A Game Theoretic Approach to Network Coding

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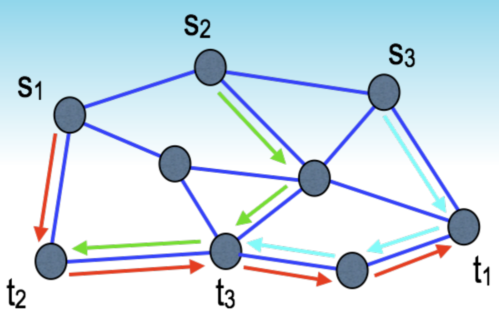
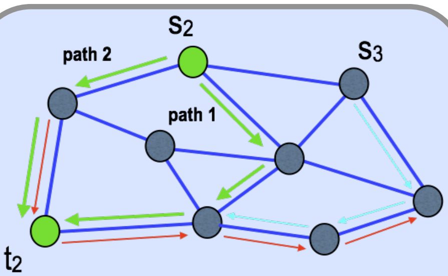
STATUS QUO



Global Objective: *Efficiently use network using network coding*

Approach: *Centralized solutions. (e.g., opportunistic coding) Fix paths, use coding opportunities if available*

NEW INSIGHTS



*What about **distributed** solutions?*

What if flows were allowed to select path in response to local "cost"?

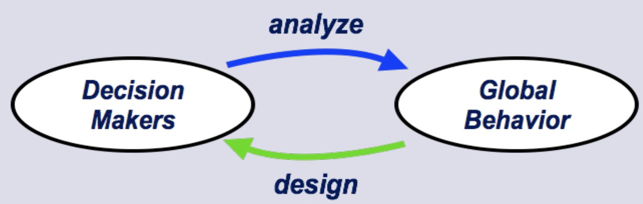
***Goal**: Let users create coding opportunities to improve efficiency*

ACHIEVEMENT DESCRIPTION

MAIN ACHIEVEMENT:

Introduced game theory as a distributed tractable mechanism to obtain good network performance

game theory for social sciences: "descriptive agenda"



game theory for engineering: "prescriptive agenda"

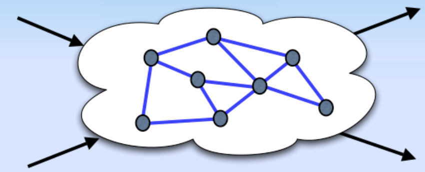
HOW IT WORKS:

- Model interactions as a non-cooperative game
  - players (unicast flows)
  - actions (available paths)
- Assign each player a "cost" function
- Analyze efficiency of equilibrium behavior

ASSUMPTIONS AND LIMITATIONS:

- Limited form of network coding (reverse carpool)
- Players have knowledge of available paths
- Players equilibrate faster than network changes

IMPACT



Approach provides guarantees **independent** of network structure.

Guarantees existence of an equilibrium that achieves a system cost of at most 50% higher than the optimal.

This offers an improvement over opportunistic coding.

