Finding the best mismatched detector

for channel coding and hypothesis testing

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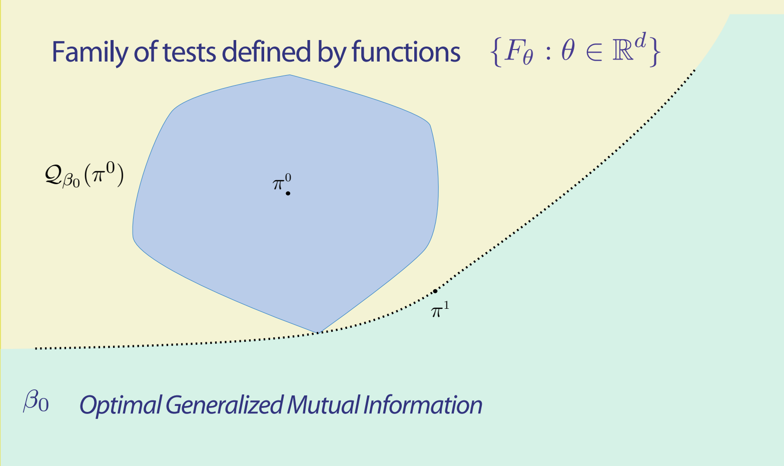
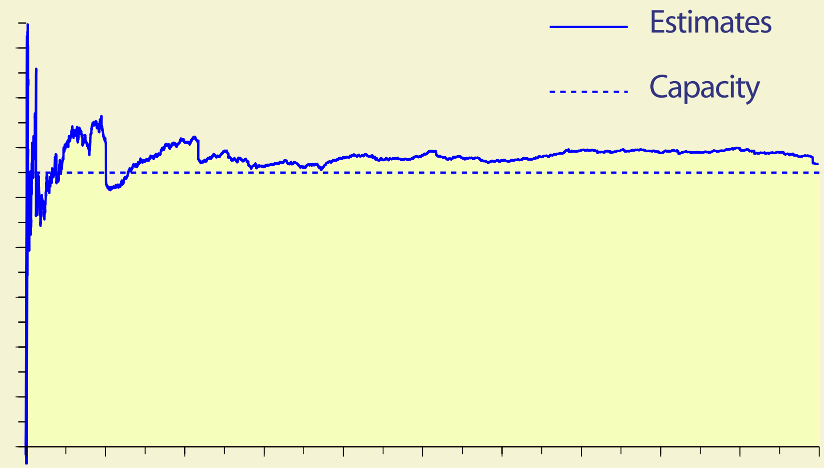
Question:

Given a family of linear detectors, *how to find the best*?





Goal: Robustness

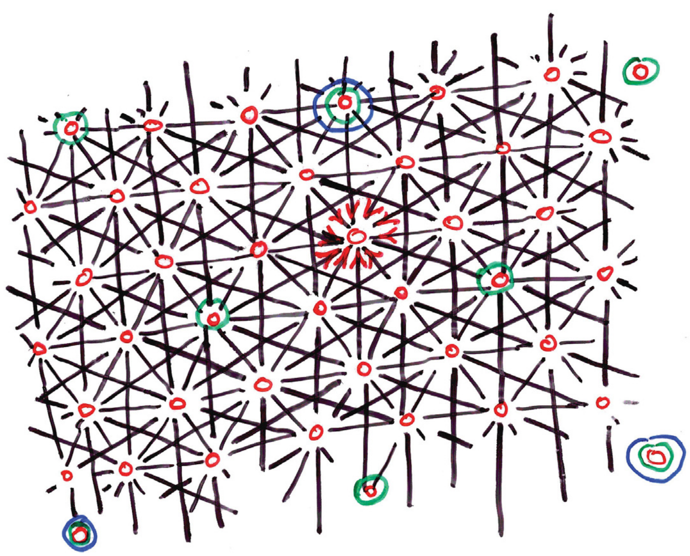
Original motivation for the research on mismatched detectors

What is the impact on capacity given a poor model?
How should codes be constructed
to take into account uncertainty?

Goal: Complexity

Detection and capacity - easy. *Its just mutual information*!!

Linear detectors suggest relaxation techniques for multiuser and network settings



Goal: Adaptation

A new point of view!

