Fiber Aided Wireless Network Architecture: A SISO wireless-optical channel*

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Abstract

We introduce the concept of a fiber aided wireless network architecture (FAWNA), which allows high-speed mobile connectivity by leveraging the speed of optical networks. As a first step towards designing such network architectures, we consider a single-input, single-output (SISO) wireless-optical communication link. This link consists of a wireless channel and a fiber optic channel, connected to each other by a wireless-optical interface. We propose a scheme where the received signal at the wireless-optical interface is quantized before being sent over the fiber. The achievable rate for this scheme approaches the SISO capacity exponentially with fiber capacity. We show that for fixed fiber capacity, there is an optimal wireless bandwidth of operation when our scheme is used. The wireless-optical interface has low complexity and does not require knowledge of the transmitter code book. Moreover, the loss in "soft" information, due to quantization, goes to 0 asymptotically with increase in fiber capacity. These properties make our scheme extendable to FAWNAs with large number of transmitters, radio-optic converters, variable rates, changing channel conditions and node positions.

1 Introduction

There is a considerable demand for increasingly high-speed communication networks with mobile connectivity. Traditionally, high-speed communication has been efficiently provided through wireline infrastructure, particularly based on optical fiber, where bandwidth is plentiful and inexpensive. However, such infrastructure does not support mobility. Instead, mobile communication is provided by wireless infrastructure, most typically over the radio spectrum. However, limited available spectrum and interference effects limit mobile communication to lower data rates.

We introduce the concept of a fiber aided wireless network architecture (FAWNA), which allows high-speed mobile connectivity by leveraging the speed of optical networks. In the proposed architecture, the network coverage area is divided into zones such that an optical fiber "bus" passes through each zone. Connected to the end of the fiber is a bus controller/processor, which coordinates use of the fiber as well as connectivity to the outside world. Along the fiber are radio-optical converters, which are access points consisting of simple antennas directly connected to the fiber. Each of these antennas harvest the energy from the wireless domain to acquire the full radio bandwidth in their local environment and place the associated waveform onto a subchannel of the fiber. Within the fiber, the harvested signals can be manipulated by the bus controller/processor and made available to all other antennas. In each zone, there may be one or more active wireless nodes. Wireless nodes communicate between one another, or to the outside world, by communicating to a nearby antenna. Thus any node in the network is at most two hops away from any other node, regardless of the size of the network. In general, each zone is generally covered by several antennas, and there may also be wired nodes connected directly to the fiber. This architecture has the potential to reduce dramatically the interference effects that limit scalability and the energy-consumption characteristics that limit battery life, in pure wireless infrastructure. In particular, the architecture makes use of the fact that areas with high densities of users, such as urban areas, or indoor business and educational settings, generally have both the most severe interference problems, and the most dense wireline infrastructure. Moreover, while wireless channels exhibit significant congestion, generally in the form of interference, in areas with a high density of users, the optical fiber infrastructure

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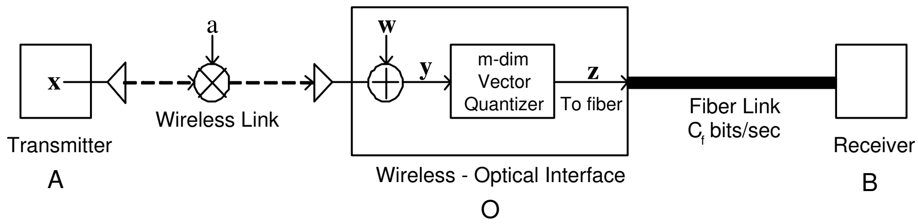

Figure 1: Point-to-point communication over a wireless-optical link.

typically has significant over provisioning, with abundance of fiber that is not lit or only very partially used. FAWNA thus uses the wireline infrastructure to provide a distributed means of aggressively harvesting energy from the wireless medium in areas where there is a rich, highly vascularized wireline infrastructure and distributing in an effective manner energy to the wireless domain by making use of the proximity of transmitters to reduce interference.

As a first step towards designing such network architectures, we consider a single-input, single-output (SISO) wireless-optical communication link. Figure 1 shows such a link between two points A and B. The various quantities in the figure will be described in detail in the next section. Optimal relay design for the serial relay channel where, both links across the interface are AWGN links, has been considered in [13, 14]. In this paper, we consider two hop communication, where, the first hop is over a wireless channel and the second, over a fiber optic channel. The links we consider are ones where the fiber optic channel capacity is larger than the wireless channel capacity.