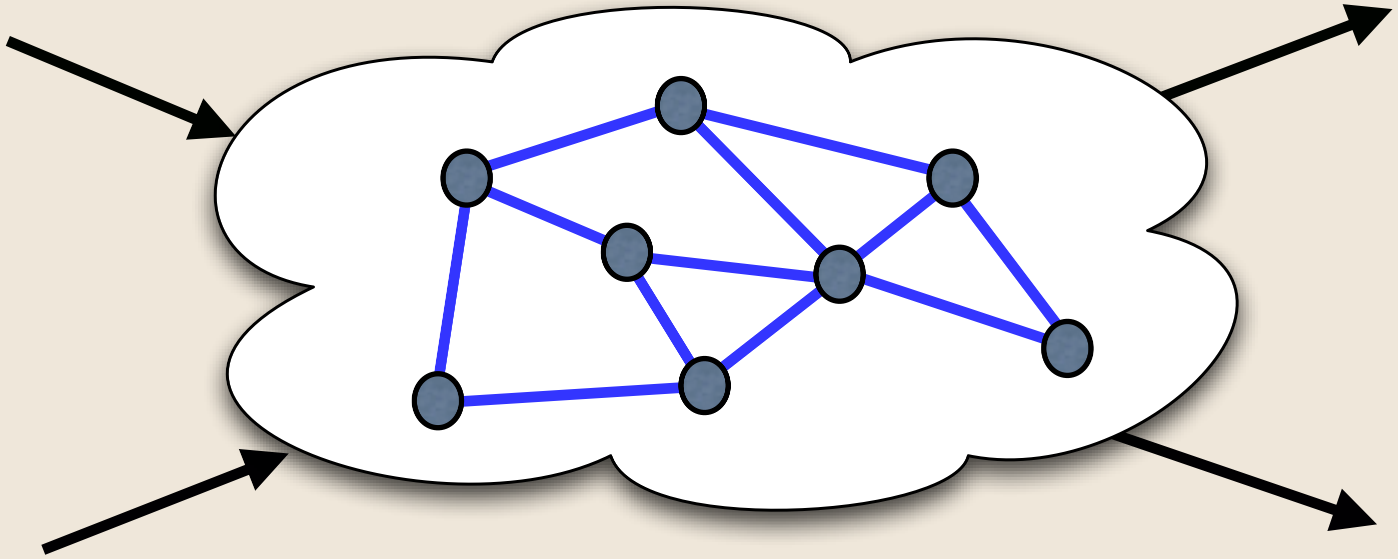
NEXT-PHASE GOALS

Understand the potential of game theory in network coding problems

*Establish **desirable** distributed learning algorithms with good convergence rates*

Extend game theoretic approach to more general network coding problems

