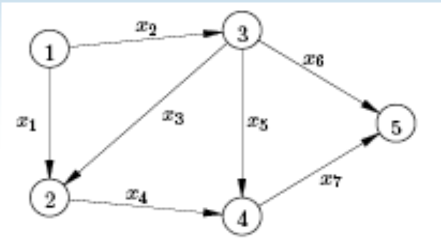
Stochastic resource allocation
Boyd, Akuiyibo

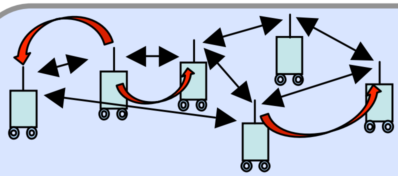
STATUS QUO



Current resource allocation research focus on iterative methods.

These automatically adapt to changing data assuming they are held constant.

NEW INSIGHTS



Formulate as stochastic control problem

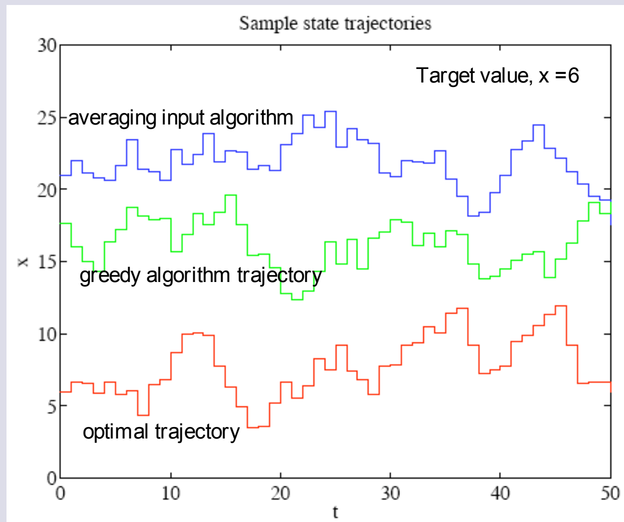
•Resource limits are random

•Allocate resources based on availability and system state

ACHIEVEMENT DESCRIPTION

MAIN RESULT:

Explicit optimal control laws for resource allocation in a system with quadratic cost, linear dynamics, and random linear constraints.



ASSUMPTIONS AND LIMITATIONS:

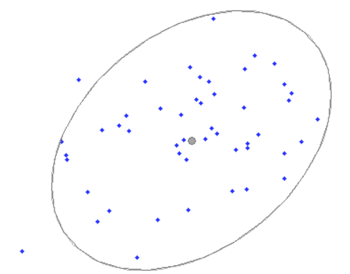
- Assumes that the first and second moments of the resources are know n
- Utility is quadratic; dynamics must be linear

IMPACT

Stochastic allocation of competing network resources i.e., bandwidth, power, flow rates, etc.

Simple control laws (linear coefficients can be computed ahead of time).

NEXT-PHASE GOALS



Utility maximizing estimation techniques

•Decentralized solutions

Optimal dynamic resource allocation with heterogeneous flows

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MÜNCHEN

Multi-Period Stochastic Control of Networks

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ITMANET PI meeting 03/5–6/09

The problem

- wireless networks are characterized by extreme variation in availability of resources, e.g., bandwidth, power, link capacities, connectivity, etc. These issues have been dealt with in an ad hoc way in the past (iterative methods)
- the goal: multi-period resource ($e.g.$, flow rate, power) allocation
- resources ($e.g.$, link capacities, channel states) vary randomly

