

# Error Exponents for Wideband Multipath Fading Channels — a Strong Coding Theorem

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*Abstract* — The wideband multipath fading channel has been considered from both an information-theoretic and implementation-oriented standpoint. Developments from the two perspectives have, however, been largely independent. We aim to find a connection between the information-theoretic results and actual coding and modulation schemes by producing a strong coding theorem for the channel. We calculate explicit upper bounds on the probability of error for general multipath and Rayleigh fading, and study their behavior numerically.

## I. INTRODUCTION

It is known that spread-spectrum signals such as those in direct-sequence CDMA systems are not optimal for very large bandwidth multipath fading channels: Médard and Gallager [1] have shown that, under the assumption of independence amongst fading processes at different frequencies, the mutual information of such a system approaches zero with increasing bandwidth. In addition, Telatar and Tse [2] have studied the wideband fading channel with a finite number of paths and have shown that the mutual information is inversely proportional to the number of resolvable paths if white-like input signals (such as those common in spread-spectrum systems) are used and signal energy is evenly divided among the resolvable paths. Thus the mutual information approaches zero as the number of resolvable paths becomes large. Moreover, if there is a very large number of underlying paths with differing delays, then the number of resolvable paths will increase as bandwidth increases.

The capacity of the infinite-bandwidth multipath fading channel, however, is non-zero and equal to the capacity of the infinite-bandwidth AWGN channel with the same constraint on the average received power. This result has been presented by Kennedy [3] and by Gallager [4, §8.6] for the case of Rayleigh fading, and most recently by Telatar and Tse [2] for general multipath fading. It is further shown in [2] that the capacity can be directly achieved using frequency-shift keying and non-coherent detection by transmitting at a low duty cycle. This capacity-achieving signaling scheme is “peaky” both in time (as the duty cycle is low) and frequency (as frequency-shift keying is used), in contrast to spread-spectrum schemes that spread energy evenly over a wide band. Indeed, Hajek and Subramanian [5] have demonstrated that there is a maximum possible mutual information associated with the “fourthegy” (a metric related to the fourth order of the output signal magnitude) of a signal. Direct-sequence spread-spectrum

signals have a fourthegy-to-energy ratio that is inversely proportional to the bandwidth, but this is not the case for signals that are peaky in time and frequency.

In another approach, Telatar [6] derived the capacity of the energy-limited Rayleigh fading channel by utilizing Gallager’s results for energy-limited channels [7]. Such channels, which are characterized by a very small energy per degree of freedom, can be modeled as discrete-time and discrete-input. He showed that, using random block codes and 0-1 signaling, the capacity of the Rayleigh fading channel is the same as that of the AWGN channel in the limit of large bandwidth and large signal-to-noise ratio. The Rayleigh fading channel, however, approaches this limit much more slowly than the Gaussian channel.

The information-theoretic results for fading channels outlined above seem to suggest that good performance is possible for very large bandwidth fading channels and moreover that, in this regime, signaling that is peaky in both the time and frequency domains is preferable to that which is continuous in time or broadband.

Proposed implementations have taken the form of ultra-wideband (UWB) radio — a spread-spectrum wireless system that promises to be a viable technology for indoor or other dense multipath environments. Most notably, impulse radio is a time-hopping spread-spectrum multiple-access system that employs pulse-position modulation (PPM) [8, 9, 10] or pulse-amplitude modulation (PAM) [11] for data modulation. The impulse radio system communicates by way of a time-hopping baseband signal comprised of sub-nanosecond pulses, thereby occupying a frequency band from near-DC to several gigahertz, albeit with a low power-spectral density. Therefore, interference to other narrow-band systems should be minimal. In addition, owing to the large bandwidth that it transmits on, such a system should be capable of accommodating many users and allow for fine multipath resolution.

Nevertheless, the characteristics of the signaling scheme appear to be sub-optimal from the point of view of approaching capacity. Indeed, signal energy is spread more or less evenly over its frequency band rather than being peaky. On the other hand, though we know that the information-theoretic results hold under the assumption of very large (perhaps near-infinite) bandwidths, we have little indication of exactly how large the bandwidth must be for the results to be applicable.

