Upper bounds and capacity limits

E. Abbe, T.Coleman, R. Koetter, M. Medard, S. Meyn, P. Moulin, L. Zheng

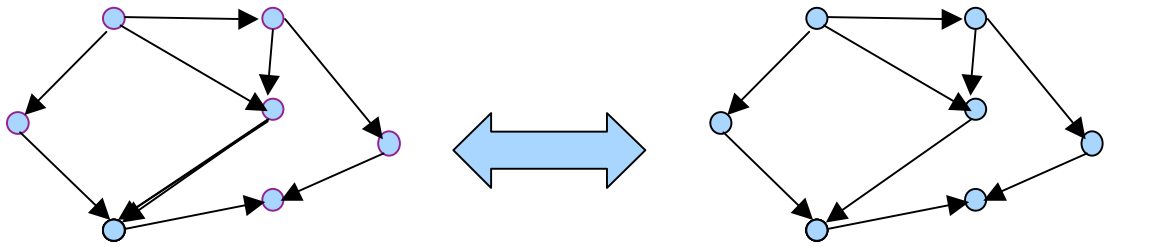
Approaches to upper bounds:

--- Min-cut max-flow theorem: [Cover & Thomas, ch. 14]

--- Fano's inequality

--- genie aided approaches

--- Equivalence classes of networks (last meeting)



noisy links

error free bit-pipes

Further issues in **wireless** ad hoc networks:

Throughput limits under jamming and hostile environments

How do practical issues, i.e. limitied knowledge and capability affect the upper bounds?

What do “accepted” wireless constraints (as e.g. half-duplex operation) really mean?

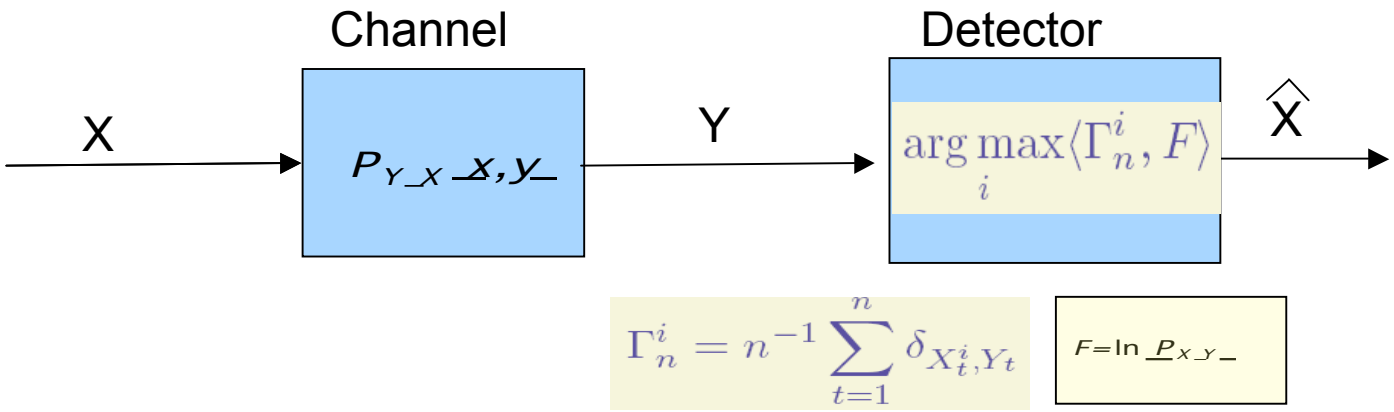
Our vision - thrust mission statement:
Develop new and creative approaches to the central problem of understanding the ultimate limits of information transmission in wireless ad-hoc networks.

- creative, original approaches (e.g. a comparative theory of upper boun
- absolute limits under constraints of imperfect operation.
- absolute limits under adversary conditions

1. In collaboration with Muriel Medard, Pierre Moulin will investigate fundamental performance limits for jamming in a network setting. Our recent work (submitted for publication at ISIT 2006) explored such limits for wideband fading channels, and uncovered some surprising findings: under some technical assumptions, a jammer is unable to significantly affect capacity or even probability of error exponents for suitably randomized transmission schemes. We would like to explore extensions of these results in a network setting, and see what type of randomization would be appropriate to obtain high resilience to jamming.
2. In Information Theory, the problem of identification of a message is fundamentally different from the usual transmission problem that was treated by Shannon. In the ID problem, the receiver is to perform a binary decision: the message is viewed as a signature, and the receiver's task is to verify the authenticity of a given signature. Ahlswede and Dueck have derived information-theoretic bounds on the performance of ID codes. Unlike the methods routinely used in computational cryptography, these methods offer fundamental information-theoretic guarantees of security. Moreover, there are practical code constructions that can approach the fundamental performance bounds, with security guarantees. Pierre Moulin and Ralf Koetter have recently explored several aspects of this problem and would like to extend them to a network setting. This would be useful in applications involving multiple senders and/or receivers, who may not necessarily trust each other.