Game theory is an applicable tool for distributed optimization in network coding



Features:

- *large common network*
- *large # of users*
- *different network demands*

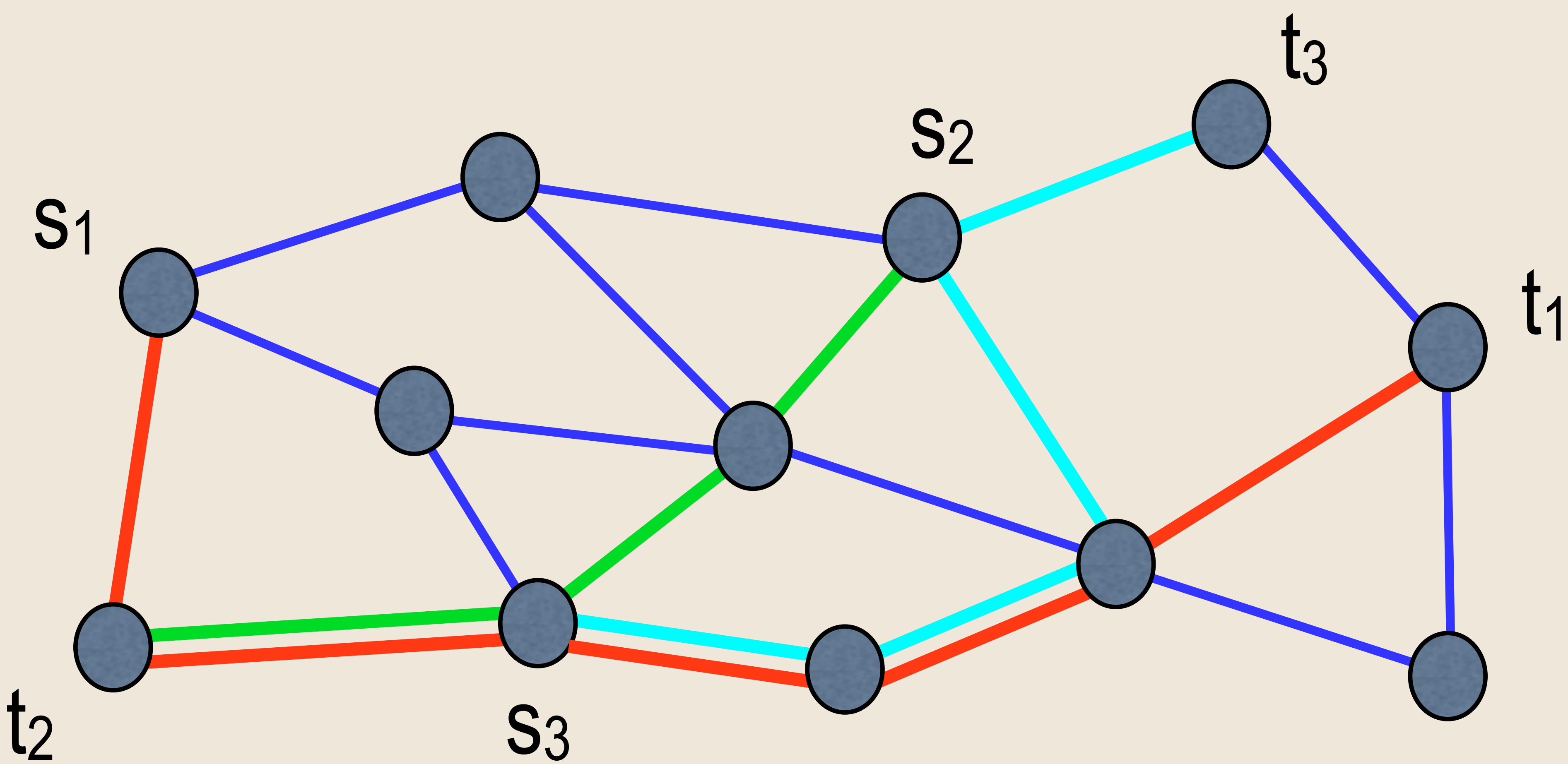
Global objective: *Allocate users efficiently over network (utilizing network coding)*