The question then naturally arises as to what signaling schemes are good for the wideband multipath fading channel. That is, given our theoretical insights, can we find actual coding and modulation schemes with good performance? One way to find a connection between theoretical capacity results

and actual coding schemes is by strong coding theorems (see, for example, [4, §5.6]); which not only delimit the capacity of a channel, but also, by directly applying a particular coding scheme and calculating explicit upper bounds on its probability of error, show how and how fast the capacity can be approached. Traditional coding arguments for the strong coding theorem use random block coding to relate the length of the code, a quantity associated with the coding delay, to the probability of error. Channel capacity is reached by extending the length of the code to infinity. In the coding argument that we present, each code word is represented by a sinusoid at a particular frequency rather than a string of symbols, thus it is not so much the coding delay that is pertinent, but rather limitations on the available bandwidth and peak power. The existing theoretical results for the multipath fading channel are essentially only concerned with finding the capacity, and in that sense, are weak coding theorems.

In this paper, using the capacity-achieving scheme expounded in [2], we derive upper bounds on the probability of error for the wideband multipath fading channel under the assumption of independent fading over separate coherence-time intervals for general multipath and Rayleigh fading. We utilize these bounds to study the interplay amongst the error probability, bandwidth, rate, and “peakiness” of the scheme.

## II. STRONG CODING THEOREM FOR MULTIPATH FADING CHANNELS

In this section, we derive upper bounds on the error probability for the capacity-achieving scheme presented in [2], which we summarize below. We adhere to similar notation.

### CAPACITY-ACHIEVING SCHEME

We begin with a general multipath fading channel. Hence the channel output  $y(t)$  to an input waveform  $x(t)$  is given by

$$y(t) = \sum_{l=1}^L a_l(t)x(t - d_l(t)) + z(t), \quad (1)$$

where  $L$  is the number of paths,  $a_l(t)$  and  $d_l(t)$  are the gain and delay on the  $l$ th path at time  $t$  respectively, and  $z(t)$  is white Gaussian noise with power spectral density  $N_0/2$ . Let  $T_c$  and  $T_d$  be the coherence time and delay spread of the fading channel respectively. We assume that the processes  $\{a_l(t)\}$  and  $\{d_l(t)\}$  are constant and i.i.d. over time intervals of  $T_c$  (block-fading model in time), and that  $T_d \ll T_c$  (an underspread channel).

Suppose that the average power constraint is  $P$ , and let  $\theta \in (0, 1]$ . Suppose further that we have a code-book of size  $M$ . The  $m$ th code word is represented at baseband as a complex sinusoid of amplitude  $\sqrt{P/\theta}$  at frequency  $f_m$ , that is

$$x_m(t) = \begin{cases} \sqrt{P/\theta} \exp(j2\pi f_m t) & 0 \leq t \leq T_s, \\ 0 & \text{otherwise;} \end{cases} \quad (2)$$

where  $T_s$  satisfies  $2T_d < T_s \leq T_c$ . The frequency  $f_m$  is chosen such that it is an integer multiple of  $1/T_s'$ , where  $T_s' = T_s - 2T_d$ .

Let us consider the channel output over the interval  $[T_d, T_s - T_d]$  (the time axis at the receiver is shifted by the shortest path delay). During this interval,  $\{a_l(t)\}$  and  $\{d_l(t)\}$  are constant owing to the assumptions of the model, and we

denote their values by  $\{a_l\}$  and  $\{d_l\}$  respectively. Hence by (1), the received signal when message  $m$  is sent is

$$y(t) = \sum_{l=1}^L a_l \sqrt{P/\theta} \exp(j2\pi f_m(t - d_l)) + z(t) \\ = G \sqrt{P/\theta} \exp(j2\pi f_m(t - d_l)) + z(t) \quad (3)$$

where  $G = \sum_{l=1}^L a_l \exp(-j2\pi f_m d_l)$  is a complex-valued random variable. We define signal power in the conventional sense as the received signal power, and thus normalize the channel gain so that  $\mathbf{E}[|G|^2] = 1$ .