The transmitter at A transmits information to an intermediate wireless-optical interface (radio-optical converter) at O over a wireless channel. The wireless-optical interface then relays this information to the destination, B, over a fiber optic channel. The end-to-end design is done to maximize the transmission rate between A and B. Since, in general, a FAWNA has a large number of radio-optical converters, an important design objective is to keep the wireless-optical interface as simple as possible without sacrificing too much in performance.

Our problem has a similar setup, but a different objective than the CEO problem [10, 11, 12] with a single agent. In the CEO problem, the rate-distortion tradeoff is analyzed for a given source that needs to be conveyed to the CEO. Rate-distortion theory, which uses infinite dimensional vector quantization, is used to analyze the problem. We analyze the maximum rate at which reliable communication is possible between the transmitter and receiver when, decoding is not performed at the wireless-optical interface. The interface needs to be practically implementable and has to have low complexity, extendability to FAWNAs with large number of transmitters and radio-optic converters and, adaptability to variable rates, changing channel conditions and node positions. We use finite-dimensional quantizers at the interface and use high-resolution quantizer theory for analysis.

Let us denote the capacities of the wireless and optical channels as $C_w(P,W)$ and C_f bits/sec, respectively, where, P is the average transmit power at A and W is the wireless transmission bandwidth. Since, as stated earlier, we consider links where $C_w(P,W) \le C_f$, [7] shows that the capacity of the two hop SISO link, $C_{\mathsf{SISO}}(P,W)$, is

$$C_{\mathsf{SISO}}(P,W) = \min\left\{ C_w(P,W), C_f \right\} = C_w(P,W) \ \text{ bits/sec.} \tag{1}$$

One way of achieving this capacity is to decode and re-encode at the wireless-optical interface. However, decoding results in the wireless-optical interface having high complexity and the interface requires knowledge of the transmitter code book. A major drawback of the decode/re-encode scheme is significant loss in optimality as we go to FAWNAs with multiple transmitters and radio-optical converters. This happens because "soft" information in the wireless signal is completely lost by decoding at the wireless-optical interface. Hence, multiple antenna gain is not possible with this scheme.

In this paper, we propose a scheme in which the wireless signal at the radio-optical converter is sampled and quantized using a fixed-rate, memoryless, vector quantizer, before being sent over the fiber. Hence, the wireless-optical interface uses a forwarding scheme. Since, transmission of continuous values over the fiber is practically not possible using commercial lasers, quantization is necessary for the implementation of a forwarding scheme.

We show that the capacity using this quantization scheme approaches the SISO capacity, $C_{\mathsf{SISO}}(\mathsf{SNR})$, exponentially with fiber capacity. The scheme is thus near-optimal with respect to decoding, since, the fiber capacity is large. For fixed fiber capacity, there is an optimal wireless bandwidth of operation when

our scheme is used. We show that low dimensional (or even scalar) quantization can be performed at the interface without significant loss in performance. We compute an upper bound to this loss as 4.35 dB. Not only does this result in low complexity, but also smaller (or no) buffers are required, thereby further simplifying the radio-optical interface. Moreover, Hui and Neuhoff [9] show that asymptotically optimal quantization can be implemented with complexity increasing at most polynomially with the rate. Knowledge of the transmitter code book is not required at the wireless-optical interface. In our scheme, the loss in "soft" information, due to quantization of the wireless signal, goes to 0 asymptotically with increase in fiber capacity. These properties make our scheme extendable to FAWNAs with large number of transmitters and radio-optic converters. Since the wireless-optical interfaces do not require knowledge of the transmitter code book, our scheme offers easy adaptability to variable rates, changing channel conditions and node positions.
The paper is organized as follows: In section 2, we describe our wireless and fiber channel models. We introduce our scheme in section 3 and analyze its performance in section 4. We conclude in section 5. Unless specified otherwise, all logarithms in this paper are to the base 2.

2 Channel models

Wireless Channel: We use a linear model for the wireless channel between A and the wireless-optical interface:

$$\mathbf{y} = a\mathbf{x} + \mathbf{w}, \tag{2}$$

where, $\mathbf{x}, \mathbf{w}, \mathbf{y} \in \mathcal{C}$ are the channel input, noise and output, respectively. The channel gain (state), $a \in \mathcal{C}$, is deterministic. The noise, $\mathbf{w}$, is a zero mean circularly symmetric complex Gaussian random variable, $\mathbf{w} \sim \mathcal{CN}(0, N_0)$, and is independent of the channel input. $N_0/2$ is the double-sided white noise spectral density. The channel input, $\mathbf{x}$, satisfies the average power constraint $E[|\mathbf{x}|^2] = P/W$, where, P and W are the average transmit power at A and wireless bandwidth, respectively. Hence, the wireless channel capacity is

$$C_w(P, W) = W \log\left(1 + \frac{|a|^2 P}{N_0 W}\right) \text{ bits/sec}, \tag{3}$$

and W symbols/sec are transmitted over the wireless channel.