Our approach

- allocate resources (random resource limits) based on availability and system state
- we form a (nonstandard) stochastic control problem to handle resource allocation and dynamic utilites
- maximize utility (or minimize cost) that reflects different
  - weights (priorities)
  - desired/required target levels
  - **averaging time scales**

  for different flows
- unifies what we called before WNUM, DNUM, SNUM

Dynamic utility and averaging time scale

- f_t is flow in period t, $t = 0, 1, \ldots$
- U : **R** $\to$ **R** gives utility derived for a flow value
- $\tilde{f}_t = (1-\theta)\sum_{\tau=0}^{t} \theta^\tau f_{t-\tau}$ is (first order) **smoothed** or **averaged** flow
- $T_{\mathrm{avg}} = 1/\log(1/\theta)$ gives smoothing time scale
- average smoothed utility is $\bar{U} = \lim_{t\to\infty}(1/t)\,\mathbf{E}\sum_{\tau=0}^{t} U(\tilde{f}_\tau)$
- when U is not linear, $\bar{U}$ depends on smoothing time scale T_{avg}
- our claim: flow utility should be judged dynamically, *i.e.*, by a **utility function** and an **averaging time scale**

Stochastic flow control

- must have $f_t \in \mathcal{R}_t$ (rate region)
- in general, $\mathcal{R}_t$ is random process on sets; we'll assume $\mathcal{R}_t$ are IID
- stochastic flow control:

  choose f_t as function (policy) of $\mathcal{R}_t$, x_t, to maximize $\bar{U}$
- can solve in principle via DP
- can solve exactly in only a few special cases
- lots of approximate solution methods

Linear quadratic formulation: (nonstandard) stochastic control problem

$$\begin{array}{ll} \text{maximize} & \bar{U} \\ \text{subject to} & x_{\tau+1} = A_\tau x_\tau + B_\tau f_\tau, \quad \mathbf{1}^T f_\tau = c_\tau, \quad \tau = 0, 1, \ldots, T-1 \end{array}$$

with variables $f_0, \ldots, f_{T-1}, x_1, x_2, \ldots, x_T$ where

- $\bar{U} = \mathbf{E} \sum_{\tau=0}^{T-1} U(x_\tau)$, $U(x_\tau)$ are concave quadratic functions
- $x_\tau \in \mathbf{R}^n$ are utility states
- $f_\tau \in \mathbf{R}^m$ are flows
- $c_\tau \in \mathbf{R}^p$ are random capacities
- rate region is defined by (random) capacity c_τ: $\mathcal{R}_\tau = \{z \mid \mathbf{1}^T z = c_\tau\}$

problem data are
$A_\tau \in \mathbf{R}^{n \times n}$, $B_\tau \in \mathbf{R}^{n \times m}$; first and second moments of c_τ

Solution via dynamic programming

- let $V_t(z)$ be expected utility to go, in state $x_t = z$, at time t, before c_t is revealed
- V_t is quadratic, defined by a (nonstandard) backward recursion
- expectation is over c_t
- optimal policies are affine in x_t, c_t:

$$f_t = Kx_t + wc_t + s$$

  - K, w, s found by backward recursion
  - $\mathbf{1}^T K = 0$, $\mathbf{1}^T w = 1$, $\mathbf{1}^T s = 0$, so we have $\mathbf{1}^T f_t = c_t$

