Distributed Resource Allocation for MANETs

Asu Ozdaglar

Electrical Engineering and Computer Science
Massachusetts Institute of Technology

ITMANET Poster
January, 2007

Resource Allocation in Manets
Main Challenges

- No centralized access to information
  - All computations need to be performed in a decentralized and asynchronous manner using local information and observations

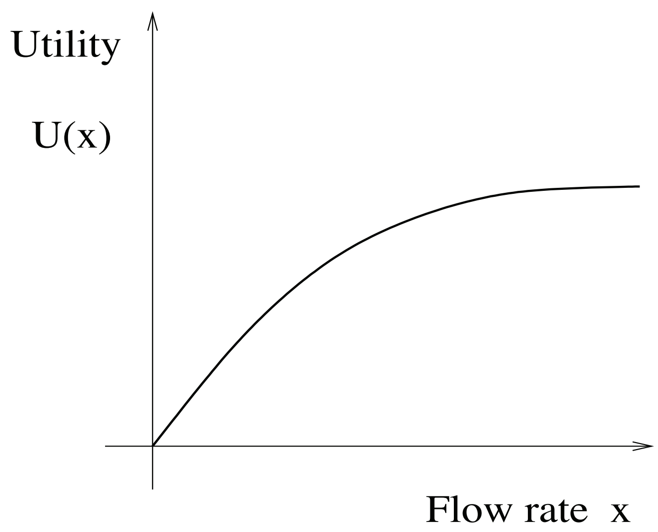
- Agents heterogeneous in terms of their local objective functions and/or information

- The operation and the structure of networks necessitate considering nonconvex optimization problems for resource allocation.
  - Interference effects
  - Applications with nonconvex preferences (e.g. voice)

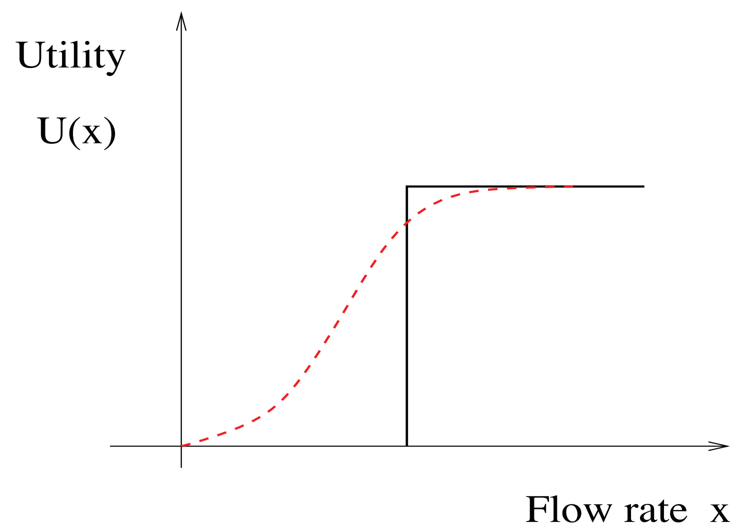
Resource Allocation among Heterogeneous Agents

- Utility-based framework of economics
  - Represent user preferences by utility functions over the resource allocated to the user (e.g., rate, power)
- Based on application service requirements, utility functions take different forms (Shenker 95):

Elastic Application



Inelastic Application



Utility Maximization

- Allocation of a divisible resource among heterogeneous agents with no externalities in the system (i.e., no interference or congestion effects)
- System problem for rate control for elastic applications:

$$\text{maximize} \quad \sum_{i=1}^{m} U_i(x_i)$$
$$\text{subject to} \quad \sum_{i \in \mathcal{I}(l)} x_i \leq c_l, \quad \forall\ l$$

- Distributed implementation using traditional Lagrangian duality:
  - Given price p_i, user i solves: $\max_{x_i} U_i(x_i) - p_i x_i$
  - Optimum $x_i^* = (U_i')^{-1}(p_i)$
  - "Incentive-compatible" from user viewpoint
- Similar extensions to elastic rate control for network coding

Shortcomings of This Approach

- Relies heavily on the:
  - Separable structure of the objective and constraint functions (congestion externalities?)
  - Convexity of the problem (inelastic applications?)