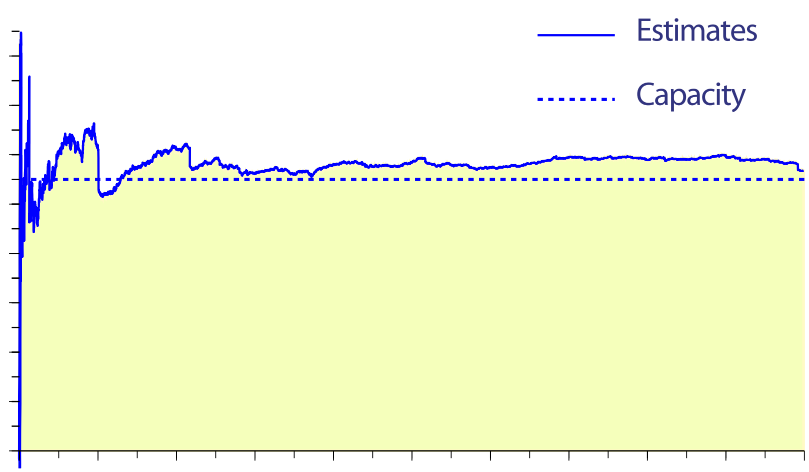
How can a detector be tuned on-line

in a dynamic environment?

This will depend on SNR

Other users

Network conditions ...

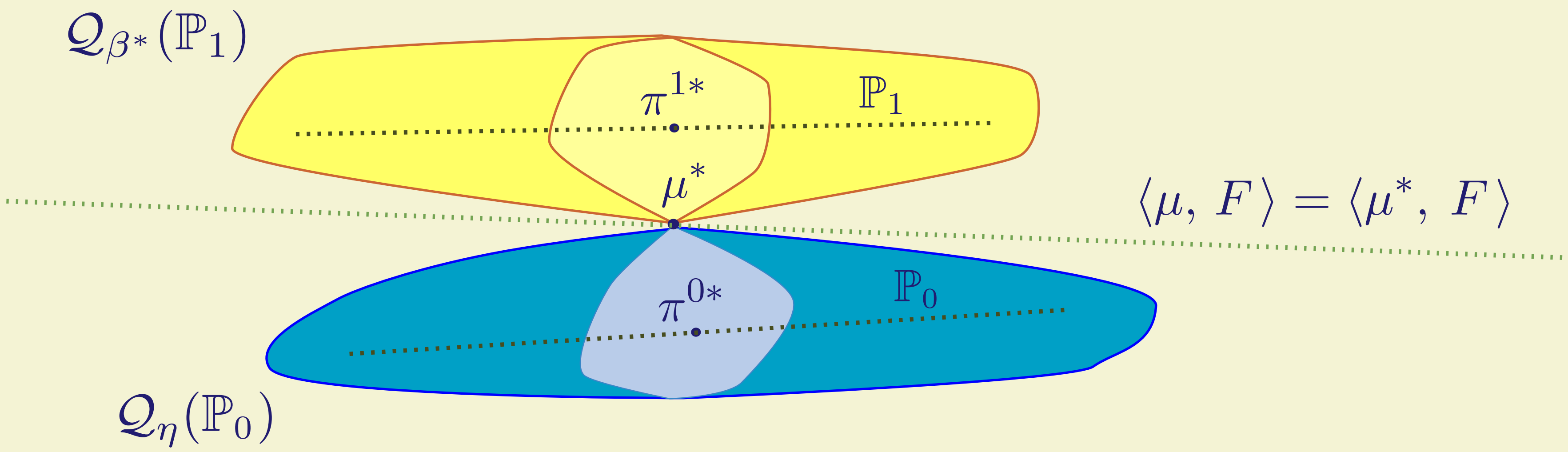


Background: Robust Hypothesis Testing

Uncertainty classes defined by moment constraints

There exists $\pi_0^* \in \mathbb{P}_0, \pi_1^* \in \mathbb{P}_1$ and μ^* solving,

$$\beta^* = \inf_{\pi_1 \in \mathbb{P}_1} \inf_{\mu \in \mathcal{Q}_\eta(\mathbb{P}_0)} D(\mu \,\|\, \pi_1)$$



F : Generalized Log Likelihood Ratio

Background: Mismatched channel

A given function $F(x,y)$ defines a surrogate ML detector:

$$i^* = \arg\max_i \langle \Gamma_n^i, F \rangle = \arg\max_i \Big\{ \sum_{t=1}^{n} F(X_t^i, Y_t) \Big\}$$

Empirical distributions for ith codeword:

$$\Gamma_n^i = n^{-1} \sum_{t=1}^{n} \delta_{X_t^i, Y_t}$$

Reliably received rate (in general a lower bound):