Fiber Optic Channel: The fiber optic channel between the wireless-optical interface and the receiver, B, can reliably support a rate of C_f bits/sec. Fiber channel coding is performed at the wireless-optical interface in order to achieve this. Note that the code required for the fiber is a very low complexity one. An example of a code that may be used is the 8B10B code, which is commonly used in Ethernet. Hence, fiber channel coding does not significant increase the complexity at the wireless-optical interface.

3 Proposed Scheme

The input to the wireless channel, $\mathbf{x}$, is a zero mean circularly symmetric complex Gaussian random variable, $\mathbf{x} \sim \mathcal{CN}(0, P/W)$. Note that it is this input distribution that achieves the capacity of our wireless channel model. The output from the antenna at the wireless-optical interface is first converted from passband to baseband and then sampled at the Nyquist rate of W complex samples/sec. The random variable, $\mathbf{y}$, represents the output from the sampler. Fixed-rate, memoryless, m-dimensional vector quantization is performed on these samples at a rate of l bits/complex sample. The quantized complex samples are subsequently sent over the fiber after fiber channel coding and modulation.
The fiber is thus required to reliably support a rate of Wl bits/sec[1]. Hence, we get the following constraint on l:

$$l \leq \frac{C_f}{W}. \tag{4}$$

The quantizer noise, $\mathbf{q}_{m,l}$, is modelled as being additive. Thus, the two-hop channel between A and B is modelled as:

$$\mathbf{z} = a\mathbf{x} + \mathbf{w} + \mathbf{q}_{m,l}.$$

[1]The samples that are being quantized are correlated across time and the correlation depends on the dimension of the vector being quantized, m. If source coding is performed on the quantized samples, the rate requirement on the fiber is lowered for fixed distortion. However, this comes at a cost of increased complexity at the wireless-optical interface.

The quantizer used is an optimal fixed rate, memoryless m-dimensional, high resolution vector quantizer and hence, its distortion-rate function is given by the Zador-Gersho function [1, 3, 5]:

$$E[|\mathbf{q}_{m,l}|^2] = E[|\mathbf{y}|^2] M_m \beta_m 2^{-l} = \left(N_0 + \frac{|a|^2 P}{W}\right) M_m \beta_m 2^{-l}. \quad (5)$$

M_m is Gersho's constant, which is independent of the distribution of $\mathbf{y}$. β_m is the Zador's factor that depends on the distribution of $\mathbf{y}$. Since the fiber channel capacity is large, the assumption that the quantizer is a high resolution one, is valid. Note that, since this quantizer is an optimal fixed rate, memoryless vector quantizer, references [2, 3, 4, 6, 8] show that it has the following properties:

$$\begin{aligned} E[\mathbf{q}_{m,l}] &= 0, & (6)\\ E[\mathbf{z}\mathbf{q}^*_{m,l}] &= 0, & (7)\\ E[\mathbf{y}\mathbf{q}^*_{m,l}] &= -E[|\mathbf{q}_{m,l}|^2]. & (8) \end{aligned}$$

From (6, 7, 8), we obtain

$$E[|\mathbf{z}|^2] = E[|\mathbf{y}|^2] - E[|\mathbf{q}_{m,l}|^2]. \quad (9)$$

Let the capacity for this scheme (in bits/sec) be denoted as $C_{\mathsf{Q}}(P, W, m, l)$, where, $m \in \{1, 2, \ldots\}$ and $l \leq \frac{C_f}{W}$. We establish the following theorem:

Theorem 1

$$C_{\mathsf{SISO}}(P, W) - \Phi(P, W, m, l) \leq C_{\mathsf{Q}}(P, W, m, l) \leq C_{\mathsf{SISO}}(P, W),$$

where,

$$\Phi(P, W, m, l) = W \log\left(1 + \frac{|a|^2 P M_m \beta_m 2^{-l}}{N_0 W}\right).$$