At the receiver, we form the correlator outputs

$$R_k = \frac{1}{\sqrt{N_0 T_s'}} \int_{T_d}^{T_s - T_d} \exp(-j2\pi f_k t) y(t) dt \quad (4)$$

for  $1 \leq k \leq M$ . Therefore,

$$R_k = \delta_{km} G \sqrt{\frac{PT_s'}{\theta N_0}} + W_k, \quad (5)$$

where  $\{W_k\}$  is a set of i.i.d. circularly-symmetric complex Gaussian random variables, each satisfying  $\mathbf{E}[|W_k|^2] = 1$ .

The message is then repeated over  $N$  disjoint time intervals to obtain time diversity. Hence for  $1 \leq k \leq M$  and  $1 \leq n \leq N$ , we have

$$R_{k,n} = \delta_{km} G_n \sqrt{\frac{PT_s'}{\theta N_0}} + W_{k,n}, \quad (6)$$

where  $\{G_n\}$  is a sequence of i.i.d. complex random variables and  $\{W_{k,n}\}$  is a set of i.i.d. circularly-symmetric complex Gaussian random variables of unit variance. We construct the decision variables

$$S_k = \frac{1}{N} \sum_{n=1}^N |R_{k,n}|^2 \quad (7)$$

and use a threshold decoding rule: Let

$$A = 1 + (1 - \epsilon) \frac{PT_s'}{\theta N_0} \quad (8)$$

(where  $\epsilon \in (0, 1)$  is an arbitrary parameter) be the threshold. If  $S_k$  exceeds  $A$  for one value of  $k$  only, then we estimate  $\hat{m} = k$ ; otherwise we declare an error.

We transmit using the above scheme for a fraction of time  $\theta$  and then transmit nothing for the remainder of the time. Hence the average power is  $P$ . Note that the scheme transmits  $\ln M$  nats in  $NT_s/\theta$  seconds, so the rate  $R$  is given by

$$R = \frac{\theta}{NT_s} \ln M. \quad (9)$$

### UPPER BOUND ON THE ERROR PROBABILITY

An error occurs if  $S_m < A$  or if  $S_k \geq A$  for some  $k \neq m$ . Furthermore, by symmetry, the error probability is the same regardless of which message was sent. So

$$p_e \leq \Pr \left\{ \bigcup_{\substack{k=1 \\ k \neq m}}^M S_k \geq A \right\} + \Pr \{S_m < A\} \\ \leq M \Pr \{S_1 \geq A\} + \Pr \{S_m < A\} \quad (10)$$

where  $l \neq m$ . An upper bound to the first term is found in [2]. The Chernoff bound is applied to obtain

$$M \Pr\{S_l \geq A\} \leq \exp\left(-\frac{\ln M}{RT_s} \left[\frac{(1-\epsilon)PT'_s}{N_0} - RT_s - \theta \ln\left(1 + \frac{(1-\epsilon)PT'_s}{\theta N_0}\right)\right]\right) \quad (11)$$

$$\triangleq p_{e,1}(M, R, \theta, \epsilon).$$

To upper bound the second term, we let  $\sigma^2 = \text{var}(|G_n|^2)$  and observe that

$$\overline{S_m} = \mathbf{E} \left[ \left| G_1 \frac{PT'_s}{\theta N_0} + W_{m,1} \right|^2 \right] = A + \epsilon \frac{PT'_s}{\theta N_0} \quad (12)$$

and

$$\begin{aligned} \text{var}(S_m) &= \frac{1}{N} \text{var} \left( \left| G_1 \frac{PT'_s}{\theta N_0} + W_{m,1} \right|^2 \right) \\ &= \frac{1}{N} \left[ \frac{P^2 T_s'^2 \sigma^2}{\theta^2 N_0^2} + \frac{2PT'_s}{\theta N_0} + 1 \right]. \end{aligned} \quad (13)$$