- *maximize throughput*
- *minimize # of transmissions*

Challenges:

- *centralized optimization is not feasible*
- *network coding capacity is unsolved*

Multiple unicast flows in shared network environment

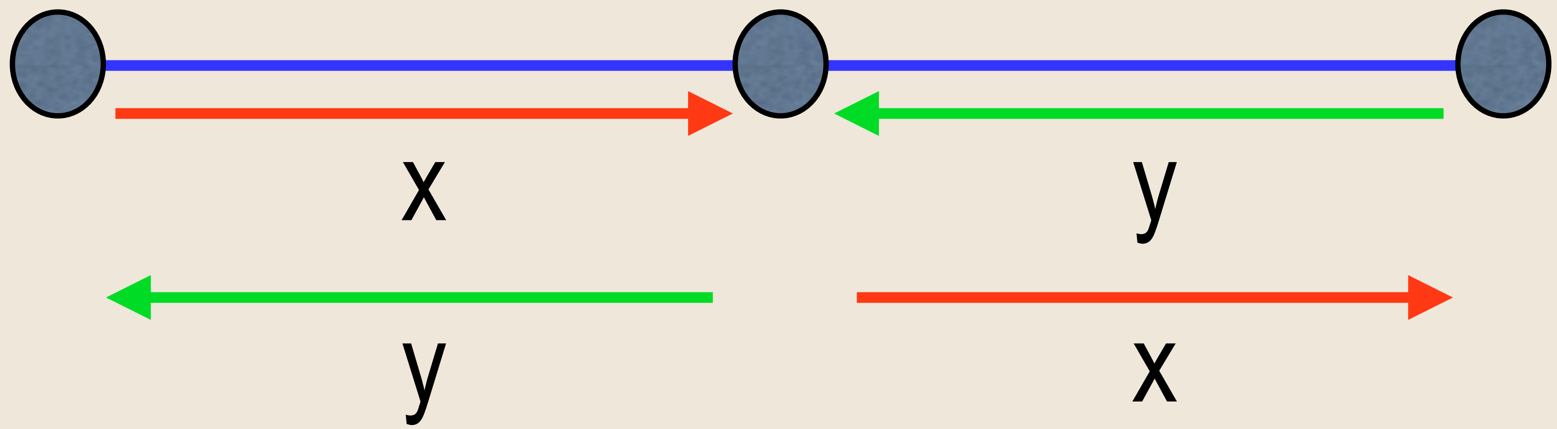


possible transmissions highlighted by edges on graph

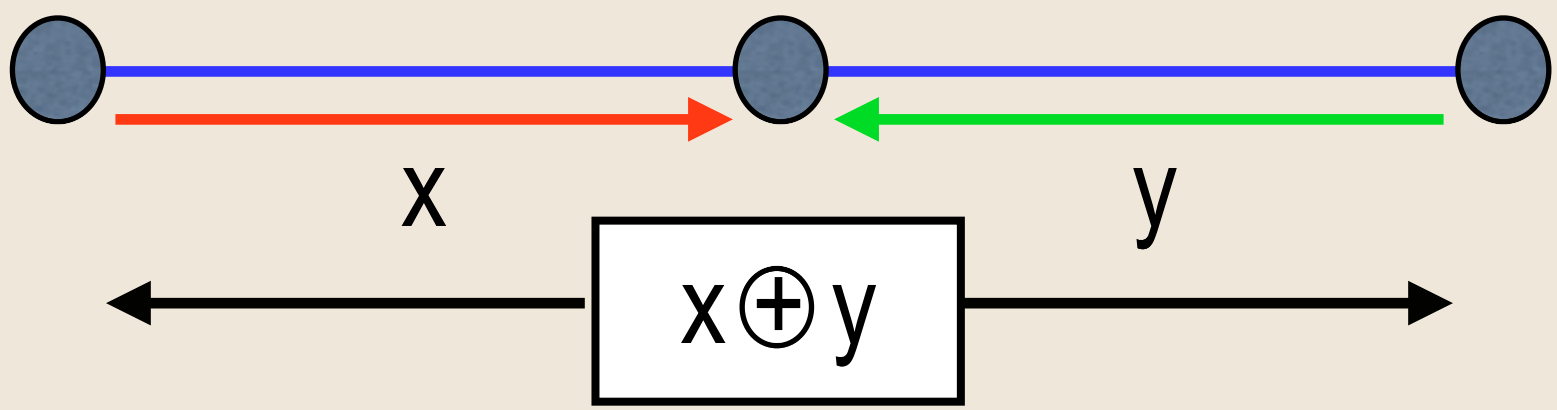
Cost of allocation = number of transmission

Limited form of network coding: **"Reverse carpooling"**

opportunity for network coding arises when
two unicasts traverse the same node in opposite directions



without network coding - 4 transmissions required



with network coding - 3 transmissions required

Setup

Approach: *Model interactions as a non-cooperative game*

- Players (unicast flows): $\{1, 2, \ldots, n\}$ (s_i, t_i)
- Actions (available paths): $a_i \in \mathcal{A}_i$

$$\mathcal{A} = \mathcal{A}_1 \times \ldots \times \mathcal{A}_n$$

- System cost: $C(a) = \sum_e \max\{\#_e^0(a), \#_e^1(a)\}$

Goal: *Design local players' cost functions so that equilibrium behavior is desirable*

Cost functions: $J_i(a) = J_i(a_i, a_{-i})$

Equilibrium behavior: *Pure Nash equilibrium*

$$J_i(a_i^*, a_{-i}^*) \le J_i(a_i, a_{-i}^*)$$

Let

$$\mathcal{E}(G) := \{a \in \mathcal{A} : a \text{ is a Nash equilibrium of game } G\}$$

$$a^{\text{opt}} \in \arg\min_{a \in \mathcal{A}} C(a)$$

Price of Anarchy

$$POA = \sup_{G} \max_{a \in \mathcal{E}(G)} \frac{C(a)}{C(a^{\text{opt}})}$$

worst case performance of any NE

Price of Stability

$$POS = \sup_{G} \min_{a \in \mathcal{E}(G)} \frac{C(a)}{C(a^{\text{opt}})}$$

worst case performance of best NE

(independent of network structure or demands)

