Rolling Horizon Control for Networks with Random Link Capacities

Stephen Boyd Argyris Zymnis Dan O'Neill Andrea Goldsmith
Electrical Engineering Department, Stanford University

The problem

- multi-period network resource allocation
- unicast, split flows
- stochastically varying link capacities
- buffer limits at nodes
- linear time-varying utilities

roughly: we're doing (centralized) multi-period joint routing/scheduling

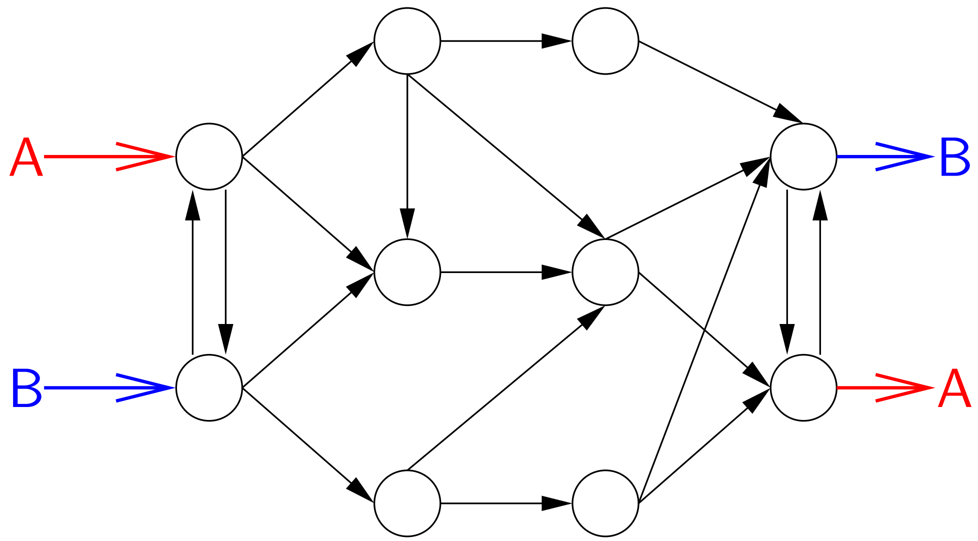
Variables, constraints, objective

- **specification:** graph incidence matrix, sources, destinations, buffer size limits, input flow limits, capacity evolution model
- **variables:** flows, buffer sizes, input and output flows
- **constraints:** flow conservation, nonnegativity, buffer limits, link capacity, source buffer limit, source limits
- **objective:** linear in output flows with time-varying weights

Approaches

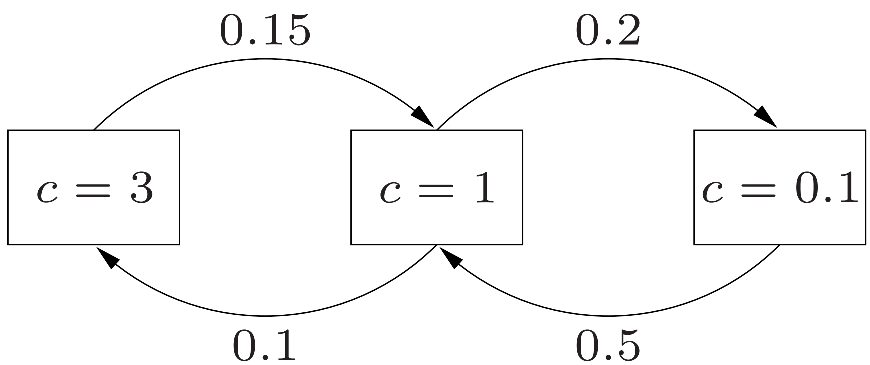
- **prescient relaxation**
  - ignore causality
  - problem becomes large LP
  - empirical utility mean gives bound on expected utility
- **optimal control**
  - current control function of current and past states
  - easily described by the Bellman recursion
  - hard to compute in general
- **rolling horizon**
  - assume future capacity equal to conditional mean
  - solve large LP to get full plan
  - use only first action