Then using the Chebyshev inequality and recalling (9), we get

$$\begin{aligned} \Pr\{S_m < A\} &\leq \Pr \left\{ |S_m - \overline{S_m}| > \epsilon \frac{PT'_s}{\theta N_0} \right\} \\ &\leq \text{var}(S_m) \frac{\theta^2 N_0^2}{\epsilon^2 P^2 T_s'^2}. \\ &= \frac{RT_s}{\epsilon^2 \ln M} \left( \frac{\sigma^2}{\theta} + \frac{2N_0}{PT'_s} + \frac{\theta N_0^2}{P^2 T_s'^2} \right). \end{aligned} \quad (14)$$

A tighter bound can be obtained if we assume that the fading is Rayleigh — namely that  $G_n$  are i.i.d. circularly-symmetric complex Gaussian random variables — then it follows that  $|G_n \sqrt{PT'_s}/(\theta N_0) + W_{k,n}|^2$  are i.i.d. exponentially distributed random variables with mean  $PT'_s/(\theta N_0) + 1$ . Applying the Chernoff bound yields

$$\begin{aligned} \Pr\{S_m < A\} &= \Pr\{NS_m < NA\} \\ &\leq \exp\left(-N \sup_{r < 0} \{rA - \ln(\mathbf{E}[\exp(r|Z_1 + W_{1,1}|^2)])]\right\} \\ &= \exp\left(-N \sup_{r < 0} \{rA - \ln(1 - [1 + PT'_s/(\theta N_0)]r)\}\right) \\ &= \exp\left(-\frac{\theta \ln M}{RT_s} \left[ \frac{-\epsilon PT'_s}{\theta N_0 + PT'_s} - \ln\left(1 - \frac{\epsilon PT'_s}{\theta N_0 + PT'_s}\right) \right]\right) \\ &\triangleq p_{e,2}(M, R, \theta, \epsilon). \end{aligned} \quad (15)$$

Note that this bound decays faster than (14) in  $M$ , but very slowly nevertheless. In particular, observe that since  $\ln(1 - z) = -z + O(z^2)$ , if  $z = (\epsilon PT'_s)/(\theta N_0 + PT'_s) \ll 1$ , then the entire exponent will be very small, which is the case for  $\epsilon \ll 1$  or  $\theta \gg PT'_s/N_0$ . It is also evident that the exponent will be small if  $\theta$  is small.

Combining (10), (11), and (14) or (15) gives us a relationship among the upper bound on the error probability, the size of the code-book  $M$  (which is directly proportional to the

bandwidth  $W = M/T'_s$ ), the transmission rate  $R$  (which is related to the SNR per bit  $E_b/N_0 = (P/N_0) \cdot (\ln 2/R)$ ), the duty factor  $\theta$ , and the parameter  $\epsilon$ . Moreover, as long as  $R$  does not exceed

$$\left(1 - 2\frac{T_d}{T_c}\right) \frac{P}{N_0} \quad (16)$$

then by letting  $T_s = T_c$  and taking  $M \rightarrow \infty$ ,  $\epsilon \rightarrow 0$ ,  $\theta \rightarrow 0$  independently, the bounds (11), (14), and (15) all converge to zero. Recall that the capacity of the infinite-bandwidth AWGN channel is given by

$$C = \frac{P}{N_0}. \quad (17)$$

We have not exactly shown that this capacity can be reached, but since we have made the assumption that  $T_d \ll T_c$ , we can come very close to it.

Observe that (11), (14), and (15) all remain constant if  $P/\theta$  and  $P/R$  are constant. Now  $P/R = E_b/\ln 2$ , so the error probability bound (10) will remain invariant for variations of the average power  $P$  as long as the peak power  $P/\theta$  and the energy per bit  $E_b$  are constant.

The parameter  $\epsilon$  may be freely chosen over its domain as it is a characteristic of the decision rule with no implication on physical quantities of interest. The parameters  $M$ ,  $R$ , and the error probability have clear physical interpretations, and the duty factor  $\theta$  is often restricted by a limitation on the peak power. Therefore, for a given  $M$  and  $R$ , we wish to find the smallest possible bound by optimizing over  $\epsilon$  and  $\theta$  (within its restricted domain).

