Iterative Power Control in Wireless Networks with Interference

Stephen Boyd   Nikolaos Trichakis   Dan O'Neill
Andrea Goldsmith   Argyris Zymnis   Sachin Adlakha

Electrical Engineering Department, Stanford University

ITMANET PI meeting 01/07

The Questions

- what's the rate region for a wireless network with interference and power control?
- how hard is it to find the powers that maximize some utility function such as the sum of the rates?
  - for high SINRs ($i.e.$, small interference), *easy*
  - otherwise, *hard*
- how well do heuristics do?
- we don't consider: interference cancellation, network coding, distributed methods, unknown and/or random path gains, . . .

Wireless Network with Interference

- n links (transmitter-receiver pairs)
- G_{ij}: gain from transmitter of link j to receiver of link i
- P_i: link i transmit power, with constraint $0 \le P_i \le P_i^{\max}$
- σ_i^2: link i receiver noise
- γ_i: link i SINR

$$\gamma_i = \frac{G_{ii}P_i}{\sigma_i^2 + \sum_{j \neq i} G_{ij}P_j}$$

- $R_i = \log(1+\gamma_i)$: link i rate
- (no interference cancellation, etc.)

Sum Rate Maximization

- we take the sum rate

$$R^{\text{tot}} = \sum_{i=1}^{n} R_i = \log \prod_{i=1}^{n} (1 + \gamma_i)$$

as an overall utility measure (could take others)

- maximizing R^{tot} is a hard, nonconvex problem
- maximizing sum high SINR approximation

$$\hat{R}^{\text{tot}} = \sum_{i=1}^{n} R_i = \log \prod_{i=1}^{n} \gamma_i$$

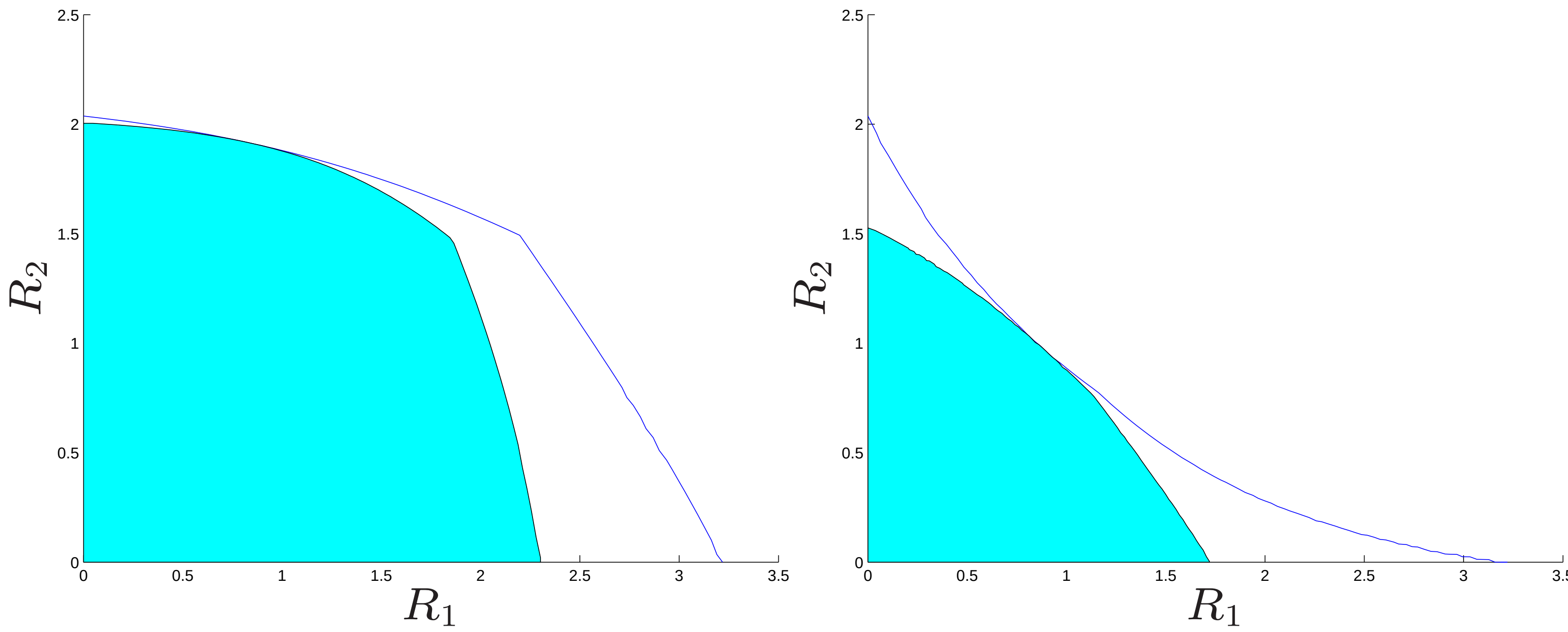
is, however, an easy problem (GP); $\hat{R}^{\text{tot}}$ is a concave function of $\log P_i$

