utilities are concave quadratic: $U(z) = -(x - x^{\rm tar})^T Q(x - x^{\rm tar})$
  ($e.g.$, negative mean square deviation from target values)
- leads to (nonstandard) linear quadratic stochastic control problem
  - has random equality constraints
  - value function is quadratic; can be found from a Riccati-like recursion
  - optimal control law is messy, but affine in x_t, c_t

Optimal policy

- optimal policy is $f_t = Kx_t + wc_t + s$
- K, w, s found by complicated formulas and iterations
- $\mathbf{1}^T K = 0$, $\mathbf{1}^T w = 1$, $\mathbf{1}^T s = 0$, so we have $\mathbf{1}^T f_t = c_t$ $(i.e.,\ f_t \in \mathcal{R}_t)$
- interpretations:
  - w gives the capacity allocation
  - s is an offset
  - K is the utility state to flow gain matrix

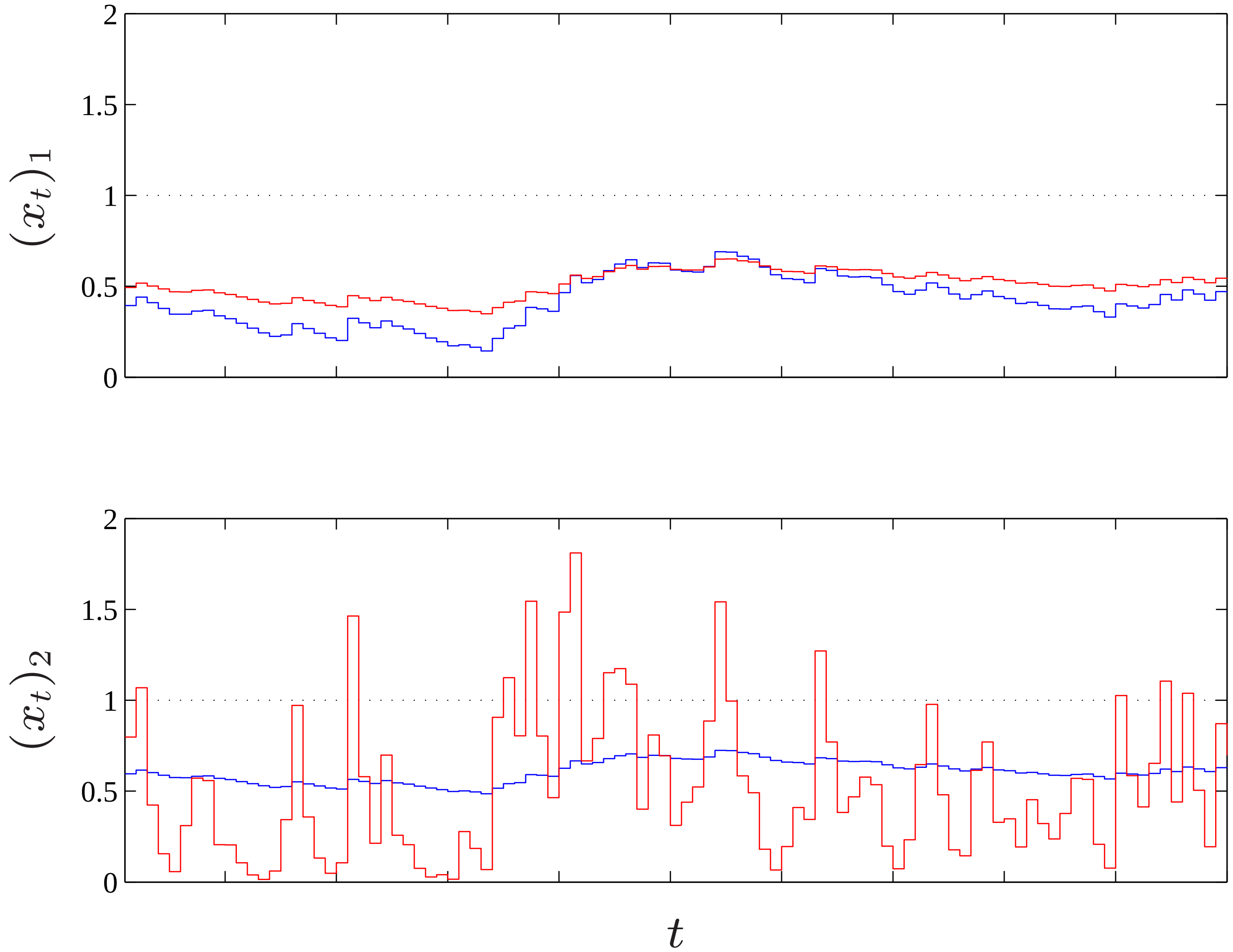
(Simplest possible) example

- 2 flows share one link
- $U(a) = -(a-1)^2$ ($i.e.$, target flow values are one)
- smoothing times $T_{\mathrm{avg}} = 1$, $T_{\mathrm{avg}} = 30$
- link capacity c_t is exponential with mean 1.5
- optimal policy:

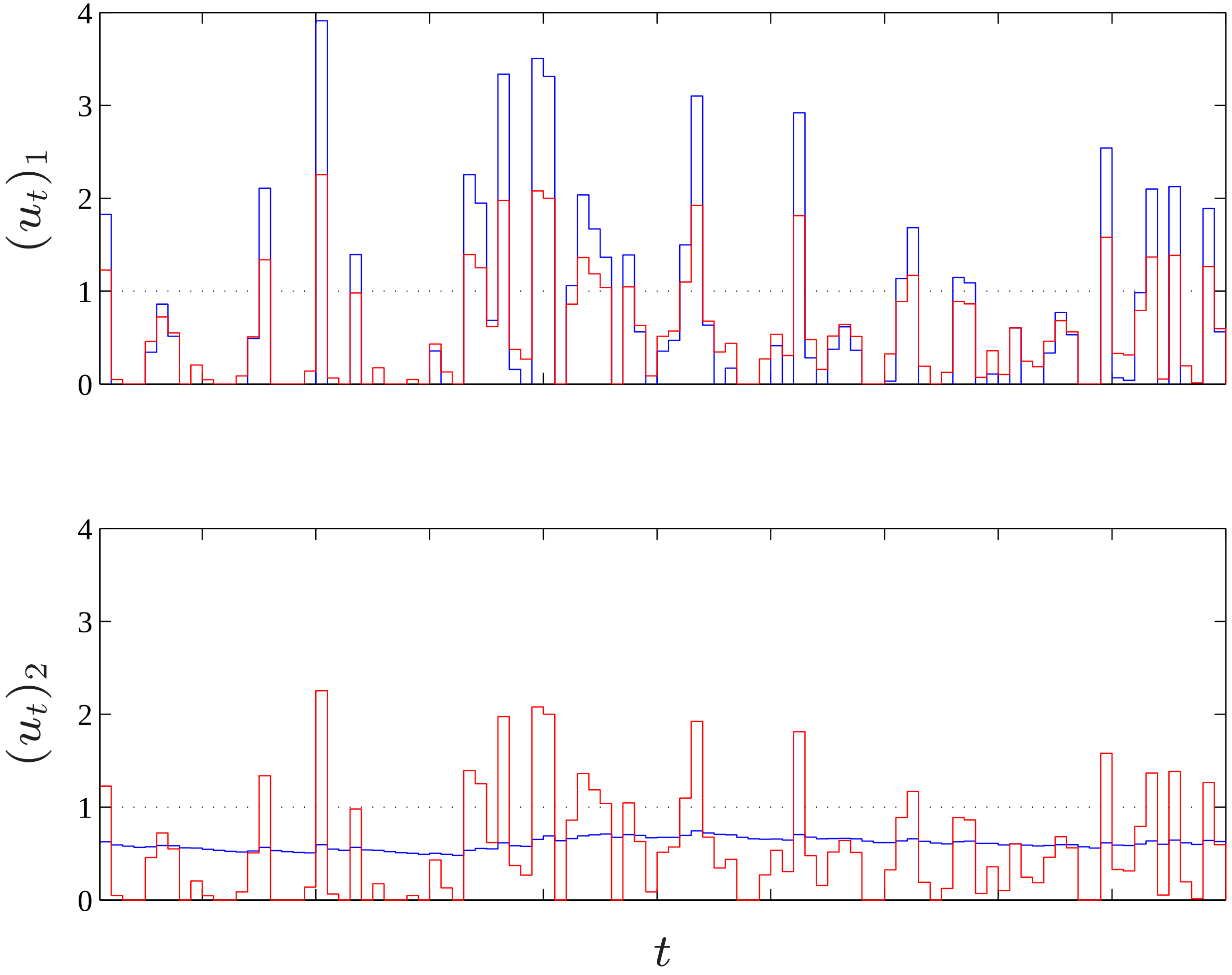
$$f_t = \begin{bmatrix} -0.652 & 0.570 \\ 0.652 & -0.570 \end{bmatrix} x_t + \begin{bmatrix} 0.978 \\ 0.022 \end{bmatrix} c_t + \begin{bmatrix} -0.655 \\ 0.655 \end{bmatrix}$$