Proof: The upper bound follows trivially from the fact that the rate achieved using the proposed scheme cannot exceed the capacity of the two hop wireless-optical channel, $C_{\mathsf{SISO}}(P, W)$.
We now consider the lower bound. Let $\hat{\mathbf{x}}_{llse}(\mathbf{z})$ be the linear least-squares error (LLSE) estimate of $\mathbf{x}$ from $\mathbf{z}$ and $\mathbf{e}_{llse}$, the estimation error. Thus

$$\mathbf{x} = \hat{\mathbf{x}}_{llse}(\mathbf{z}) + \mathbf{e}_{llse}.$$

We denote the variance of the estimation error as $\lambda_{llse} = E[|\mathbf{e}_{llse}|^2]$. Now

$$\begin{aligned} &\frac{1}{W} C_{\mathsf{Q}}(P, W, m, l) \\ &= I(\mathbf{x}; \mathbf{z}) \\ &= h(\mathbf{x}) - h(\mathbf{x}|\mathbf{z}) \\ &= h(\mathbf{x}) - h(\mathbf{e}_{llse}|\mathbf{z}) \\ &\geq h(\mathbf{x}) - h(\mathbf{e}_{llse}) & (10)\\ &\geq h(\mathbf{x}) - h(\mathbf{e}^G_{llse}) & (11)\\ &= \log\left(\frac{P}{\lambda_{llse} W}\right) \\ &= \log\left(\frac{1}{1 - \rho^2_{\mathbf{x},\mathbf{z}}}\right). & (12) \end{aligned}$$

Since conditioning reduces entropy, the inequality in (10) follows. In (11), we replace $\mathbf{e}_{llse}$ by another random variable, $\mathbf{e}^G_{llse} \sim \mathcal{CN}(0, \lambda_{llse})$. Since the Gaussian distribution maximizes entropy for fixed variance, $h(\mathbf{e}_{llse}) \leq h(\mathbf{e}^G_{llse})$, and the inequality in (11) follows. Equation (12) follows since

$$\lambda_{llse} = E[|\mathbf{x}|^2] - \frac{E[\mathbf{x}\mathbf{z}^*] E[\mathbf{x}^*\mathbf{z}]}{E[|\mathbf{z}|^2]} = \frac{P}{W} - \frac{P \rho^2_{\mathbf{x},\mathbf{z}}}{W} = \frac{P(1 - \rho^2_{\mathbf{x},\mathbf{z}})}{W}.$$

We now compute $\rho_{\mathbf{x},\mathbf{z}}$. From our channel and quantizer models, we have the following Markov chain: $(\mathbf{x},\mathbf{w}) \leftrightarrow \mathbf{y} \leftrightarrow \mathbf{q}_{m,l}$. Using this Markov chain, we obtain the following relation:

$$\begin{aligned}
&E[\mathbf{x}\mathbf{q}^*_{m,l}] \\
&\quad = E_{\mathbf{y}}\left[E[\mathbf{x}\mathbf{q}^*_{m,l}|\mathbf{y}]\right] \\
&\quad = E_{\mathbf{y}}\left[E[\mathbf{x}|\mathbf{y}]E[\mathbf{q}^*_{m,l}|\mathbf{y}]\right] \\
&\quad = E_{\mathbf{y}}\left[\frac{E[\mathbf{x}\mathbf{y}^*]}{E[|\mathbf{y}|^2]}\mathbf{y}E[\mathbf{q}^*_{m,l}|\mathbf{y}]\right] \qquad (13)\\
&\quad = \frac{E[\mathbf{x}\mathbf{y}^*]E[\mathbf{y}\mathbf{q}^*_{m,l}]}{E[|\mathbf{y}|^2]} \\
&\quad = -\frac{a^* P M_m \beta_m 2^{-l}}{W}. \qquad (14)
\end{aligned}$$

Since $\mathbf{x}$ and $\mathbf{y}$ are jointly Gaussian random variables, we obtain (13). Equation (14) follows from the quantizer properties (5, 8), and our wireless channel model (2). Now

$$\begin{aligned}
&\rho^2_{\mathbf{x},\mathbf{z}} \\
&\quad = \frac{|E[\mathbf{x}\mathbf{z}^*]|^2}{E[|\mathbf{x}|^2]E[|\mathbf{z}|^2]} \\
&\quad = \frac{|E[\mathbf{x}\mathbf{y}^*] + E[\mathbf{x}\mathbf{q}^*]|^2}{E[|\mathbf{x}|^2]E[|\mathbf{z}|^2]} \\
&\quad = \frac{|a|^2\frac{P}{W}(1 - M_m\beta_m 2^{-l})^2}{E[|\mathbf{z}|^2]} \qquad (15)\\
&\quad = \frac{|a|^2\frac{P}{W}(1 - M_m\beta_m 2^{-l})}{N_0 + |a|^2\frac{P}{W}}. \qquad (16)
\end{aligned}$$