Focusing on the tighter bound obtained under the additional Rayleigh fading assumption, we see that if we choose  $\epsilon$  optimally, we have

$$p_e \leq \min_{\epsilon > 0} \{p_{e,1}(M, R, \theta, \epsilon) + p_{e,2}(M, R, \theta, \epsilon)\}. \quad (18)$$

Note that, as functions of  $\epsilon$ ,  $p_{e,1}$  is strictly increasing whilst  $p_{e,2}$  is strictly decreasing. In addition,  $p_{e,1}(M, R, \theta, \epsilon) = p_{e,2}(M, R, \theta, \epsilon)$  when

$$\epsilon = \epsilon_0 \triangleq \frac{\theta N_0 + PT'_s}{PT'_s} \left[ 1 - \frac{RT_s N_0}{PT'_s} - \frac{\theta N_0}{PT'_s} \ln\left(1 + \frac{PT'_s}{\theta N_0}\right) \right], \quad (19)$$

which is in the interval  $(0, 1)$  if

$$0 \leq R < \frac{T'_s P}{T_s N_0} - \frac{\theta}{T_s} \ln\left(1 + \frac{PT'_s}{\theta N_0}\right). \quad (20)$$

Therefore, given that (20) is satisfied, we can upper bound (18) by

$$\begin{aligned} p_e &\leq 2 \min_{\epsilon > 0} \{\max(p_{e,1}(M, R, \theta, \epsilon), p_{e,2}(M, R, \theta, \epsilon))\} \\ &= 2p_{e,1}(M, R, \theta, \epsilon_0) \\ &= 2 \exp(-\ln M E_r(R, \theta)), \end{aligned} \quad (21)$$

where

$$\begin{aligned} E_r(R, \theta) &= \frac{\theta}{RT_s} \left\{ \frac{RT_s N_0}{PT'_s} + \frac{\theta N_0}{PT'_s} \log\left(1 + \frac{PT'_s}{\theta N_0}\right) - 1 \right. \\ &\quad \left. - \log\left(\frac{RT_s N_0}{PT'_s} + \frac{\theta N_0}{PT'_s} \log\left(1 + \frac{PT'_s}{\theta N_0}\right)\right) \right\}. \end{aligned} \quad (22)$$

It now only remains to maximize the exponent (22) for a given rate. For this, we resort to numerical analysis.

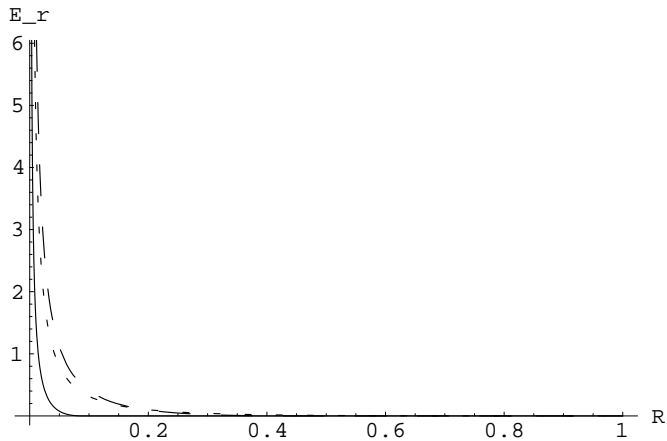


Fig. 1: Error exponent  $E_r(R, \theta)$  as a function of  $R$  for  $\theta = 10^{-2}$  (solid),  $\theta = 10^{-3}$  (dashed), and  $\theta = 10^{-4}$  (dotted).