- we'll compare with simple sharing: $f_t = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} c_t$

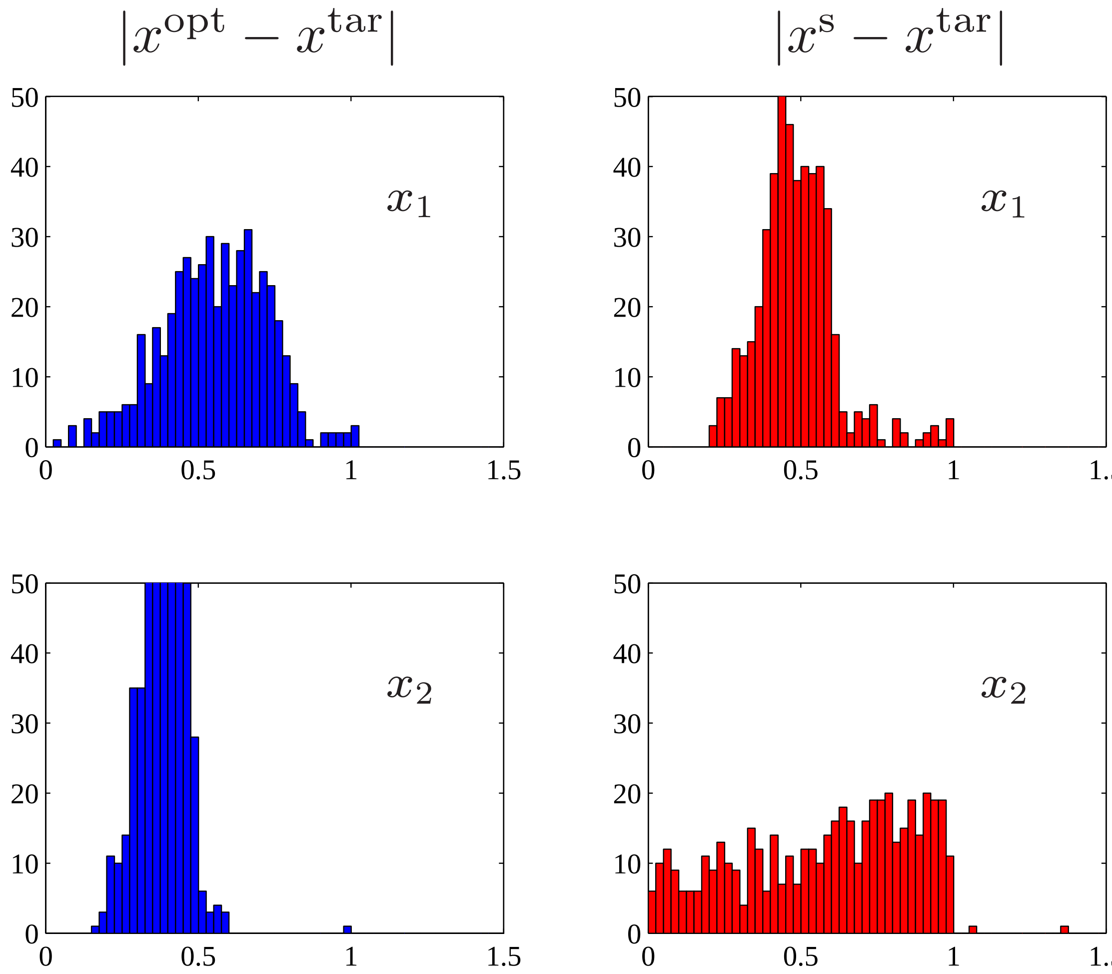
Sample trajectories—smoothed flow



Sample trajectories—flow



Error histograms



What's next

- handling inequality constraints ($e.g.$, $f_t \geq 0$, $Rf_t \leq c_t$) via control Lyapunov methods
- extension to multi-flow, multi-hop, store-and-forward (easy)
- approximating general concave utilities as quadratic
- decentralization