Example



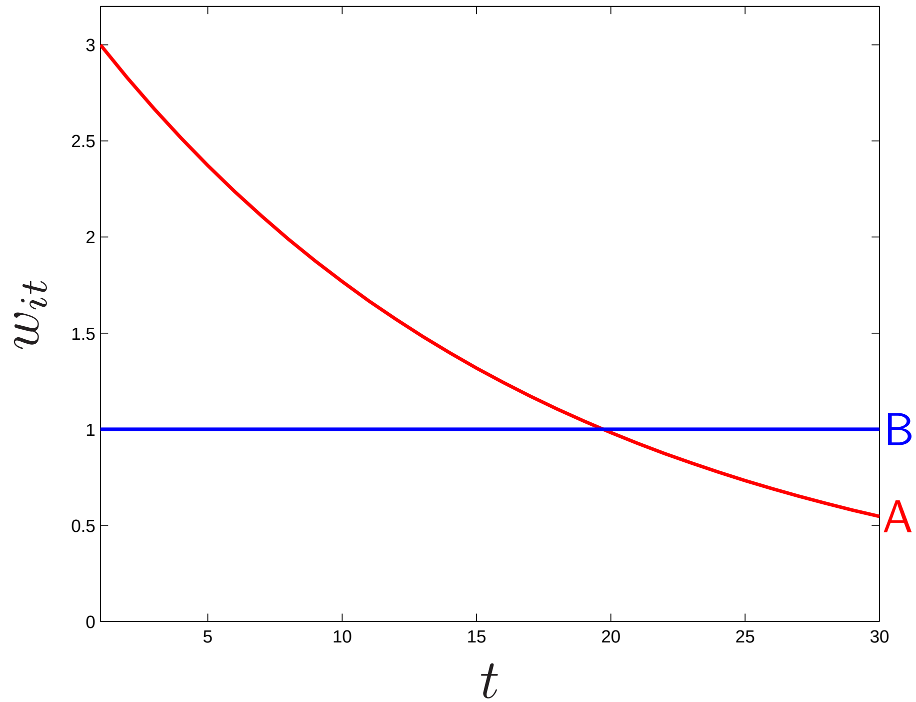
- 10 nodes, 19 links, 2 flows (A and B)
- we consider $T = 30$ steps
- buffer limits $Q^{\max} = 1.5$; input flow limit $s_i^{\mathrm{in,max}} = 2$

Markov link capacity model



- three states: good ($c = 3$), OK ($c = 1$), bad ($c = 0.1$)
- link capacities evolve independently
- mixing time about 3 periods
- equilibrium distribution is 0.3, 0.5, 0.2; average capacity is $\overline{c} = 1.42$
- all links start in OK state

