Minimum-Cost Subgraphs for Joint Distributed Source and Network Coding

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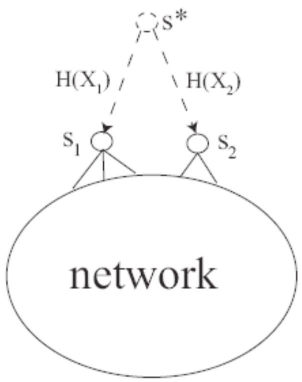
Introduction

- Network coding decouples into two components
  - Subgraph construction
  - random network coding
- Ho et al. showed random network coding works for correlated sources
- find min-cost subgraph for network coding
- Lun et al. developed algorithms for min-cost subgraph construction
- We look at minimum cost subgraph construction for multicasting of correlated sources
- show benefits of joint source-network coding over separation of two

Problem Formulation

Given

- a graph $G = (N, A)$
- edge weights $w_{ij} : (i,j) \in A \to R^+$
- edge capacities $c_{ij} : (i,j) \in A \to R^+$
- set of two source nodes S
- sources X_i generated at s_i with rate $H(X_i)$
- joint rate $H(X_1, X_2)$
- set of receiver nodes T.



Let

$$
\begin{aligned}
G^* &= (N^*, A^*) \\
N^* &= N \cup s^* \\
A^* &= A \cup E,
\end{aligned}
$$

$$R = H(X_1, X_2)$$

where

$$
\begin{aligned}
E &= \{(s^*, j) | j \in S\} \\
c_{s^*j} &= H(X_j) \\
w_{s^*j} &= 0.
\end{aligned}
$$

Problem Formulation

Minimize

$$\sum_{(i,j)\in A^*} w_{ij} z_{ij},$$

subject to

$$c_{ij} \geq z_{ij}, \qquad \forall (i,j) \in A, \qquad (1)$$

$$c_{ij} \geq x_{ij}, \qquad \forall (i,j) \in E, \qquad (2)$$

$$z_{ij} \geq x_{ij}^{(t)} \geq 0, \qquad \forall (i,j) \in A^*, t \in T, \qquad (3)$$

$$\sum_{\{j|(i,j)\in A^*\}} x_{ij}^{(t)} - \sum_{\{j|(j,i)\in A^*\}} x_{ji}^{(t)} = \sigma_i^{(t)}, \quad \forall i \in N^*, \ t \in T, \qquad (4)$$

where

$$\sigma_i^{(t)} = \begin{cases} R & i = s^*, \\ -R & i = t, \\ 0 & \text{otherwise.} \end{cases}$$

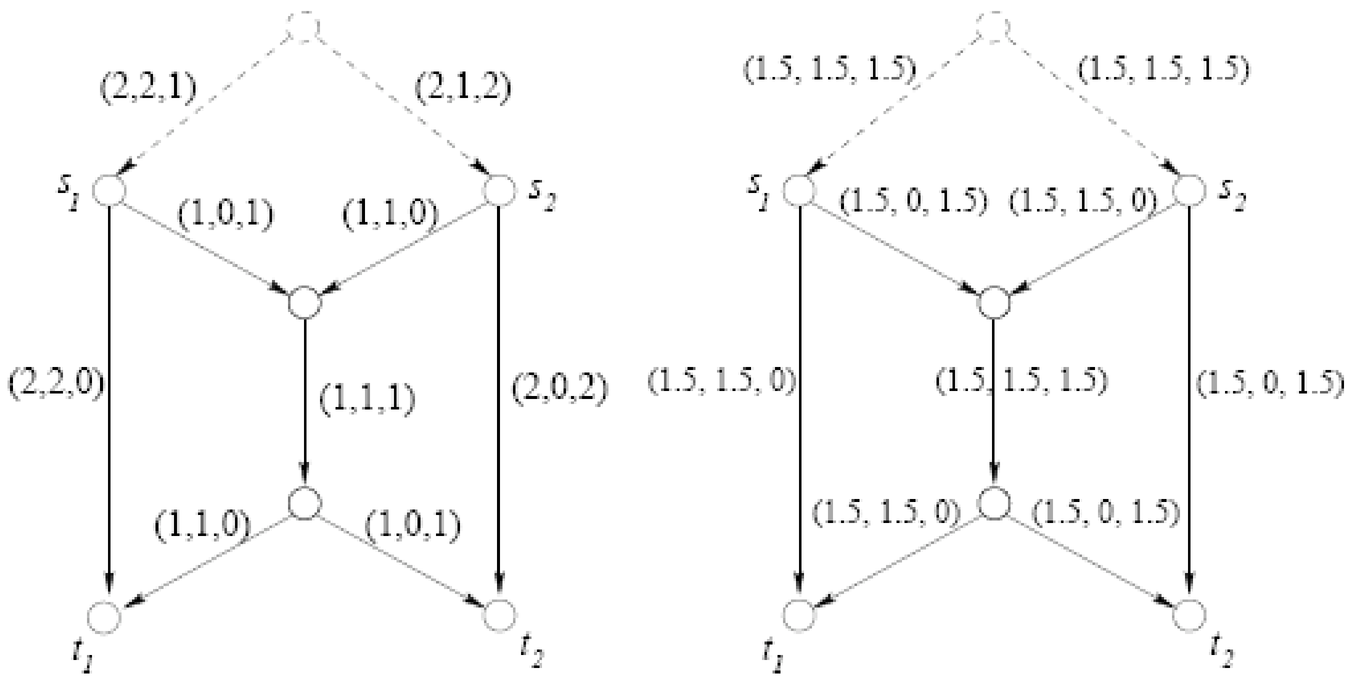
desired subgraph: $\{z_{ij} | (i,j) \in A\}$