- Convergence rate of the resulting subgradient methods not well-understood

- Assumes dual solved to exact optimality to recover the primal optimal solution

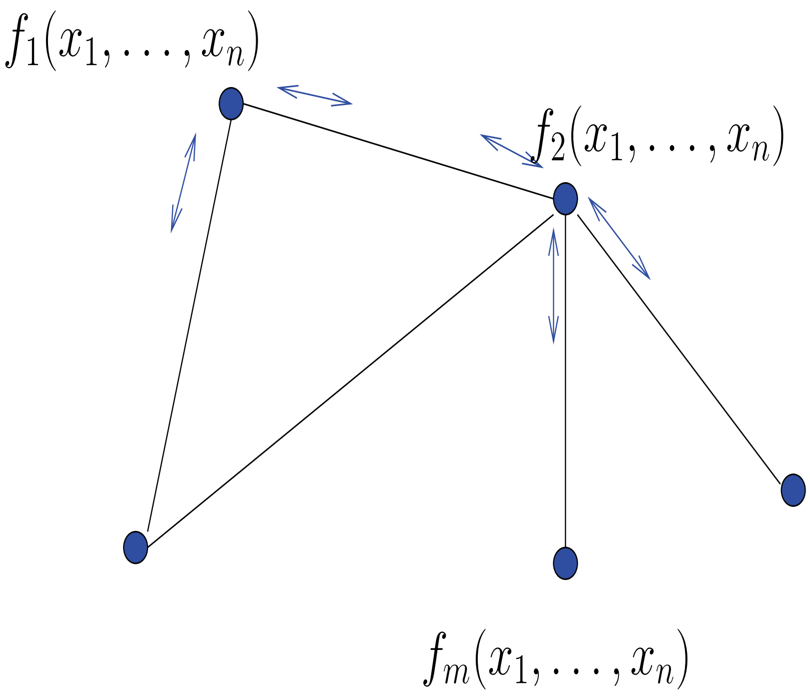
A Distributed and Asynchronous Optimization Model

- Consider a network consisting of m nodes (or agents) that cooperatively minimize a common additive cost (not necessarily separable)

$$\text{minimize} \quad \sum_{i=1}^{m} f_i(x)$$
$$\text{subject to} \quad x \in \mathcal{R}^n,$$

- Each agent has information about one cost component, and minimizes that while exchanging information locally with other agents.

- Model is similar in spirit to the distributed computation model proposed by Tsitsiklis



Algorithm

- Denote by ξ$^{\iota}$(k) vector estimate computed by agent i at time τ κ.
- Agent i generates a new estimate as follows:

$$x^i(k+1) = \underbrace{\sum_{j=1}^{m} a^i_j \; x^j(k)}_{\text{Average of estimates}} - \underbrace{\alpha(k) d_i(x(k))}_{\text{Innovation}}$$

$$\begin{aligned} a^i = (a^i_1, \ldots, a^i_m) &: \text{ vector of weights} \\ \alpha(k) &: \text{ stepsize} \\ d_i(x^i(k)) &: \text{ subgradient of } f_i \end{aligned}$$

- Estimates from other agents can be obtained asynchronously

Convergence

- Define the sequence

$$y(k+1) = y(k) - \frac{1}{m}\alpha(k)d(y(k)),$$

where $d(y(k)) = \sum_{i=1}^{m} d_i(y(k)).$

- Theorem: *[Nedich, Ozdaglar 07]* Assume that each f_i is bounded from below and continuously differentiable. For diminishing and constant stepsizes, we have:
  - For all *i*, $\lim_{k\to\infty} \|y(k) - x^i(k)\| = 0$
  - Also, $\liminf_{k\to\infty} \sum_{i=1}^{m} \nabla f_i(x^i(k)) = f^*$, where f^* is the optimal value.
  - Moreover, under some coercivity assumptions, the sequence $\{x_i(k)\}$ converges to the optimal solution.