Iterative Algorithm

- basic idea: iterate to convergence
  - form GP compatible approximation $\hat{R}_i$ of R_i, accurate at current point
  - maximize $\sum_{i=1}^{n} \hat{R}_i$
- a standard scheme, used in GP applications such as circuit design
- yields a local, but not necessarily global, optimal point
  ($i.e.$: it can fail to find the actual maximum sum rate)
- in any case, seems to give quite good choice of powers

Convex Inner Approximation of Rate Region

with monomial $1 + \gamma_i$ approximation, rate region is convex



Iterative Power Optimization Algorithm

at iteration k

- approximate $\prod_i (1+\gamma_i)$ as

$$\prod_i (1+\gamma_i^{(k)})\gamma_i^{\beta_i^{(k)}}, \qquad \beta_i^{(k)} = \frac{\gamma_i^{(k)}}{1+\gamma_i^{(k)}}$$

- solve GP

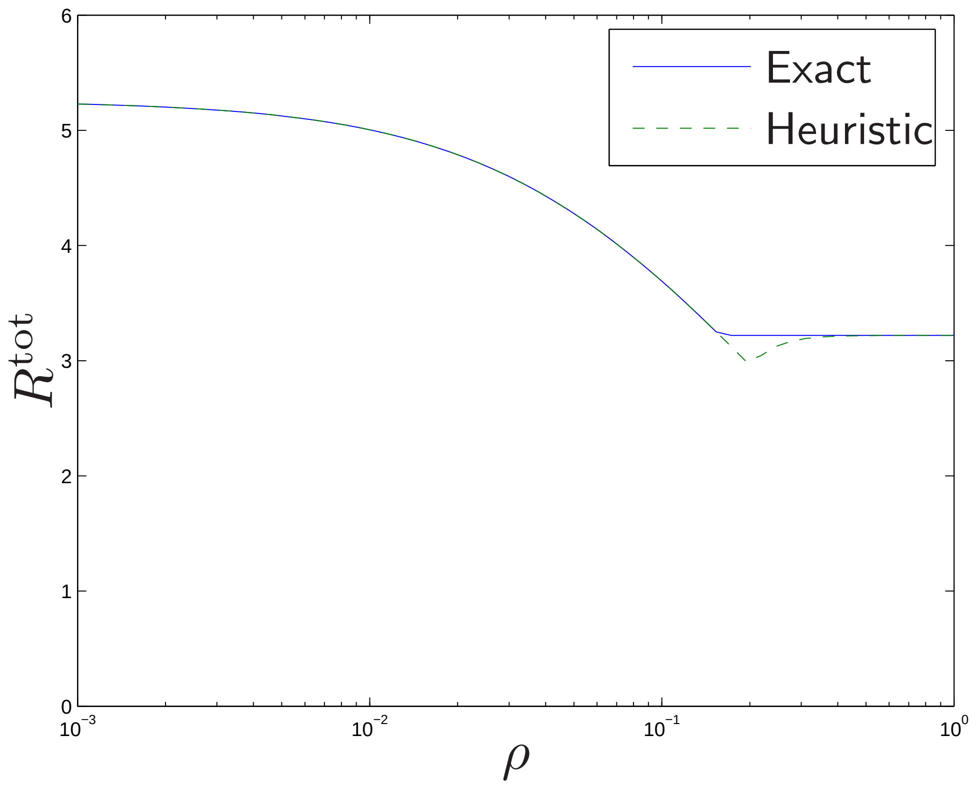
$$\begin{array}{ll} \text{maximize} & \prod_i \gamma_i^{\beta_i^{(k)}} \\ \text{subject to} & P_i \le P_i^{\max} \end{array}$$