desired subgraph cost: $\sum_{(i,j)\in A} w_{ij} z_{ij}$

Joint vs. Separable

sum of rates on virtual links > R $\rightarrow$ joint solution

= R $\rightarrow$ separable solution



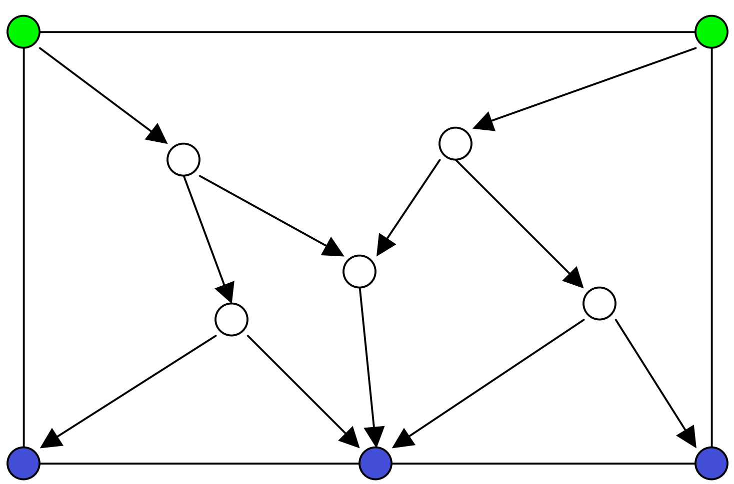
(a) Joint Solution (cost = 9) (b) Separable Solution (cost = 10.5)

$(z_{ij}, x_{ij}^{(1)}, x_{ij}^{(2)})$ for each link (R = 3)

Simulation: Random Networks

Network model:

- *n* nodes randomly placed in *h* x *w* box
- nodes within distance *r* connected by edge
- *s* sources, *t* receivers
- all edges have unit weight, no capacity constraints



sources along top edge

receivers at bottom

Simulation Results

h x *w*	R	avg. cost	% joint < separable	% cost difference	% overall saving
1 x 1	3	14.29	6.5	4.28	0.27
	2.5	11.66	6.6	9.28	0.65
	2	9.15	8.9	11.88	1.06
1 x 2	3	15.06	79.5	12.52	9.95
	2.5	11.79	79.9	12.52	10.0
	2	8.56	78.6	20.98	16.5
1 x 3	3	16.53	99.2	10.19	10.1
	2.5	12.32	99.6	18.64	18.6
	2	8.10	99.4	31.96	31.8