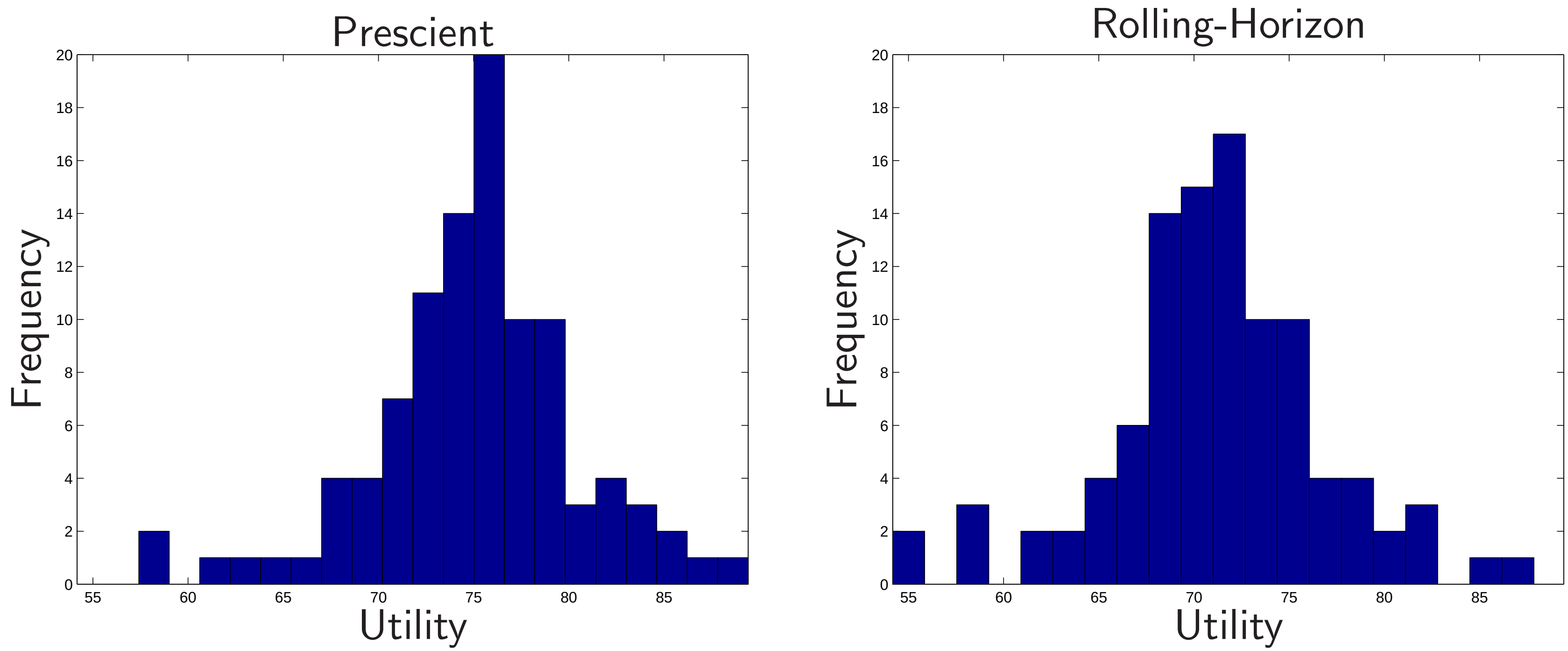
Utility weights



- flow A somewhat time-critical; flow B is best-effort

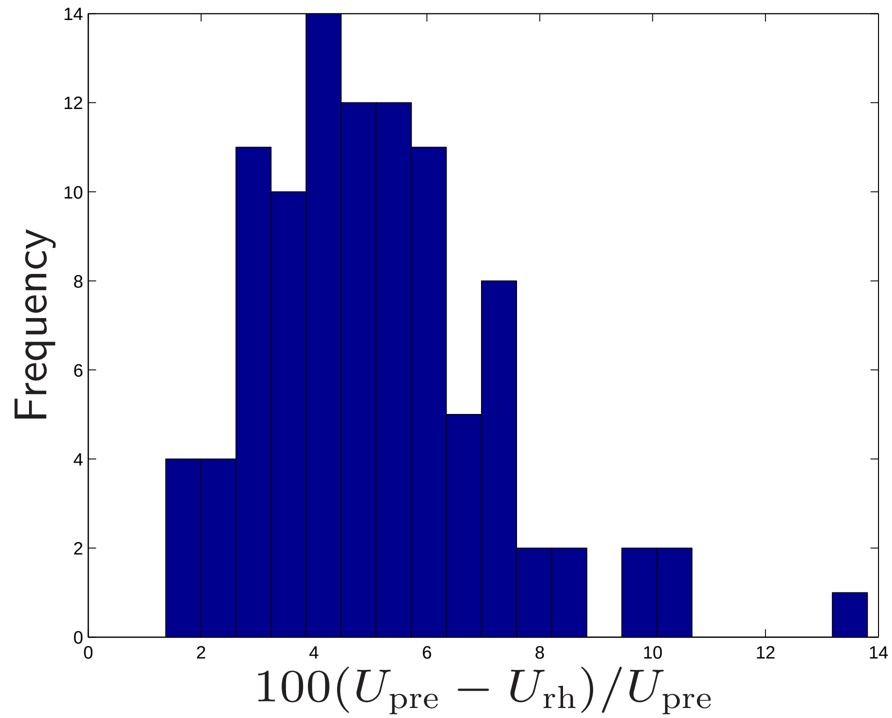
Simulation - Utility distributions

upper bound: $\mathbf{E}U \leq 92$, prescient: $\mathbf{E}U \approx 75$, rolling horizon: $\mathbf{E}U \approx 71$