Generalized Mutual Information (Lapidoth, Csiszar)

Mismatched Detector: Sanov's Theorem

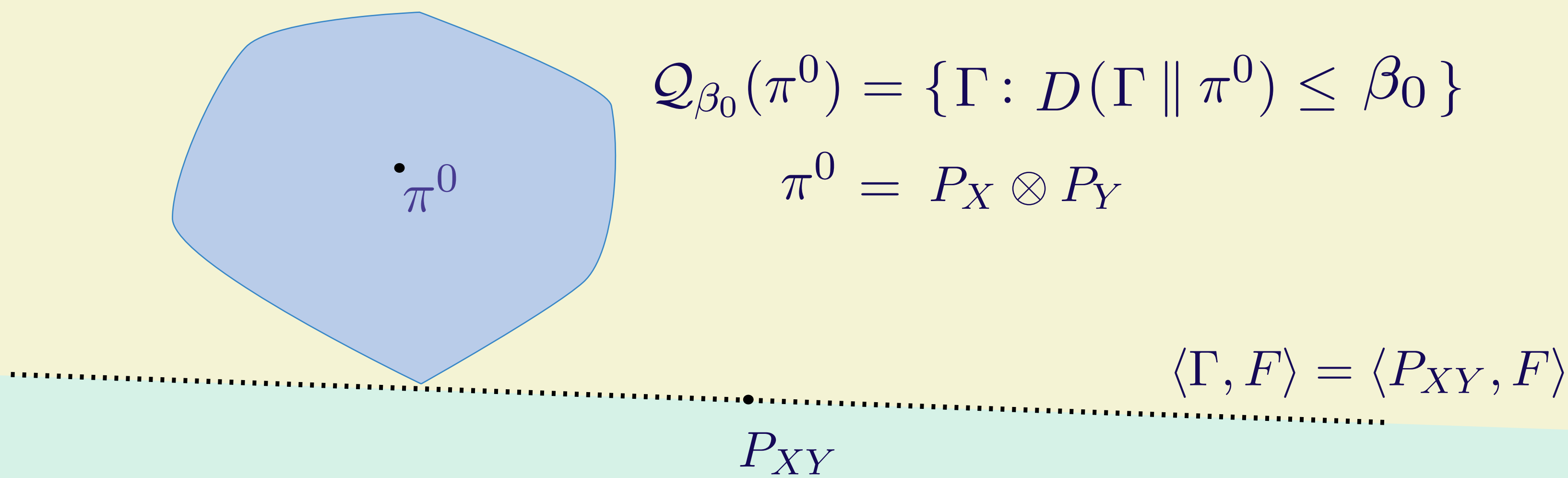
Sanov's Theorem (in form of Chernoff's Bound) gives

$$\log P_e \leq -nI_{\text{LDP}}(P_X; F)$$

$$I_{\text{LDP}}(P_X; F) := \inf\Big\{D(\Gamma \| P_X \otimes P_Y) : \langle \Gamma, F \rangle = \langle P_{XY}, F \rangle\Big\}$$

$$\mathcal{Q}_{\beta_0}(\pi^0) = \{\Gamma : D(\Gamma \,\|\, \pi^0) \leq \beta_0\}$$

$$\pi^0 = P_X \otimes P_Y$$



$\beta_0 = I_{\text{LDP}}(P_X; F)$

Mismatched Detector: Generalized Mutual Information

Generalized Mutual Information

$$\log P_e \leq -nI_{\mathrm{GMI}}(P_X; F)$$

$$I_{\mathrm{GMI}}(P_X; F) := \inf\Big\{D(\Gamma \| P_X \otimes P_Y) : \langle \Gamma, F \rangle = \langle P_{XY}, F \rangle \,,\, \Gamma_2 = P_Y\Big\}$$

Derivation of GMI bound using Sanov: For any function $G(y)$,

$$i^* = \arg\max_i \langle \Gamma_n^i, F + G \rangle = \arg\max_i \Big\{ \sum_{t=1}^{n} [F(X_t^i, Y_t) + G(Y_t)] \Big\}$$

$$\boxed{\log P_e \leq -nI_{\mathrm{LDP}}(P_X; F + G) \quad \textit{any } G(y)}$$