2 sources

2 receivers

$H(X_i) = 2$

$H(X_1,X_2) = R$

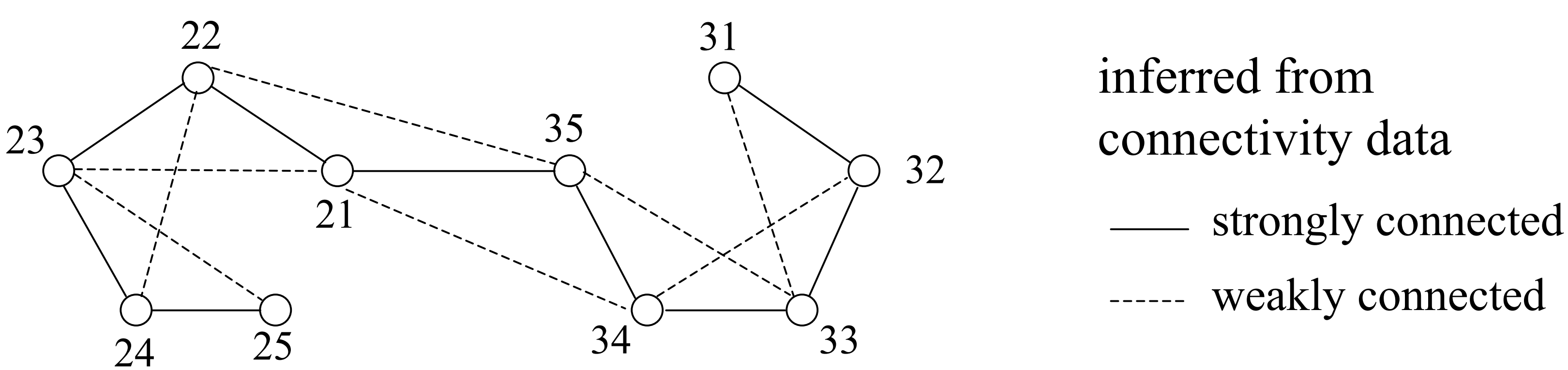
benefit of joint coding increases as network widens, and sources are more correlated.

Simulation: FCS Data

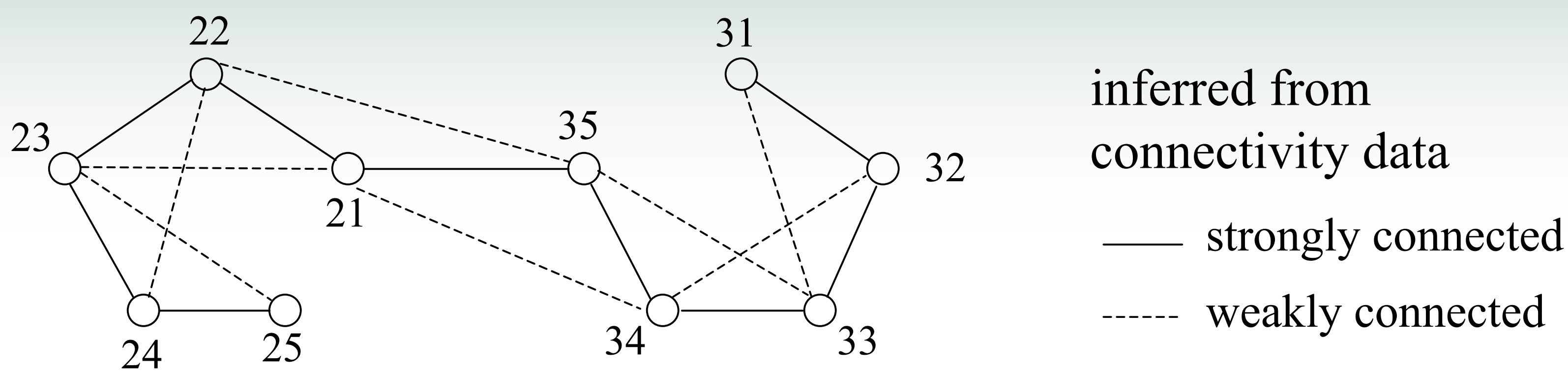
Given: 5000 connectivity graphs

pair wise correlation (ρ) of data at nodes

- randomly choose 2 sources, 2 receivers
- source model:
  - $H(X_i) = 1$,
  - $H(X_1,X_2) = 1+H((\rho+1)/2)$
- edges given unit weight, no capacity constraints



Simulation Results



sample results

sources	receivers	% feasible	avg cost	% joint < separable	% cost difference	% overall saving
23, 32	25, 31	65.76	3.05	30.99	40.33	8.22
24, 32	25, 31	80.50	3.30	58.51	38.42	18.09
22, 32	24, 34	81.72	3.90	52.67	28.66	12.34

Further Work

- practical codes for joint source network coding
- extend low-complexity Slepian-Wolf codes to incorporate network coding
- design placement of source nodes and correlation, given network and receivers