  to get $P_i^{(k+1)}$

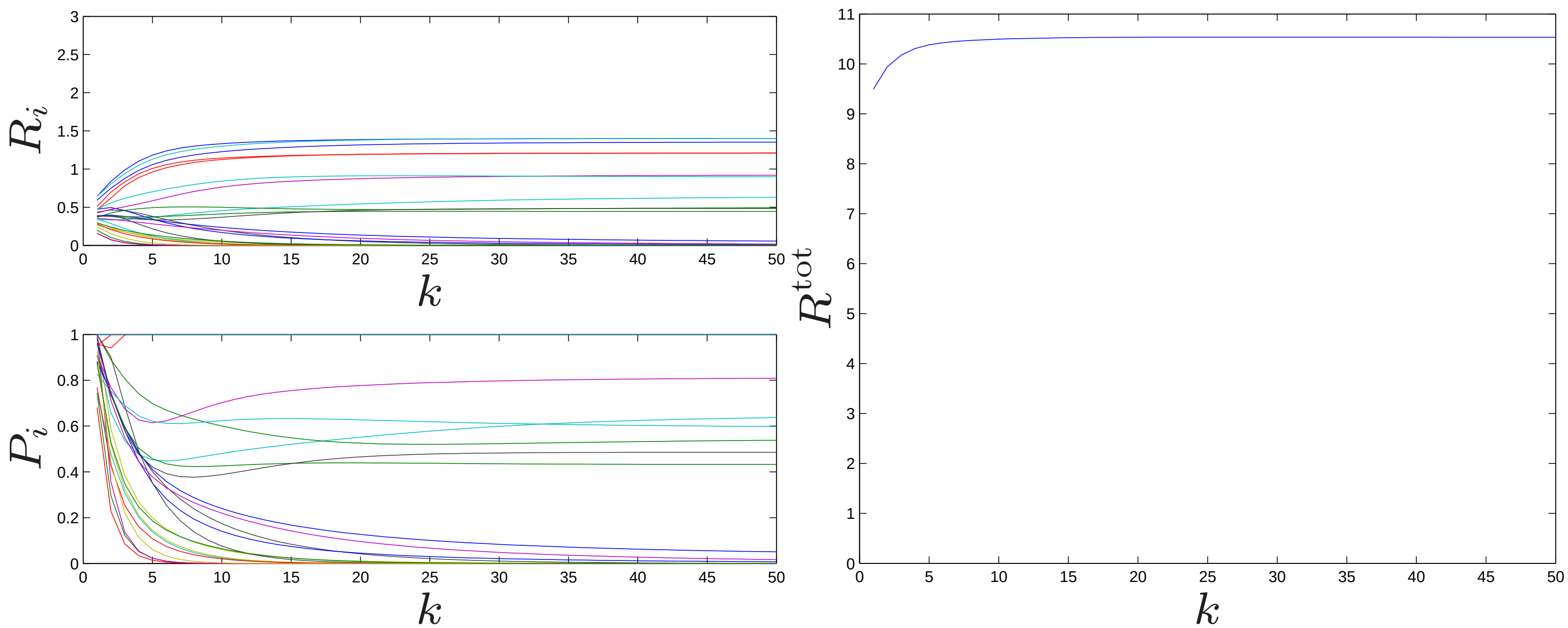
can start from powers found using high SINR approximation

Simple(st) Example — Sum Rate Maximization

$$G = \begin{bmatrix} 1.2 & \rho \\ 1.4\rho & 1 \end{bmatrix} \qquad \sigma^2 = \begin{bmatrix} 0.05 \\ 0.15 \end{bmatrix} \qquad P^{\max} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