Mismatched Detector: Generalized Mutual Information

Generalized Mutual Information

$$\log P_e \leq -nI_{\mathrm{GMI}}(P_X; F)$$

$$I_{\mathrm{GMI}}(P_X; F) := \inf\Big\{D(\Gamma \| P_X \otimes P_Y) : \langle \Gamma, F\rangle = \langle P_{XY}, F\rangle\,,\ \Gamma_2 = P_Y\Big\}$$

$$\log P_e \leq -nI_{\mathrm{LDP}}(P_X; F + G\,)$$ *any* $G(y)$

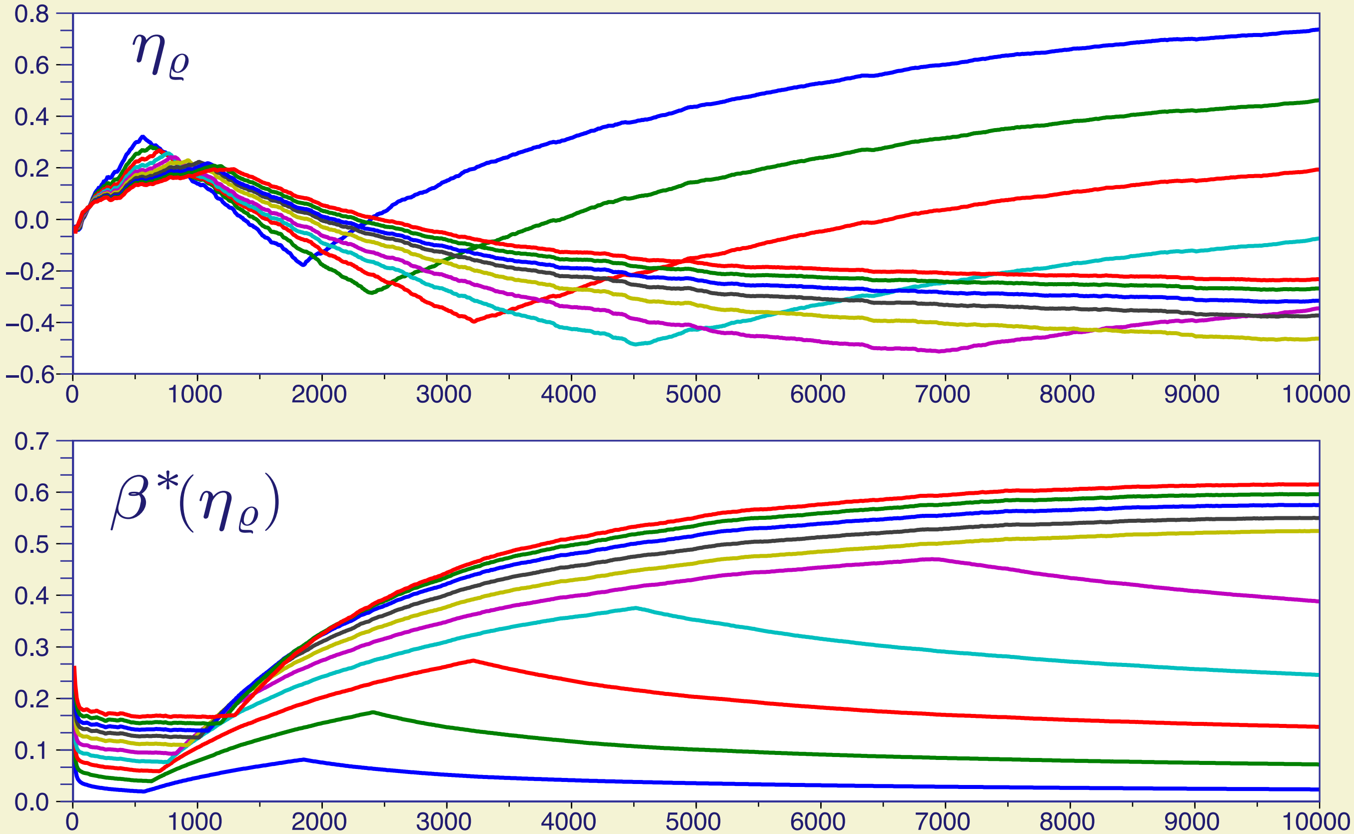
$$\sup_{G(y)} I_{\mathrm{LDP}}(P_X; F + G) = I_{\mathrm{GMI}}(P_X; F)$$