We obtain (15) using (14) and, (16) using (5, 9). Combining (12, 16), we obtain

$$\begin{aligned}
&C_{\mathsf{Q}}(P, W, m, l) \\
&\quad \geq W\log(1 + \frac{|a|^2 P}{N_0 W}) - \Phi(P, W, m, l) \\
&\quad = C_{\mathsf{SISO}}(P, W) - \Phi(P, W, m, l), \qquad (17)
\end{aligned}$$

where, $\Phi(P, W, m, l) = W\log\left(1 + \frac{|a|^2 P M_m \beta_m 2^{-l}}{N_0 W}\right)$ is the loss in capacity due to quantization noise. We use (1, 3) for (17). This completes the proof of the theorem. □

4 Performance Analysis

In this section, we analyze the performance of the scheme proposed in Section 3. We consider how the loss in capacity due to quantization noise, $\Phi(P, W, m, l)$, varies with the quantization dimension, m, and quantization rate, l. We also study the dependence of the capacity lower bound of Theorem 1, on transmit power and wireless bandwidth. Since, high resolution quantization is done at the wireless-optical interface and our link model is one where the fiber channel capacity is always greater than the wireless channel capacity, the following constraints must be met:

$$0 \leq W < C_f, \qquad (18)$$

$$W\log(1 + \frac{|a|^2 P}{N_0 W}) \leq C_f. \qquad (19)$$

4.1 Effect of quantization dimension

We first study the effect of quantization dimension on the performance of our scheme. We prove the following lemma:

Lemma 1

$$\Phi(P, W, \infty, l) \leq \Phi(P, W, m, l) \leq \Phi(P, W, 1, l),$$

where, $\Phi(P, W, 1, l)$ and $\Phi(P, W, \infty, l)$ correspond to loss in capacity due to fixed rate, scalar and infinite dimensional vector quantization, respectively, and

$$\Phi(P, W, 1, l) = W \log \left(1 + \frac{\pi\sqrt{3}|a|^2 P 2^{-l}}{2N_0 W}\right), \quad \Phi(P, W, \infty, l) = W \log \left(1 + \frac{|a|^2 P 2^{-l}}{N_0 W}\right).$$

Proof: Since Gaussian signaling is used for the wireless channel, the input to the quantizer is a correlated Gaussian random vector. Zador's factor and Gersho's constant obey the following property:

$$M_\infty \beta_\infty \leq M_m \beta_m \leq M_1 \beta_1 \leq M_1 \beta_1^G,$$

where, β_1^G is the Zador's factor for an i.i.d Gaussian source and, $\beta_1 \leq \beta_1^G$. Since, $M_1 = \frac{1}{12}$, $M_\infty = \frac{1}{2\pi e}$, $\beta_1^G = 6\sqrt{3}\pi$ and $\beta_\infty = 2\pi e$, [2] we obtain

$$1 \leq M_m \beta_m \leq \frac{\pi\sqrt{3}}{2}. \tag{20}$$

The lower bound corresponds to fixed rate, infinite dimensional vector quantization, whereas, the upper bound corresponds to fixed rate, scalar quantization. Equation (20), and the fact that $\Phi(P, W, m, l)$ is a concave (logarithmic) function of $M_m \beta_m$, completes the proof. □

As we reduce the quantizer dimension, we reduce the quantization complexity at the wireless-optical interface. However, we pay in terms of capacity. This lemma shows that this loss in performance is upper bounded by $10 \log(\frac{\pi\sqrt{3}}{2}) \sim 4.35$ dB. The maximum loss occurs when a fixed rate, scalar quantizer is used.

4.2 Effect of quantizer rate

We now study the effect of quantizer rate on $\Phi(P, W, m, l)$. We prove the following lemma:

Lemma 2

$$\Phi(P, W, m, l) = \Theta(2^{-l}).$$

Proof:

$$\begin{aligned}
&\Phi(P, W, m, l) \\
&\quad = W \log \left(1 + \frac{|a|^2 P M_m \beta_m 2^{-l}}{N_0 W}\right) \\
&\quad \leq |a|^2 \frac{P}{N_0} M_m \beta_m 2^{-l} \log(e) = O(2^{-l}).
\end{aligned} \tag{21}$$

The inequality $\log(1 + v) \leq v \log(e)$ results in (21). Hence,

$$\Phi(P, W, m, l) = O(2^{-l}). \tag{22}$$

We also have

$$\begin{aligned}
&\Phi(P, W, m, l) \\
&\quad = W \log \left(1 + \frac{|a|^2 P M_m \beta_m 2^{-l}}{N_0 W}\right) \\
&\quad \geq |a|^2 \frac{P}{N_0} M_m \beta_m 2^{-l} \log(e) - \frac{|a|^4 P^2 M_m^2 \beta_m^2 2^{-2l} \log(e)}{2N_0^2 W} = \Omega(2^{-l}).
\end{aligned} \tag{23}$$

We use the inequality $\log(1 + v) \geq v \log(e) - v^2 \log(e)/2$, to obtain (23). Hence,

$$\Phi(P, W, m, l) = \Omega(2^{-l}). \tag{24}$$

[2] Our quantizer model does not make use of the correlation between its input symbols. If the quantizer does so, β_∞ can be further lowered, giving a lower distortion for fixed rate.