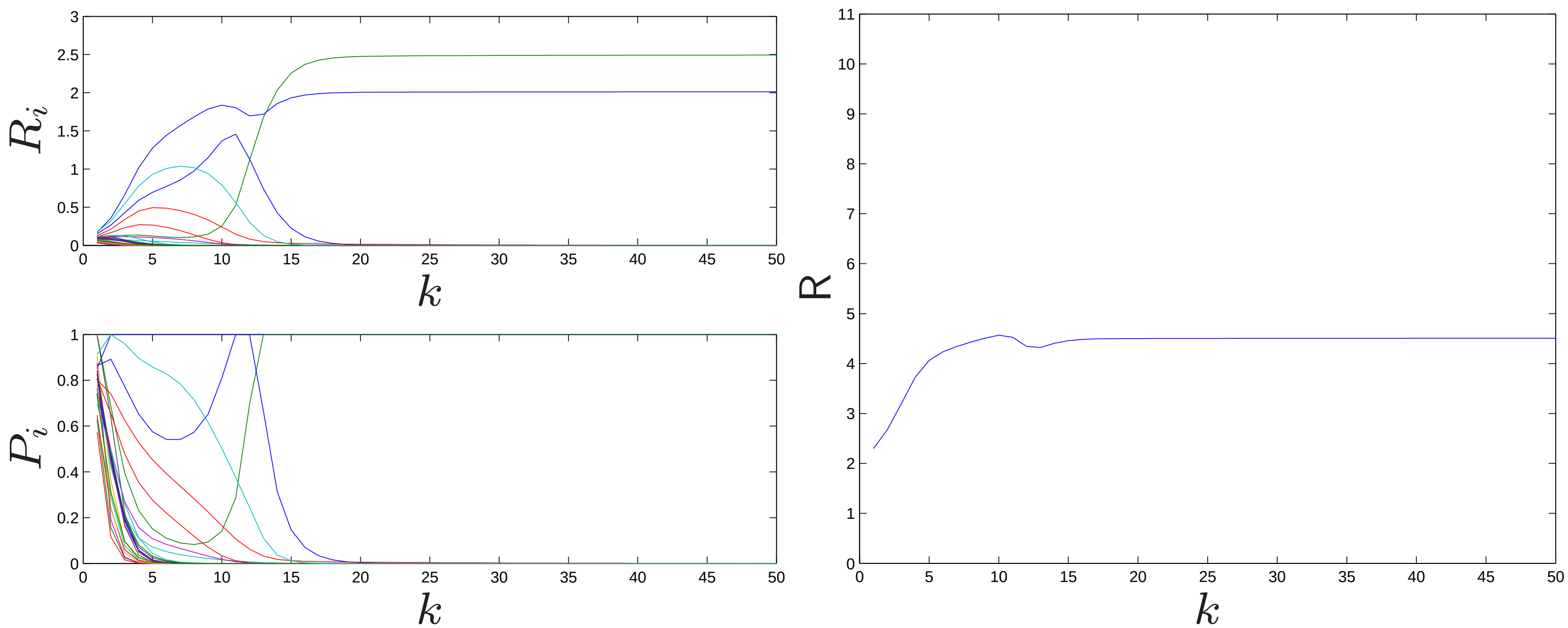


Larger Example — High SINR ($\rho = 0.1$)



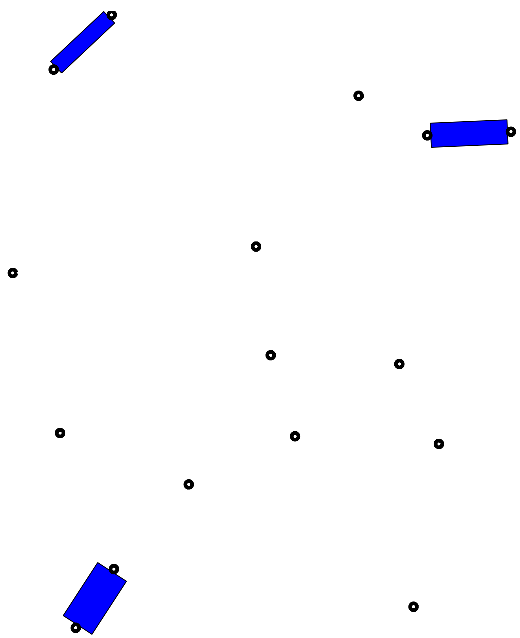
11 out of 25 links are on; 14 are off

Larger Example — Low SINR ($\rho = 0.5$)

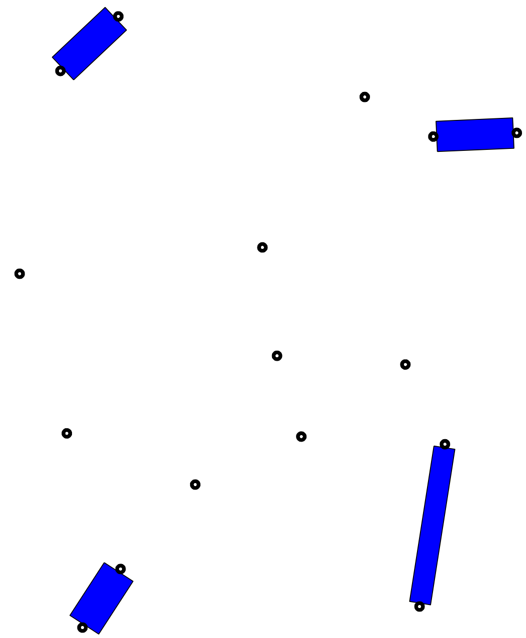


2 out of 25 links are on; 23 are off

Wireless Network Example



$G_{ij,kl} = \|x_l - x_i\|^{-2}$



$G_{ij,kl} = \|x_l - x_i\|^{-4}$

width shows power of link

Extensions & Conclusions

extensions (easy):

- any GP compatible constraints ($e.g.$, on power)
- joint allocation of bandwidth, power; flow routing

what's next:

- upper bounds on $R^{\mathrm{tot}\star}$?
- decentralized version?