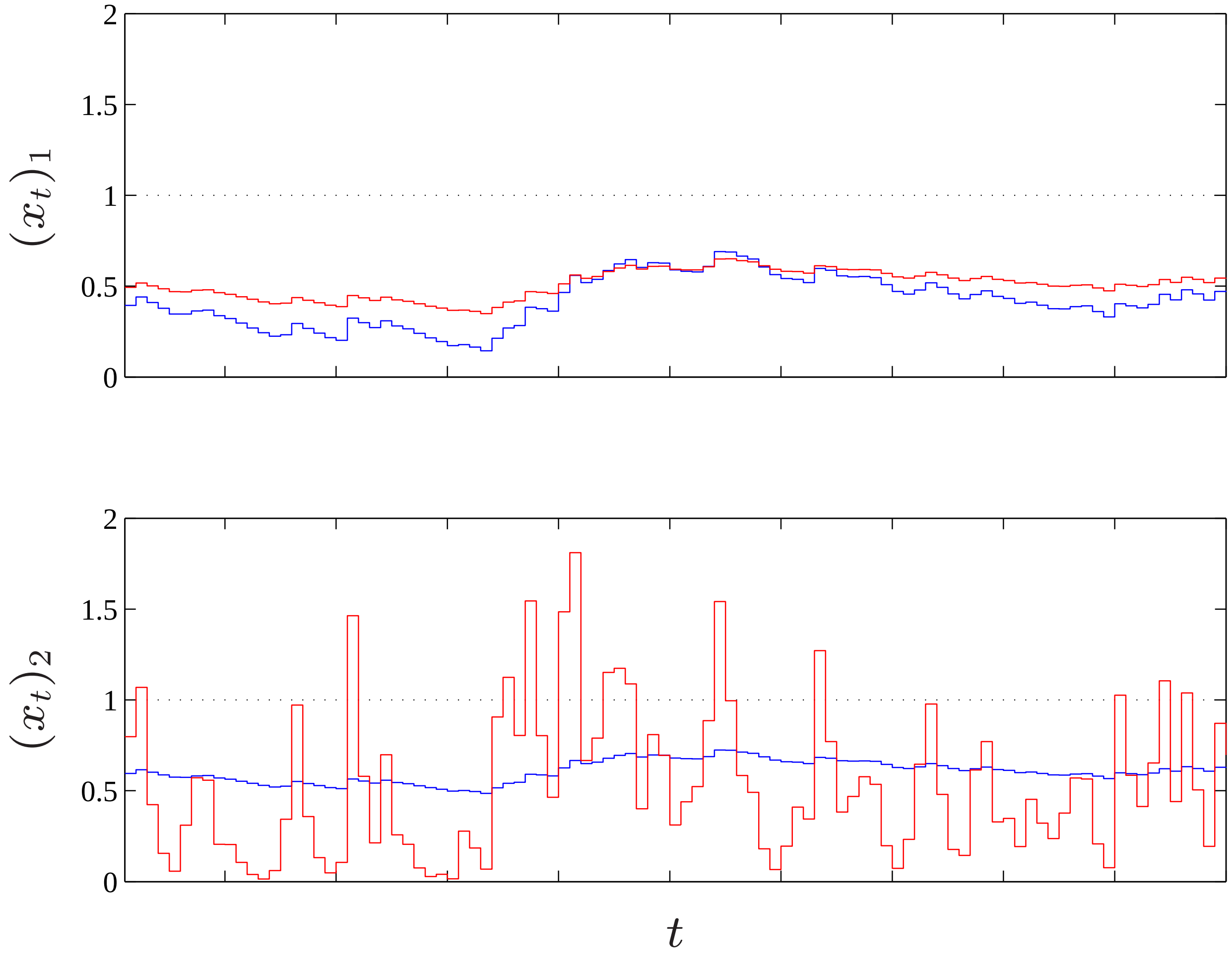
(Simplest possible) example

- 2 flows share one link
- $U(a) = -(a-1)^2$ ($i.e.$, target flow values are one)
- smoothing times $T^{\rm avg} = 1$, $T^{\rm avg} = 30$
- link capacity c_t is exponential with mean 1.5
- optimal policy:

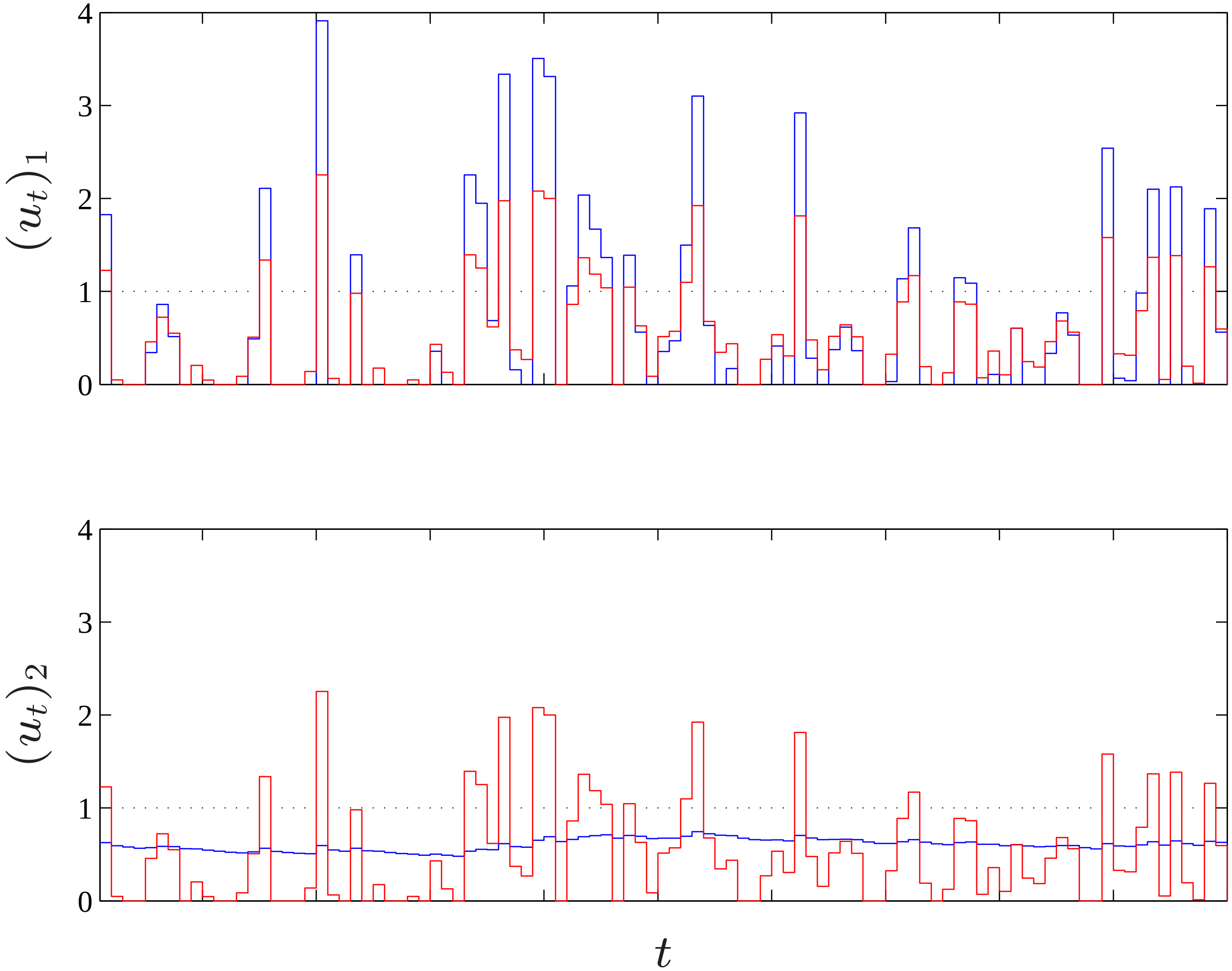
$$f_t = \begin{bmatrix} -0.652 & 0.570 \\ 0.652 & -0.570 \end{bmatrix} x_t + \begin{bmatrix} 0.978 \\ 0.022 \end{bmatrix} c_t + \begin{bmatrix} -0.655 \\ 0.655 \end{bmatrix}$$

- we'll compare with simple sharing: $f_t = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} c_t$

Sample trajectories—smoothed flow



Sample trajectories—flow



What's next

- handling inequality constraints ($e.g.$, $f_t \geq 0$, $Rf_t \leq c_t$) via control Lyapunov methods
- extension to multi-flow, multi-hop, store-and-forward (easy)
- approximating general concave utilities as quadratic
- decentralization