Combining (22) and (24), completes the proof. □

This lemma shows that the loss in capacity, $\Phi(P,W,m,l)$, decreases exponentially with the quantizer rate. Hence, the capacity using the proposed scheme approaches the SISO capacity exponentially with quantizer rate. Note that

$$\lim_{l\to\infty} \Phi(P,W,m,l) = 0.$$

This happens since the quantization loss goes to 0 asymptotically with quantizer rate. However, from (4), we see that the maximum value that l takes, depends on the fiber capacity and the wireless bandwidth. Since, $\Phi(P,W,m,l)$ decreases exponentially with l, it is minimum when $l = C_f/W$. Thus, we have the following lemma:

Lemma 3

$$\min_{l\leq\frac{C_f}{W}} \Phi(P,W,m,l) = \Phi(P,W,m,C_f) = W\log\left(1+\frac{|a|^2 P M_m\beta_m 2^{-\frac{C_f}{W}}}{N_0 W}\right).$$

The lemma shows that the capacity loss due to quantization decreases exponentially with fiber capacity. Hence, with our scheme, the rate at which reliable communication is possible approaches the SISO capacity exponentially with fiber capacity. The scheme is thus near-optimal with respect to decoding at the wireless-optical interface, since, the fiber capacity is large.
Let us define

$$C_{\mathsf{LB}}(P,W,m,C_f) \triangleq C_{\mathsf{SISO}}(P,W) - \Phi(P,W,m,C_f).$$

$C_{\mathsf{LB}}(P,W,m,C_f)$ is a lower bound to the maximum achievable rate (and SISO capacity) when quantization is done at the wireless-optical interface. From Lemma 3, we have

$$C_{\mathsf{LB}}(P,W,m,C_f) = W\log\left(1+\frac{|a|^2P}{N_0W}\right) - W\log\left(1+\frac{|a|^2 P M_m\beta_m 2^{-\frac{C_f}{W}}}{N_0 W}\right).$$

We have already seen that $C_{\mathsf{LB}}(P,W,m,C_f)$ tends to $C_{\mathsf{SISO}}(P,W)$, exponentially with fiber capacity (Lemma 2). Figure 2 is a plot of $C_{\mathsf{LB}}(P,W,m,C_f)$ versus fiber capacity, which illustrates this behavior. In the plot, we set $W = 10$ Mhz, $M_m\beta_m = 1$ and $\frac{|a|^2P}{N_0} = 25\times 10^6$ sec^{-1}.

4.3 Effect of transmit power on capacity lower bound

We analyze the dependence of $C_{\mathsf{LB}}(P,W,m,C_f)$ on transmit power, P. Examining the first derivative with respect to P:

$$\frac{\partial C_{\mathsf{LB}}(P,W,m,C_f)}{\partial P} = \frac{|a|^2 N_0(1-M_m\beta_m 2^{-\frac{C_f}{W}})\log(e)W^2}{(N_0W+|a|^2P)(N_0W+|a|^2PM_m\beta_m 2^{-\frac{C_f}{W}})} > 0,$$

where, the inequality follows since high resolution quantization is done at the wireless-optical interface, i.e., $1 \ll C_f/W$. Thus, $C_{\mathsf{LB}}(P,W,m,C_f)$ *increases monotonically* with P.
Equation (19) can be expressed as a constraint on the transmit power:

$$P \leq P^* \triangleq \frac{N_0W}{|a|^2}\left(2^{\frac{C_f}{W}}-1\right). \tag{25}$$

If the SISO link is operated at a transmit power greater than P^*, the wireless channel capacity will exceed the fiber capacity and $C_{\mathsf{SISO}}(P,W) = C_f$.[3] Hence, the optical channel will be limiting the SISO link capacity and increasing the transmit power beyond P^* does not affect SISO capacity. In this paper, we do not consider the power regime, $P > P^*$. Hence, maximizing the capacity lower bound for $P\in[0,P^*]$:

$$\max_{P\in[0,P^*]} C_{\mathsf{LB}}(P,W,m,C_f) = C_{\mathsf{LB}}(P^*,W,m,C_f) = C_f - W\log\left(1+M_m\beta_m(1-2^{-\frac{C_f}{W}})\right).$$

Figure 3 is a plot of $C_{\mathsf{SISO}}(P,W)$ and $C_{\mathsf{LB}}(P,W,m,C_f)$ versus $\frac{|a|^2P}{N_0}$. In the plot, we refer to $\frac{|a|^2P}{N_0}$ as the "Scaled Transmit Power". We set $W = 5$ Mhz, $C_f = 50$ Mbps and $M_m\beta_m = 1$.