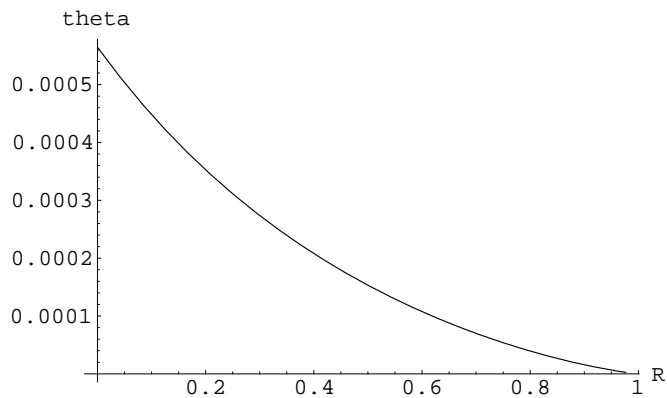


Fig. 2: Optimal duty factor  $\theta$  as a function of  $R$ .

### III. NUMERICAL ANALYSIS

We choose fading parameters that are typical for very-high frequency transmission in an indoor environment: Let  $T_d = 10^{-7}$  s and  $T_c = 2 \times 10^{-3}$  s. Let  $T_s = T_c$ . Suppose the peak power limitation is  $P/\theta \leq 250$ ; and let  $P = N_0 = 1$ , so  $C = 1$ .

We commence by looking at the behavior of the error exponent (22) for various values of the duty factor  $\theta$ , as shown in Figure 1. Note the rapid decay of the exponent. We therefore expect that the minimum required bandwidth increases very rapidly as the rate approaches capacity. It is also evident that smaller values of the duty factor are required to achieve higher rates, though the optimal  $\theta$  for a given rate is not immediately apparent. This optimization can be performed numerically and the result is shown in Figure 2. As expected, we see that the optimal duty factor gradually decreases to zero as capacity is approached. More surprising however is the fact that, even for very low rates, it is necessary that  $\theta \simeq 5 \times 10^{-4}$  for a maximal error exponent, which translates to a peak power that is approximately 2000 times larger than the average. Thus, recalling that the peak power limitation is  $P/\theta \leq 250$ , it follows that, for any rate, the duty factor is optimized over its restricted domain for  $\theta = 4 \times 10^{-3}$ .

We now turn to investigating the interplay amongst the physical parameters of interest. Figure 3 shows the band-

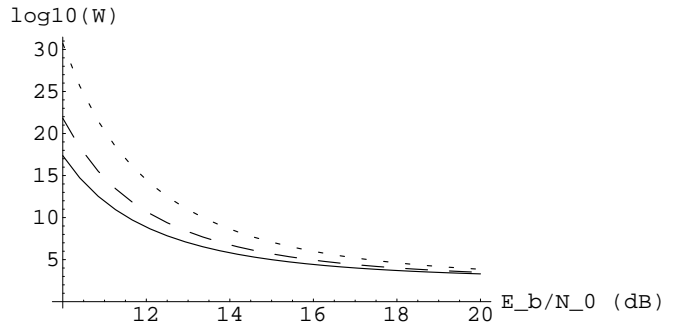


Fig. 3: Bandwidth required as a function of the SNR per bit for  $\theta = 4 \times 10^{-3}$  and error probability bounds of  $10^{-3}$  (solid),  $10^{-4}$  (dashed), and  $10^{-6}$  (dotted). The bandwidth  $W$  is in Hz.

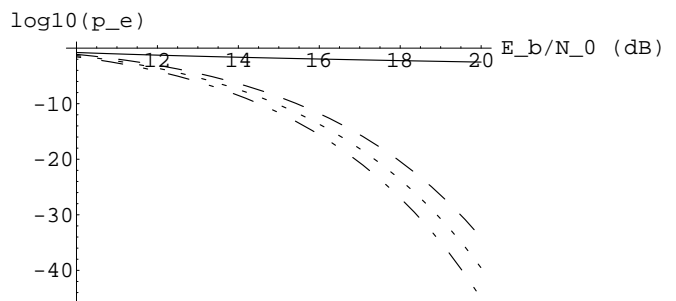


Fig. 4: Error probability upper bound as a function of the SNR per bit for the general bound with a bandwidth of 10 GHz (solid), and for the Rayleigh fading bound (21) with bandwidths of 1 GHz (dashed), 10 GHz (dotted), and 100 GHz (dot-dashed). The duty factor is  $\theta = 4 \times 10^{-3}$ .

width required (in Hz) as a function of the SNR per bit for error probability bounds of  $10^{-3}$ ,  $10^{-4}$  and  $10^{-6}$  using (21). Observe, that if we have a bandwidth of 10 GHz, then we need to transmit at an SNR per bit of around 12–14 dB to achieve error probabilities of such orders, and that the required bandwidth increases very rapidly as the SNR per bit decreases. To facilitate comparison, note that SNRs per bit increasing from 10 dB to 20 dB correspond to rates decreasing from approximately 0.069 nats/s to 0.0069 nats/s.