Capacity and constrained detection optimization:

Based on: $Pr\{Y_1^n | X_1^n\} = \prod P(Y_i | X_i)^{\#(X_i, Y_i)}$



What if we cannot realize or know the function $F = \ln P_{X,Y}$ but have to find

the best test F than can be created from a set of given elementary tests?

Finding the best mismatched detector

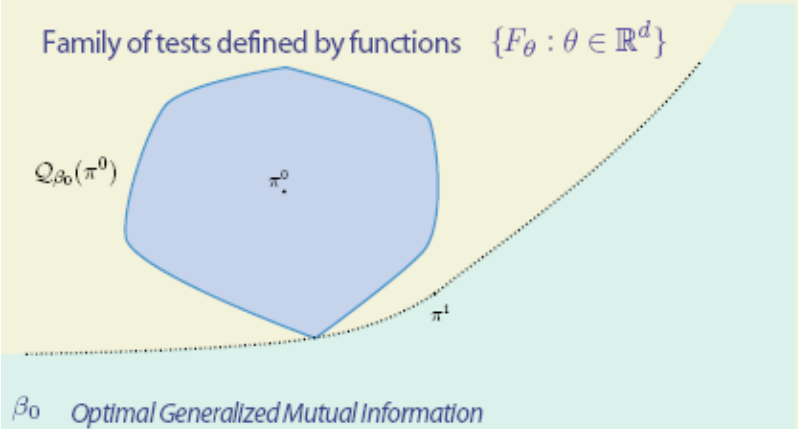
for channel coding and hypothesis testing

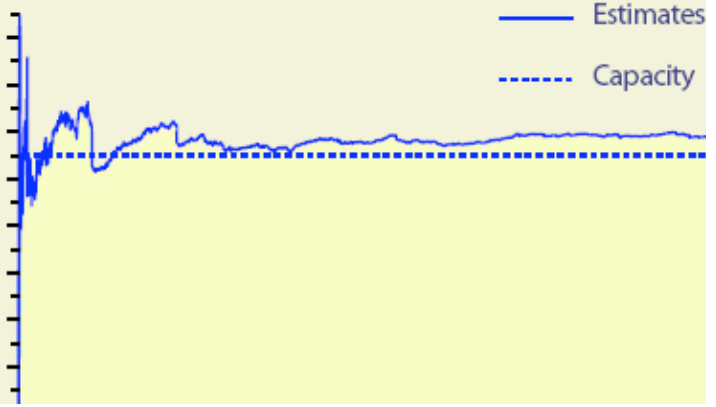
Team members: Muriel Me'dard, Sean Meyn and Lizhong Zheng

Graduate student member: Emmanuel Abbe

Publications: Finding the best mismatched detector for channel coding and hypothesis testing, UCSD ITA 2007
Geometry of Mismatched Decoders, submitted to IEEE ISIT 2007

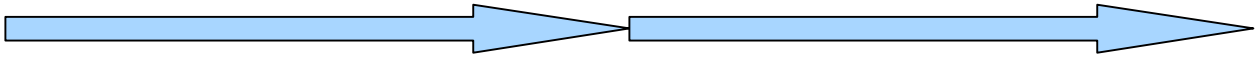
Issues: Given a family of linear detectors, how to find the best?
Generalized mutual information ⟶ bounds on capacity
Geometric insight on mismatch capacity
Stochastic approximation to compute optimal test