Decision Makers

$J_i(a_i, a_{-i})$

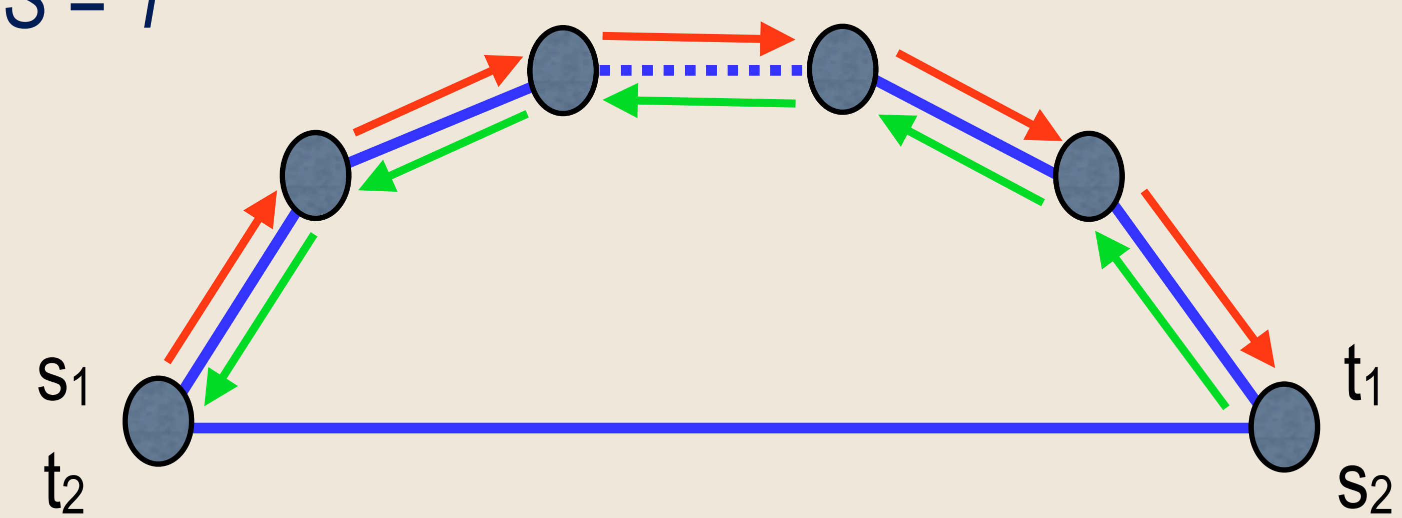
design

Global Behavior

$C(a) = \sum_e \max\{\#_e^0(a), \#_e^1(a)\}$

Wonderful Life: $J_i(a) = C(a) - C(a_i^0, a_{-i})$

Positives	Negatives
local	
NE exists (minimizer C)	*POA unbounded*
POS = 1	



Select any global cost $\phi : \mathcal{A} \rightarrow R$

$$J_i(a) = \phi(a) - \phi(a_i^0, a_{-i})$$

Positives

NE exists (minimizer ϕ)