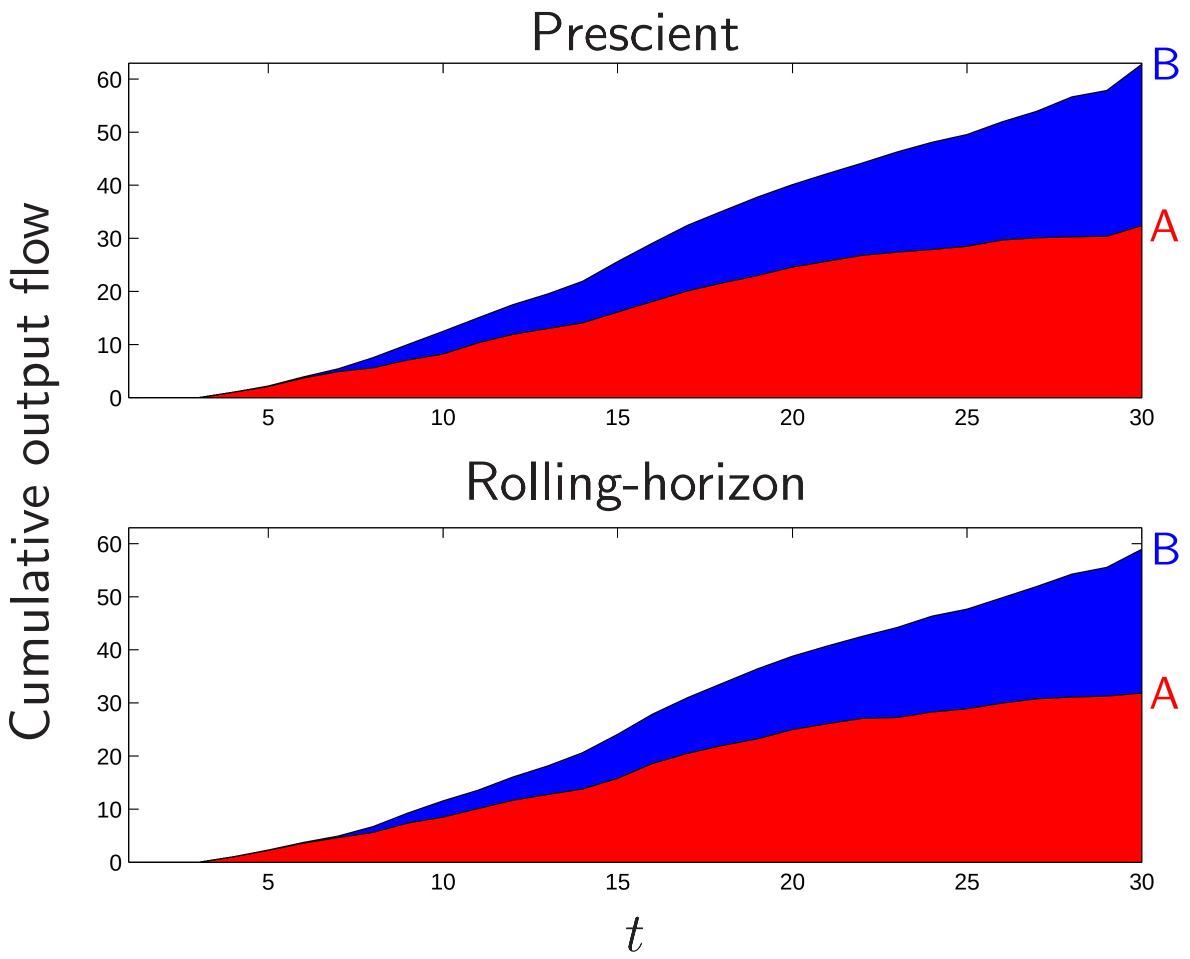


Simulation - Utility gap distribution

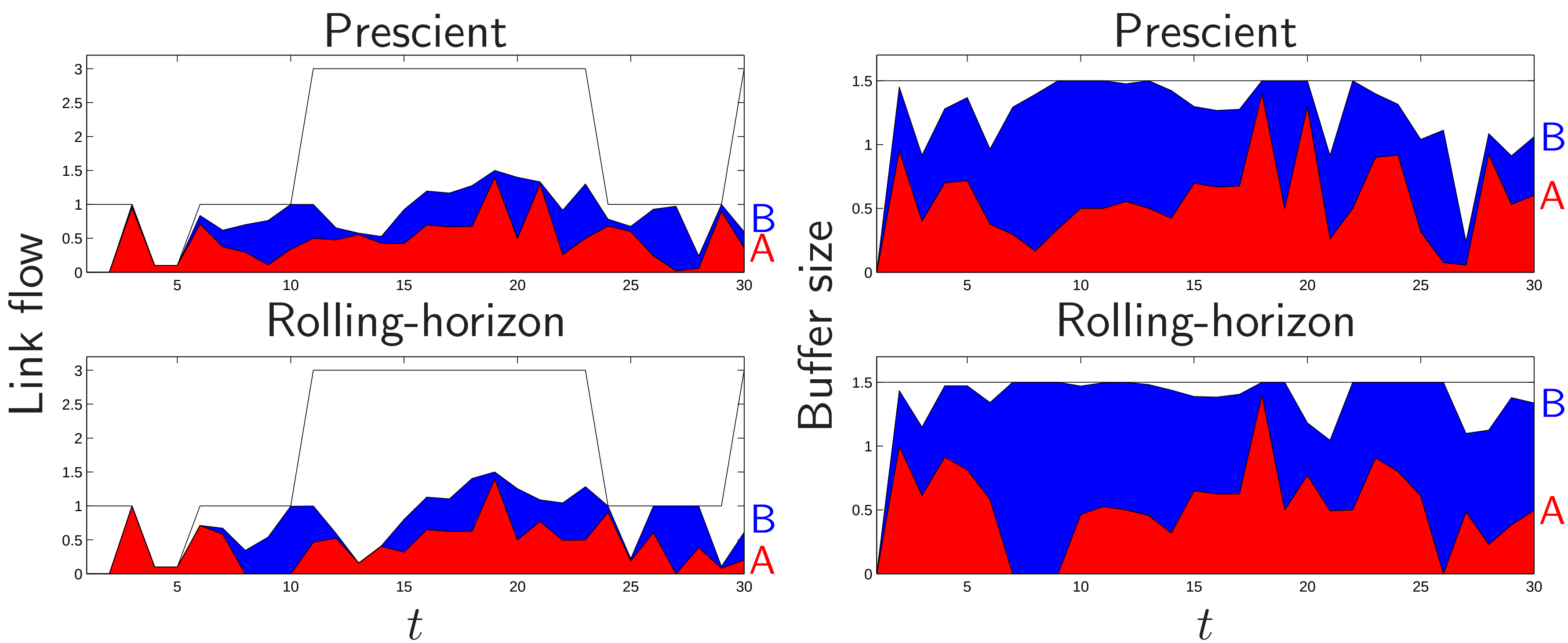
distribution of $(U_{\rm pre} - U_{\rm rh})/U_{\rm pre}$; average is 5%



Cumulative output flow (realization 1)



Central link flow and buffer size (realization 1)



Extensions

'straightforward':

- general concave utilities
- random (*e.g.*, Markovian) utility weights
  (*e.g.*, flows randomly transition from 'best-effort' to 'urgent' and back)
- more general (non-Markovian) link state model
- changing source/destination nodes for flows
- joint resource allocation (bandwidth, power, . . .)
- fixed-route and multi-route flows
- multi-cast flows with fixed routes (trees)
- comparison with existing (distributed) protocols and methods

more challenging: distributed rolling-horizon methods