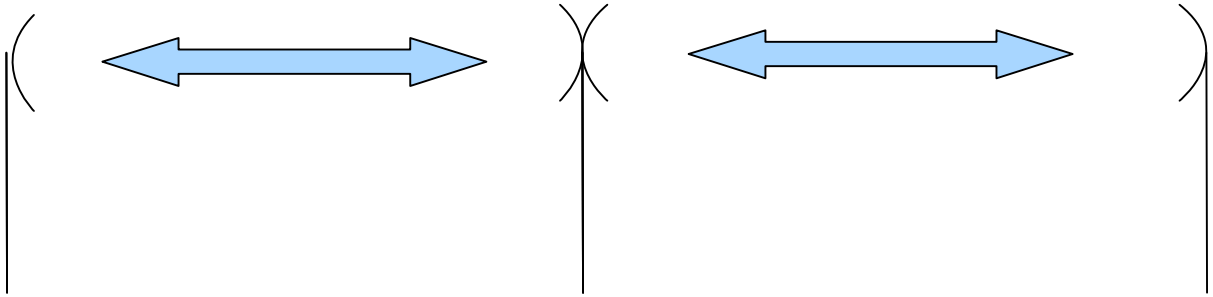




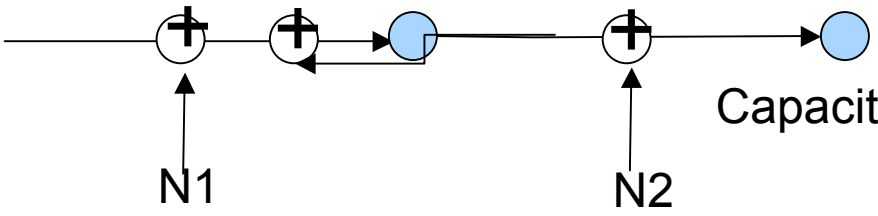
A simple (?) question: What is the capacity of a two-hop radio link

- simple links transmitting one bit per unit time.



- 

half-duplex constraint (?)

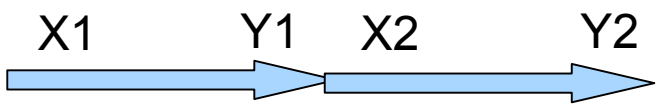
- 

Capacity: 1/2log(1+SNR) bit per unit tin

The middle node operates as a multiple access channel with strong interference

Different models for a wireless multi-hop link: half-duplex vs. multiple access channel

The half-duplex constraint is information theoretically not clear at



assuming the half-duplex constraint on binary bit p

X1 and X2 cannot be nonzero simultaneously, X1,
assume values in {1,-1,NS}

$$P(Y_1|X_1, X_2)$$

$Y_1 \backslash X_1, X_2$	NS, NS	$\{NS, \pm 1\}, \pm 1$	$\pm 1, NS$
E	1	0	0
± 1	0	1	1

This conditional pdf satiesfies the conditions for a physically degraded relay channel

$$P(Y_2, Y_1|X_1, X_2) = P(Y_1|X_1, X_2)P(Y_2|Y_1, X_2)$$

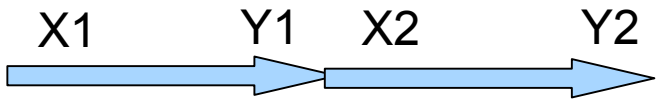
with capacity:

$$\sup_{P(X_1,X_2)} \min\{I(X_1, X_2; Y), I(X; Y_1|X_1)\}$$

For our two link half duplex example this comes out as 1.1388 bits/channel use

The capacity of the binary input half duplex chain of two links is 1.1388 bits/channel use

A second attempt at a meaningful half-duplex link chain:
We enforce the side constraint P(NS,NS)=0, i.e. at least one sen
has tor transmit a signal.

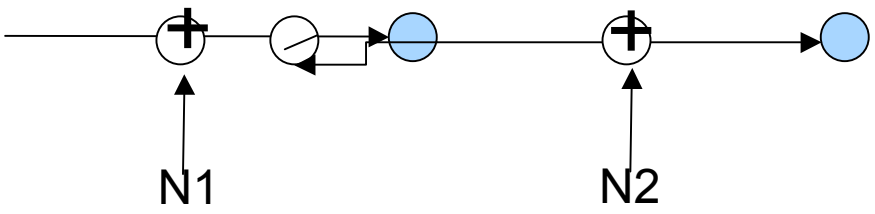


$$P((X_1, X_2) = (NS, NS)) = 0$$

$Y_1 \backslash X_1, X_2$	NS, NS	$\{NS, \pm 1\}, \pm 1$	$\pm 1, NS$
± 1	$undefined$	1	1

This is still a physically degraded relay channel: Capacity equals still 0.8295 bits/channel