[3] In general, for fixed fiber capacity, this is the best wireless performance a FAWNA can offer.

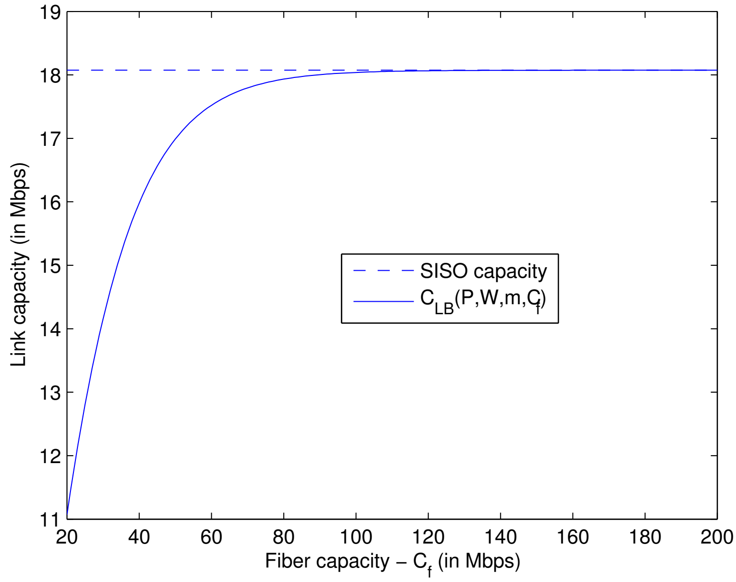

Figure 2: Dependence of SISO link capacity on fiber capacity.

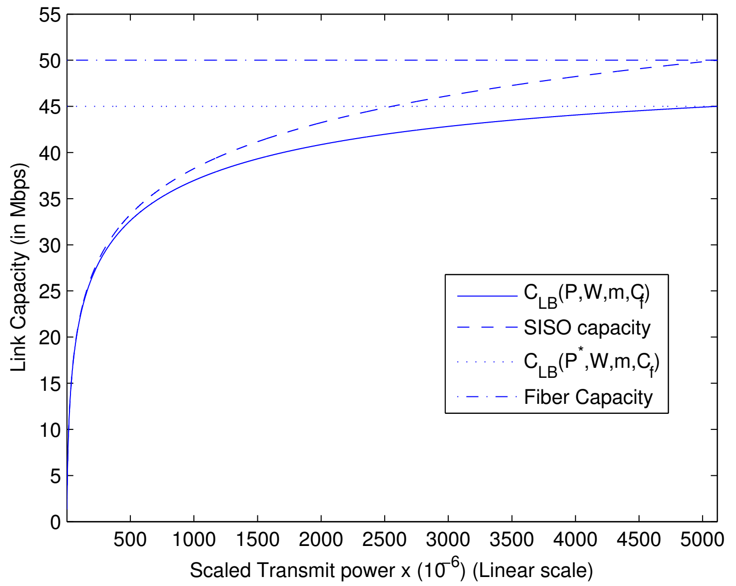

Figure 3: Dependence of SISO link capacity on transmit power.

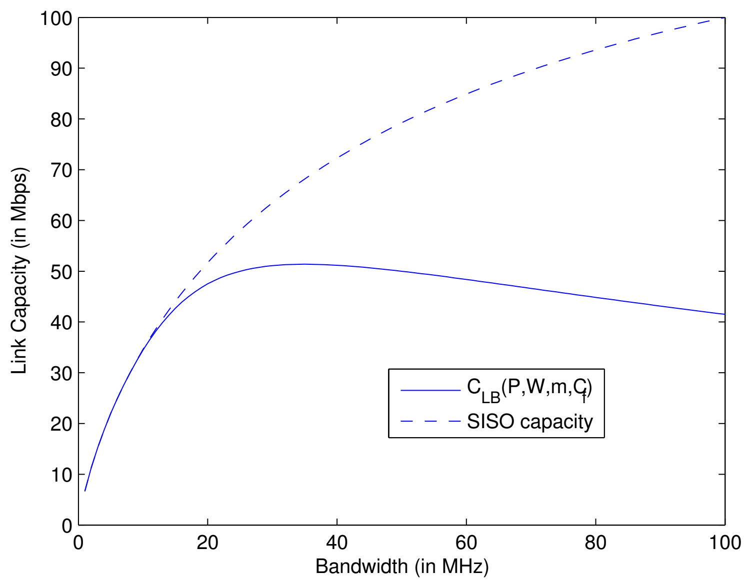

Figure 4: Dependence of SISO link capacity on bandwidth.