local

Negatives

$$\phi(a) = C(a)$$

$\Rightarrow$

POS = 1

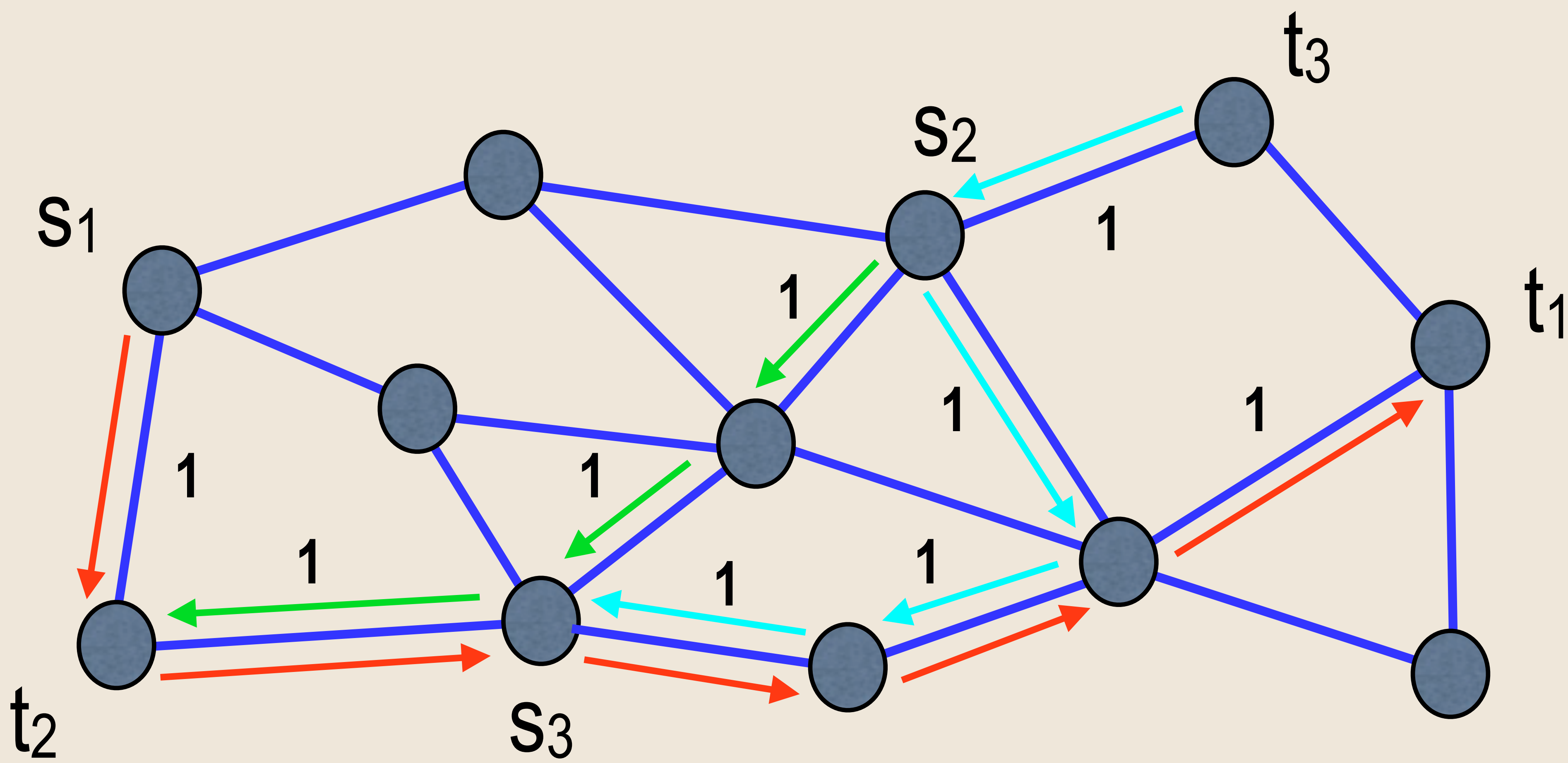
POA unbounded

Can we choose a cost that gives us better equilibrium efficiency?

Consider $$\phi(a) = (\alpha - 1)C(a) + \sum_i |a_i| \qquad \alpha \geq 0$$

New design:
$$\boxed{\begin{aligned} J_i(a) &= \phi(a) - \phi(a_i^0, a_{-i}) \\ &= \alpha N_i^{(>)}(a) + N_i^{(\leq)}(a) \end{aligned}}$$

Old design:
$$\begin{aligned} J_i(a) &= C(a) - C(a_i^0, a_{-i}) \\ &= N_i^{(>)}(a) \end{aligned}$$

