The Multicast Capacity Region of Large Wireless Networks

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Status Quo

Little is known about the $n \times 2^n$ dimensional multicast capacity region for networks with n nodes.

New Insights

Equivalence of wireless network and capacitated tree graph.

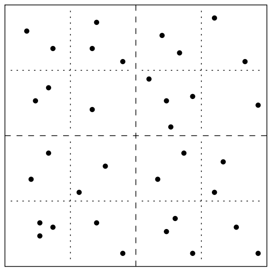
Impact

Optimal two-layer communication scheme for arbitrary multicast traffic.

Next-Phase Goals

Effects of arbitrary node placement on achievable multicast rates.

Model



- n nodes $V(n)$ placed randomly on $[0, \sqrt{n}]^2$
- $y_v[t] = \sum_{u \in V(n) \setminus \{v\}} h_{u,v}[t] x_u[t] + z_v[t]$
- $h_{u,v}[t] = r_{u,v}^{-\alpha/2} \exp(\sqrt{-1}\theta_{u,v}[t])$
- Path loss exponent $\alpha > 2$
- $\{\theta_{u,v}[t]\}_{u,v}$ i.i.d. uniform over $[0, 2\pi)$
- Fast or slow fading $\{\theta_{u,v}[t]\}_t$
- Full CSI at all nodes

Main Result

- Multicast traffic matrix $\lambda \in \mathbb{R}_+^{n \times 2^n}$
- Capacity region $\Lambda(n) \subset \mathbb{R}_+^{n \times 2^n}$ (set of all achievable λ)
- Partition $[0, \sqrt{n}]^2$ into square-grids, with spacing $2^{-\ell}\sqrt{n}$ at level ℓ
- $\{V_{\ell,i}(n)\}_{i=1}^{4^\ell}$ are the nodes in squares at level ℓ

Define

$$\Lambda_G(n) \triangleq \Big\{ \lambda \in \mathbb{R}_+^{n \times 2^n} : \sum_{u \in V_{\ell,i}(n)} \sum_{\substack{W \subset V(n): \\ W \cap V_{\ell,i}(n)^c \neq \emptyset}} \lambda_{u,W} + \sum_{u \in V_{\ell,i}(n)^c} \sum_{\substack{W \subset V(n): \\ W \cap V_{\ell,i}(n) \neq \emptyset}} \lambda_{u,W} \leq g_\alpha(4^{-\ell} n) \quad \forall \ell, i \Big\},$$

where

$$g_\alpha(r) \triangleq \begin{cases} r^{2-\min\{3,\alpha\}/2} & \text{if } r \geq 1, \\ 1 & \text{else.} \end{cases}$$

Theorem. *Under either fast or slow fading, for any $\alpha > 2$, $\varepsilon > 0$,*

$$\Omega(n^{-\varepsilon})\Lambda_G(n) \subseteq \Lambda(n) \subseteq O(n^{\varepsilon})\Lambda_G(n)$$

with probability $1 - o(1)$ as $n \to \infty$.

Examples

Broadcast From Many Sources

- n^β sources arbitrarily chosen
- Each source broadcasts an independent message at uniform rate $\rho(n)$

$$\Longrightarrow \quad \rho^*(n) = \Theta(n^{-\beta \pm \varepsilon})$$

Multicast From Many Sources

- n^{β_1} sources, chosen randomly
- Each source multicasts to n^{β_2} randomly chosen destinations at uniform rate $\rho(n)$

$$\Longrightarrow \quad \rho^*(n) = \Theta\Big(\min\{n^{\pm\varepsilon}, n^{(1-\beta_2)\tilde{\alpha} - \beta_1 \pm \varepsilon}\}\Big),$$

where

$$\tilde{\alpha} \triangleq 2 - \min\{3, \alpha\}/2.$$

$\Rightarrow$ $\beta_1 = \beta$, $\beta_2 = 1$ recovers broadcast case

$\Rightarrow$ $\beta_2 = 0$ recovers unicast case

Multiple Classes of Localized Multicast

- K classes of source nodes
- $n^{\beta_{1,k}}$ sources in class k, chosen randomly
- Each source node in class i multicasts to $n^{\beta_{2,k}}$ destination nodes chosen randomly within distance $n^{\beta_{3,k}/2}$, $\beta_{3,k} > \beta_{2,k}$
- Source nodes in class k generate multicast traffic at uniform rate $\rho_k(n)$

$$\Longrightarrow \quad \rho_k^*(n) = \Theta\Big(\min\{n^{\pm\varepsilon}, n^{(\beta_{3,k} - \beta_{2,k})\tilde{\alpha} - \max\{0, \beta_{1,k} + \beta_{3,k} - 1\} \pm \varepsilon}\}\Big)$$

for all $k \in \{1, \ldots K\}$.

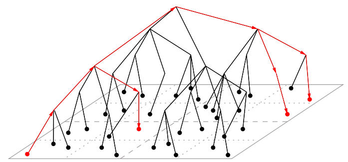
$\Rightarrow$ $K = 1$, $\beta_3 = 1$ recovers second example

$\Rightarrow$ Illustrates that the general theorem can be used to obtain closed form expressions for rather complicated settings

Communication Scheme

- Two layer scheme achieving entire capacity region (in scaling sense)
- Top or routing layer: routes data over a tree graph
- Bottom or physical layer: provides tree abstraction

Routing Layer



- Construct a tree $G = (V_G, E_G)$
- $V(n) \subset V_G$ are the leaves of G
- Intermediate nodes in G "represent" nodes in $V_{\ell,i}(n)$
- Grid structure induces hierarchy

Messages are transmitted between u and W by routing them over G.

Physical Layer

- The physical layer provides the tree abstraction G
- To send a message along an edge $e \in E_G$ towards the root, the message is "distributed" over the wireless network
- To send a message along an edge $e \in E_G$ away from the root, the message is "concentrated" over the wireless network
- This distribution/concentration is performed using cooperative communication ($\alpha \in (2, 3]$) or multi-hop communication ($\alpha > 3$)