The relationship between bandwidth and SNR per bit can instead be examined using the more general bound that does not assume Rayleigh fading (obtained by combining (10), (11), and (14) and numerically optimizing over  $\epsilon$ ), but it is very much looser. Figure 4 gives us a notion of exactly how much looser it is. The plot shows, amongst other things, both error probability bounds as a function of SNR per bit for a bandwidth of 10 GHz. Notice how slowly the general bound decays as a function of the SNR per bit. Thus we continue to focus on using (21) for the bound. In this case, we see that the error probability bound is quite large for  $E_b/N_0 \lesssim 11$  dB, but that it decays rather quickly as  $E_b/N_0$  increases. The plot also shows this bound for bandwidths of 1 GHz and 100 GHz to illustrate how much it varies with bandwidth.

Finally, it is interesting to examine the variation of  $\theta$  as a function of the SNR per bit (see Figure 5). This tells us how peaky the signal needs to be to achieve a particular proba-

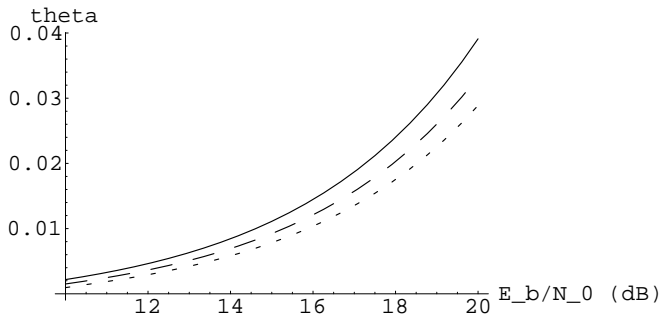


Fig. 5: Duty factor  $\theta$  as a function of the SNR per bit for a bandwidth of 10 GHz and error probability bounds of  $10^{-3}$  (solid),  $10^{-4}$  (dashed), and  $10^{-5}$  (dotted).

bility of error for a given SNR per bit and bandwidth, and therefore the peak power required. For example, for an SNR per bit around 10 dB and a bandwidth of 10 GHz, we need  $\theta \approx 10^{-3}$  to ensure an error probability less than  $10^{-5}$ , which implies that for an average power constraint of 1 mW, the instantaneous power needs to reach up to around 1 W.

#### IV. CONCLUSION

In this paper, we have calculated explicit upper bounds on the probability of error of a capacity-achieving scheme for the infinite-bandwidth multipath fading channel. These bounds can be made to decay to zero as the bandwidth goes to infinity for rates below the capacity, thus yielding a strong coding theorem for the infinite-bandwidth channel. We have calculated a loose, general bound and a tighter bound under the additional assumption of Rayleigh fading. These bounds give us a notion of how quickly the error probability decays to zero as the bandwidth approaches infinity and of the importance of the various parameters relevant to determining this rate of decay. We have investigated the interaction amongst the probability of error upper bound, the bandwidth, the SNR per bit, and the peakiness of the signaling scheme for some specific numerical cases.

Since transmission takes place at a low duty cycle, the capacity-achieving scheme can be straightforwardly extended to multiple users who are multiplexed by time-division. If these users are co-operating, then it is clear that  $[1/\theta]$  non-interfering users can be supported for a given value of  $\theta$ . If they are not co-operating, then we can incorporate a term due to interference from other users into our existing expressions for the upper bound on the probability of error. Studying the behavior of the error probability bounds in the multiple access scenario is a definite avenue for future investigations. It may also be fruitful to explore a strong converse to the coding theorem.

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