G^* : Lagrange multiplier for equality constraint $\Gamma_2 = P_Y$

Mismatched Neyman Pearson Hypothesis Testing

X^1 uniform on $[0,1]$, $X^0 = \sqrt[5]{X^1}$

Optimal Linear Test Obtained Using Stochastic Newton-Raphson



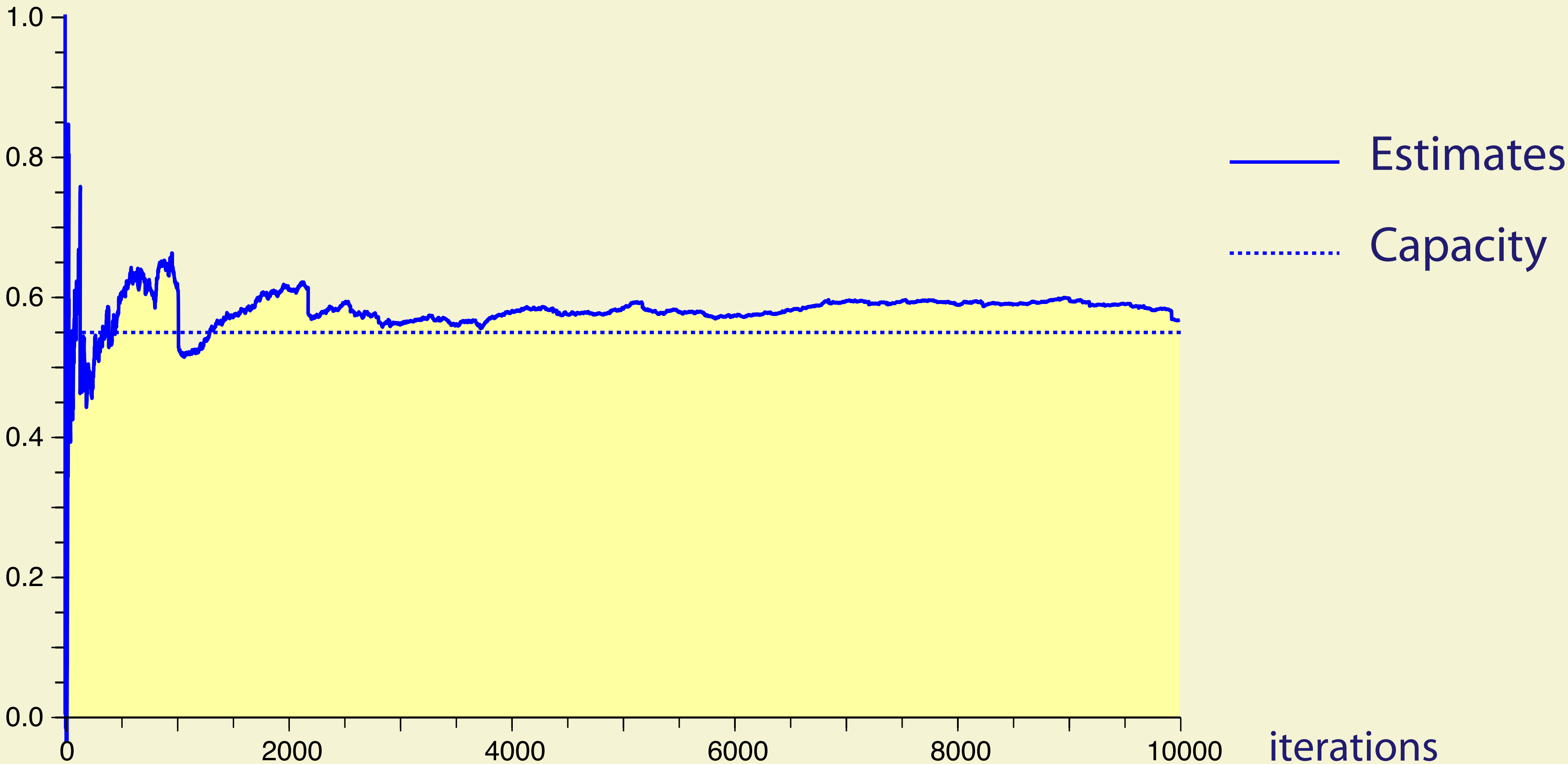
Parallel simulation for a range of η

AWGN Channel

$Y = X + N$ SNR = 2

Basis: $\psi = (x^2, y^2, xy)^{\mathrm{T}}$

Optimal Linear Test Obtained
Using Stochastic Newton-Raphson



Conclusions

Geometry based on Sanov's Theorem combined
with Stochastic Approximation provides powerful
computational tools

Future work:

Application to MIMO will simultaneously resolve coding
and resource allocation. Extensions to network coding possible?

Simulation algorithm exhibits high variance. Application of
importance sampling? Revisit robust approach?