Extensions

- Design and analysis of distributed algorithms for constrained optimization (joint with Angelia Nedich)
  - Consider gradient projection methods and use local communication to exchange locally feasible solutions
  - Primal-dual methods

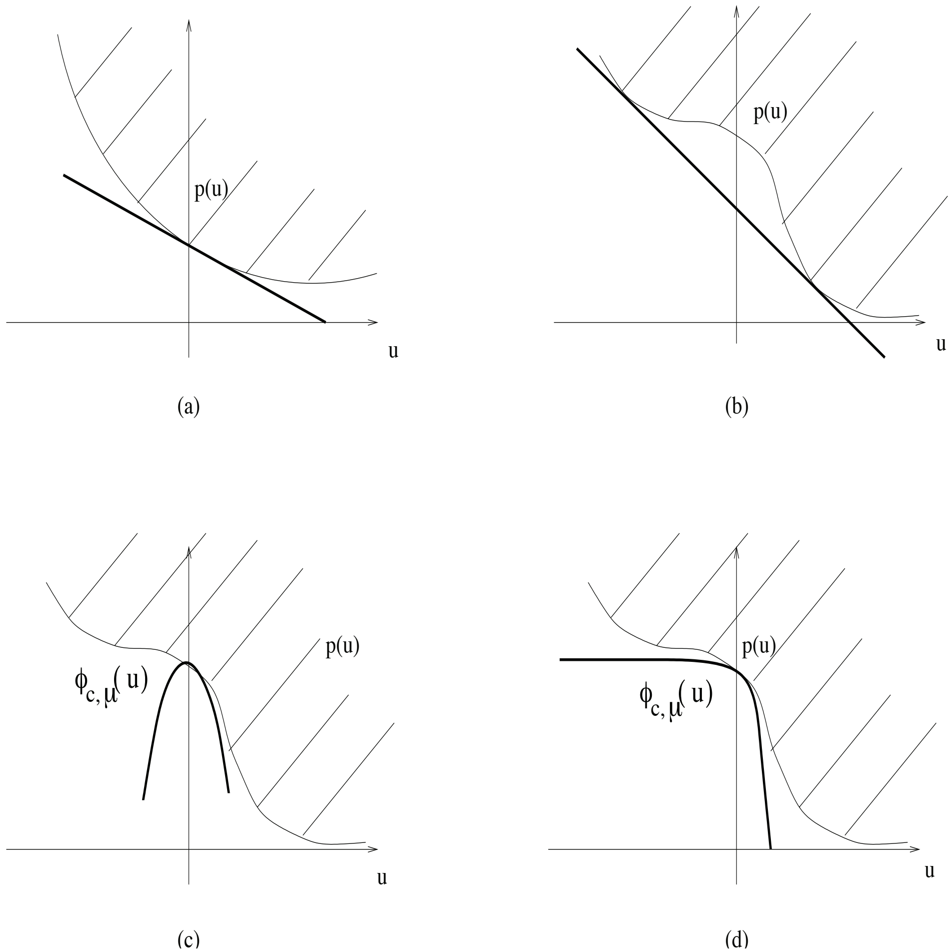
- Convergence and convergence rate analysis
  - Also crucial in applications that use synchronous dual subgradient methods (e.g., network coding with cost minimization – joint with Muriel Medard)

- Design and analysis of approximate algorithms

Extensions

- Nonconvex optimization problems
  - Focus on special classes of problems,
    e.g., sigmoidal utility maximization representing inelastic applications such as voice
  - Scaling to eliminate duality gap
  - A new framework for nonconvex optimization duality

Main Idea



raditional duality relies on supporting the epigraph of the primal function with affine surfaces (hyperplanes)

or nonconvex problems need nonlinear surfaces to "penetrate dents"

ero gap results can be established by proving separation results for nonconvex sets via general