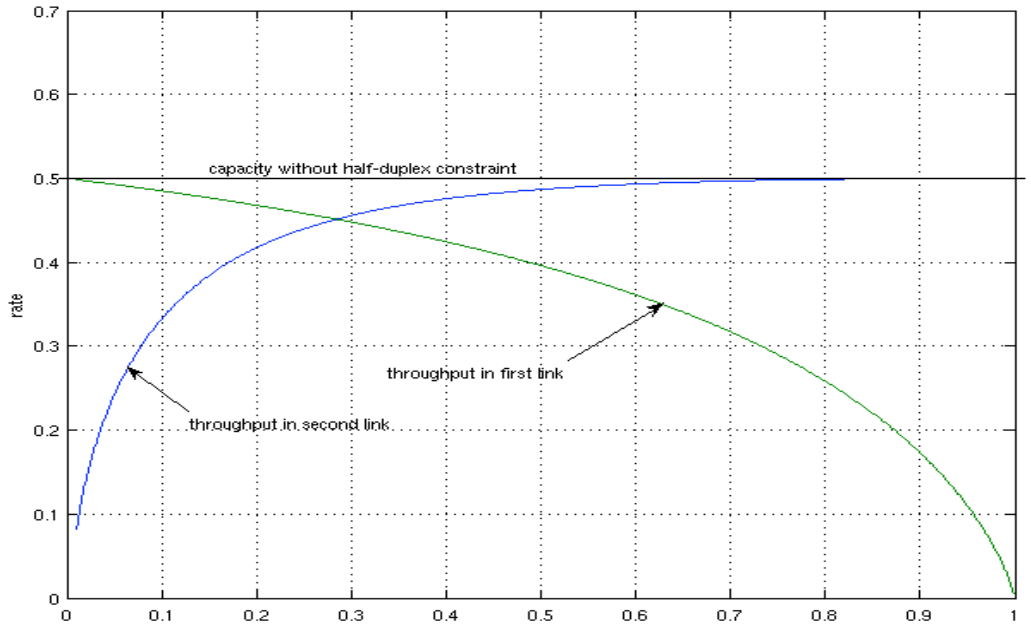
For a gaussian half-duplex channel (N0<< P2):



$$Y_1 = \begin{cases} X_1 + N_1 & X_2 = NS \\ X_2 + N_0 & \text{otherwise} \end{cases}$$

Capacity is: ½ E[1-T](log(1+SNR1/E[1-T])) where T is a Bernoulli r.v. satisfying

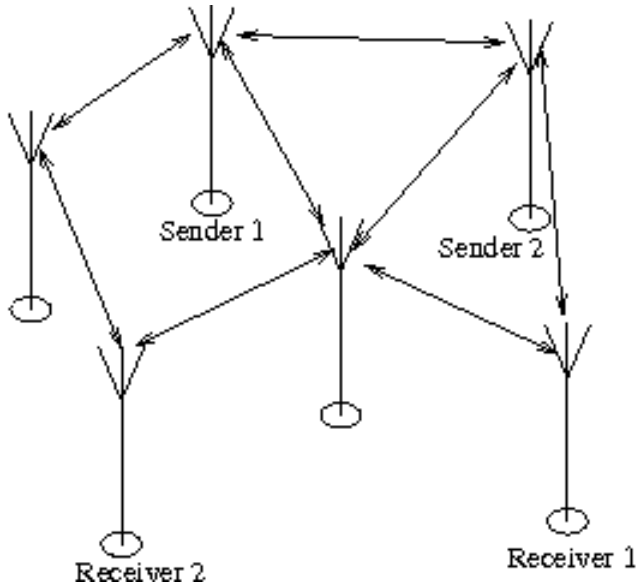
$$\frac{1}{2}(1 - E[T])(log(1 + SNR_1/(1 - E[T]))) = \max_{f_X : E[X^2]/E[T] = P_2} h(N_2 + EX)$$

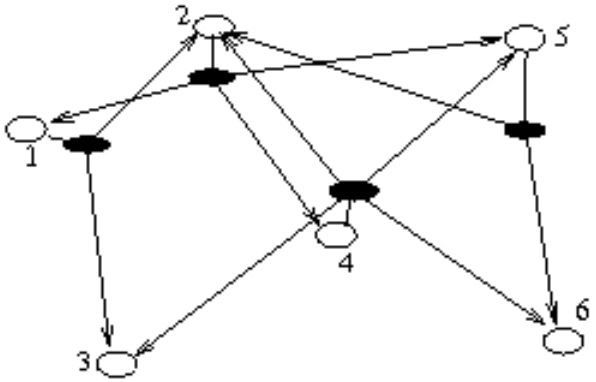


....tricky....

The wireless butterfly network

Half-duplex and interference constraint





Without network coding and time-sharing: 0.25 bits/(unit time x connections)
With network coding and time sharing: 0.33 bits/(unit time x connections)
With network coding and half-duplex constraint: ≥ 0.57 bits/(unit time x connections)

Summary: Upper bounds and Capacity is central to determining the ultimate limits

We pursue a double thrust of extending the theory in classical approaches as well
focusing on fundamental limits under various MANET induced constraint secenari

Posters relating to this area:

- "On MANET jamming," P. Moulin
- "Find the best mismatch detector for capacity and hypothesis testing," S. Meyn, L. Zheng, M. Medard
- "Information geometry and capacity limits," L. Zheng