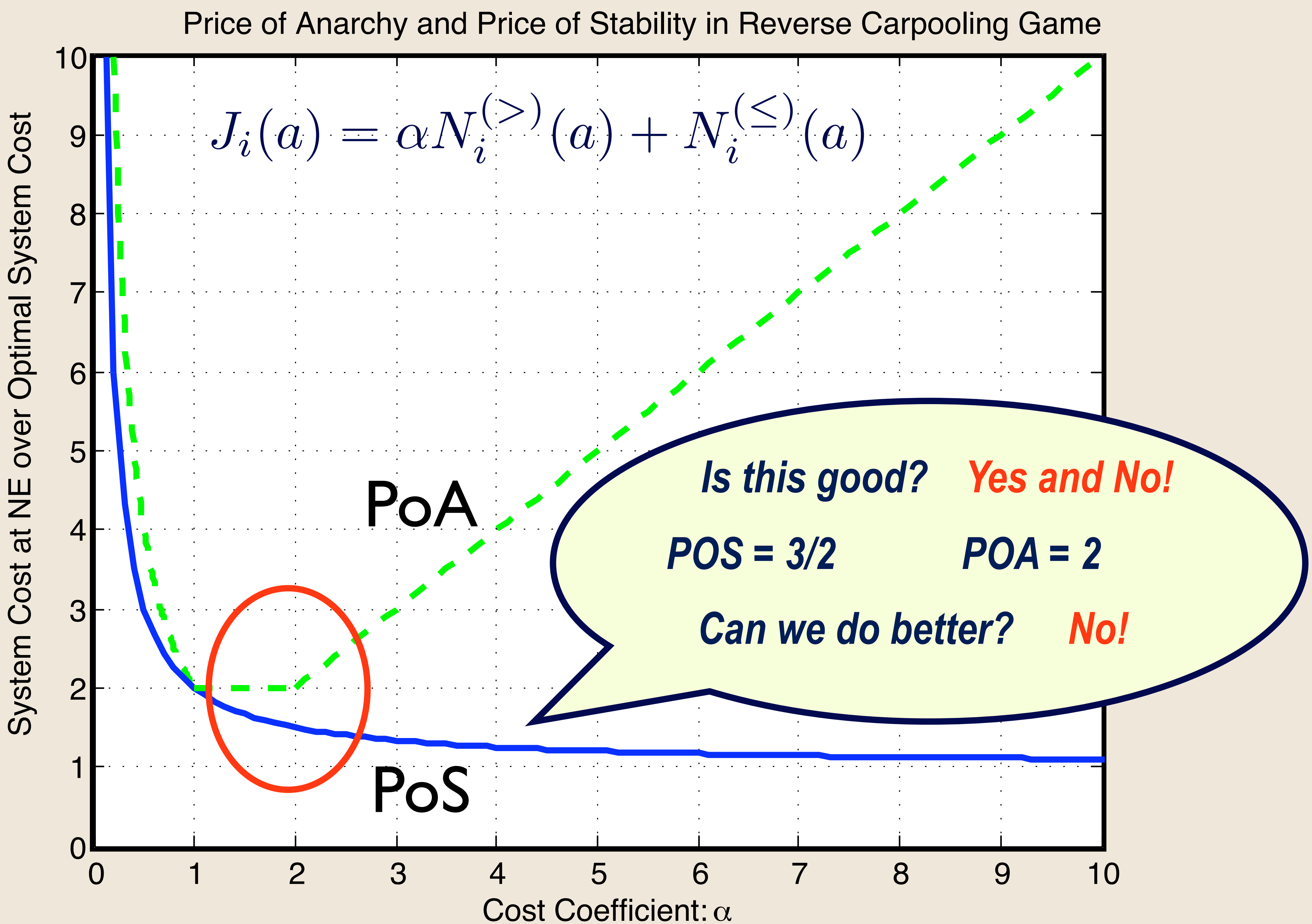


What is the cost of this assignment for player 1?

$$J_i(a) = \phi(a) - \phi(a_i^0, a_{-i})$$

$$= \alpha N_i^{(>)}(a) + N_i^{(\leq)}(a) \;\; = 2\alpha + 3$$

Main result


Price of Anarchy and Price of Stability in Reverse Carpooling Game
$J_i(a) = \alpha N_i^{(>)}(a) + N_i^{(\le)}(a)$
PoA
PoS
System Cost at NE over Optimal System Cost
Cost Coefficient: α
0
1
2
3
4
5
6
7
8
9
10
Is this good? *Yes and No!*
POS = 3/2 *POA = 2*
Can we do better? *No!*

Conclusions

Recap:

- *Derived "optimal" cost functions for a simple network coding problem (WL)*
- *The results on efficiency have implications beyond simple setup*

Goals:

- *Illustrate potential of game theory as mechanism for distributed optimization*
- *Results stem for recent research connecting "potential games" and cooperative control problems*

J. Marden, G. Arslan, and J. Shamma, "Connections between cooperative control and potential games," 2008.

Left unsaid:

- *How do players reach equilibrium in a distributed fashion?*
- *Existing literature on learning in games provides some algorithms*