4.4 Effect of bandwidth on capacity lower bound

We now analyze how the capacity lower bound, $C_{\mathsf{LB}}(P, W, m, C_f)$, varies with the wireless bandwidth, W. Considering the derivative with respect to W:

$$\begin{aligned} & \frac{\partial C_{\mathsf{LB}}(P,W,m,C_f)}{\partial W} \\ & = \log\left[\frac{1+\frac{|a|^2P}{N_0W}}{1+\frac{|a|^2PM_m\beta_m 2^{-\frac{c_f}{W}}}{N_0W}}\right] - \frac{|a|^2P\log(e)}{N_0W}.\frac{1+M_m\beta_m 2^{-\frac{c_f}{W}}\left(\frac{|a|^2PC_f\log_e(2)}{N_0W^2}+\frac{C_f\log_e(2)}{W}-1\right)}{\left(1+\frac{|a|^2P}{N_0W}\right)\left(1+\frac{|a|^2PM_m\beta_m 2^{-\frac{c_f}{W}}}{N_0W}\right)}. \end{aligned} \tag{26}$$

Setting $\frac{\partial C_{\mathsf{LB}}(P,W,m,C_f)}{\partial W} = 0$, we obtain the critical point of the function. This point is the maximum of $C_{\mathsf{LB}}(P, W, m, C_f)$, since, the second derivative is negative at this point (we omit the steps here for brevity). Hence, the optimal bandwidth of operation is:

$$W^* = \arg \max_{W\in[0,C_f)} C_{\mathsf{LB}}(P,W,m,C_f).$$

When our scheme is used, for fixed fiber capacity, quantizer distortion as well as wireless capacity (power efficiency) increases with wireless bandwidth. The quantizer distortion increases since, the quantization rate decreases inversely with bandwidth. The two effects compete and yield the optimal bandwidth. When the operating bandwidth is lowered from W^*, $C_{\mathsf{LB}}(P, W, m, C_f)$ is lowered since, the reduction in power efficiency is more than the reduction in quantizer distortion. On the other hand, when the bandwidth is increased beyond W^*, $C_{\mathsf{LB}}(P, W, m, C_f)$ is lowered since, the increase in quantizer distortion is more than the increase in power efficiency. Obtaining an analytical solution for W^* from (26), is difficult. However, the optimal bandwidth can easily be found by numerical techniques. Figure 4 shows the plot of the SISO capacity, $C_{\mathsf{SISO}}(P, W)$, and the capacity lower bound, $C_{\mathsf{LB}}(P, W, m, C_f)$, versus bandwidth for $\frac{|a|^2P}{N_0} = 10^8$ sec^{-1} and $C_f = 100$ Mbps. The optimum bandwidth for this case is numerically found out to be $W^* \sim 35$ MHz. The value of the capacity lower bound at this bandwidth is $C_{\mathsf{LB}}(P, W^*, m, C_f) \sim 51.38$ Mbps.

5 Conclusion

In this paper, we introduce the concept of a fiber aided wireless network architecture, which allows high-speed mobile connectivity by leveraging the speed of optical networks. As a first step towards designing such network architectures, we consider a single-input, single-output (SISO) wireless-optical communication link. We propose a scheme in which the wireless signal at the radio-optical converter is sampled and quantized using a fixed-rate, memoryless, vector quantizer, before being sent over the fiber. The rate at which reliable communication is possible with this scheme approaches the SISO capacity exponentially with fiber capacity. The scheme is thus near-optimal, since, the fiber capacity is large. For fixed fiber capacity, there is an optimal wireless bandwidth of operation when our scheme is used. The wireless-optical interface has low complexity and does not require knowledge of the transmitter code book. In our scheme, the loss in "soft" information, due to quantization of the wireless signal, goes to 0 asymptotically with increase in fiber capacity. These properties make our scheme extendable to FAWNAs with large number of transmitters, radio-optic converters, variable rates, changing channel conditions and node positions.

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